TAIWANESE JOURNAL OF MATHEMATICS
Vol. 11, No. 4, pp. 1063-1074, September 2007
This paper is available online at http://www.math.nthu.edu.tw/tjm/

ON CENTRALIZERS OF SEMISIMPLE H^*-ALGEBRAS

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Abstract. In this paper we prove the following result. Let A be a semisimple H^*-algebra and let $T : A \to A$ be an additive mapping satisfying the relation $2T(x^{m+n+1}) = x^m T(x)x^n + x^n T(x)x^m$, for all $x \in A$ and some nonnegative integers m, n such that $m+n \neq 0$. In this case T is a left and a right centralizer.

1. INTRODUCTION

Throughout, R will represent an associative ring with center $Z(R)$. Given an integer $n \geq 2$, a ring R is said to be $n-$torsion free, if for $x \in R$, $nx = 0$ implies $x = 0$. As usual the commutator $xy - yx$ will be denoted by $[x, y]$. Recall that a ring R is prime if for $a, b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$, and is semiprime in case $aRa = (0)$ implies $a = 0$. An additive mapping $T : R \to R$ is called a left (right) centralizer in case $T(xy) = T(x)y$ ($T(xy) = xT(y)$) holds for all pairs $x, y \in R$. One of the initial papers using the concept of centralizers (also called multipliers) is due to Wendel $[32]$ for group algebras. Helgason $[6]$ introduced centralizers for Banach algebras. Wang $[31]$ studied centralizers of commutative Banach algebras. Johnson $[8]$ introduced the concept of centralizers for rings. We refer to Busby $[4]$ for a study of so-called double centralizers in the extension of C^*-algebras. Akemann, Pedersen and Tomiyama $[1]$ have studied centralizers of C^*-algebras. Several authors have also studied spectral properties of centralizers on Banach algebras (see $[13, 14]$). Johnson $[9]$ has studied centralizers on some topological algebras. Johnson $[10]$ has studied the continuity of centralizers on Banach algebras (see also $[11]$). Husain $[7]$ has also investigated centralizers on topological algebras with particular reference to complete metrizable locally convex algebras and topological algebras with orthogonal bases. Recently, Khan,

Received April 27, 2005, accepted June 21, 2005.
Communicated by Shun-Jen Cheng.
2000 *Mathematics Subject Classification*: 16W10, 46K15, 39B05.
Key words and phrases: Prime ring, Semiprime ring, Banach space, Standard operator algebra, H^*-Algebra, Left (right) centralizer, Left (right) Jordan centraliz.
This research has been supported by the Research Council of Slovenia.

Mohammad and Thaheem $[12, 15]$ have studied centralizers and double centralizers on certain topological algebras. Centralizers have also appeared in a variety, among which we mention representation theory of Banach algebras, the study of Banach modules, Hopf algebras (see $[17, 18]$), the theory of singular integrals, interpolation theory, stohastic processes, the theory of semigroups of operators, partial differential equations and the study of approximation problems (see Larsen $[13]$ for more details). In case $T : R \to R$ is a left and a right centralizer, where R is a semiprime ring with extended centroid C, then there exists an element $\lambda \in C$ such that $T(x) = \lambda x$ for all $x \in R$ (see Theorem 2.3.2 in [3]). An additive mapping $T : R \to R$is called a left (right) Jordan centralizer in case $T(x^2) = T(x)x$ ($T(x^2) = xT(x)$) holds for all $x \in R$. Zalar [33] has proved that any left (right) Jordan centralizer on a $2-$torsion free semiprime ring is a left (right) centralizer. Molnár $[16]$ has proved that in case we have an additive mapping $T : A \to A$, where A is a semisimple H^*-algebra, satisfying the relation $T(x^3) = T(x)x^2$ ($T(x^3) = x^2T(x)$) for all $x \in A$, then T is a left (right) centralizer. Let us recall that a semisimple H^*-algebra is a semisimple Banach $^*-$algebra whose norm is a Hilbert space norm such that $(x, yz^*) = (xz, y) = (z, x^*y)$ is fulfilled for all $x, y, z \in A$ (see [2]). Vukman [21] has proved that in case there exists an additive mapping $T : R \to R$, where R is a $2-$torsion free semiprime ring, satisfying the relation $2T(x^2) = T(x)x + xT(x)$, for all $x \in R$ then T is a left and a right centralizer. For results concerning centralizers on semiprime rings operator algebras and H^*-algebras we refer to $[16, 19 - 30, 33]$. Let X be a real or complex Banach space and let $L(X)$ and $F(X)$ denote the algebra of all bounded linear operators on X and the ideal of all finite rank operators in $L(X)$, respectively. An algebra $A(X) \subset L(X)$ is said to be standard in case $F(X) \subset A(X)$. Let us point out that any standard algebra is prime, which is a consequence of Hahn-Banach theorem. We denote by X^* the dual space of a Banach space X and by I the identity operator on X.

2. The Main Results

Let us start with the following purely algebraic result proved by Vukman in $[21]$.

Theorem A. ([21], Theorem 1). *Let R be a $2-$torsion free semiprime ring and let $T : R \to R$ be an additive mapping satisfying the relation*

$$2T(x^2) = T(x)x + xT(x)$$

for all $x \in R$. In this case T is a left and a right centralizer.

Theorem A was the inspiration for the following result.

Theorem 1. *Let A be a semisimple H^*-algebra and let $T : A \to A$ be an additive mapping satisfying the relation*

$$2T(x^{m+n+1}) = x^m T(x) x^n + x^n T(x) x^m,$$

for all $x \in A$ and some nonnegative integers m, n such that $m + n \neq 0$. In this case T is a left and a right centralizer.

For the proof of the theorem above we need the result below which is of independent interest.

Theorem 2. *Let X be a Banach space over F and let $A(X) \subset L(X)$ be a standard operator algebra. Suppose there exists an additive mapping $T : A(X) \to L(X)$ satisfying the relation*

$$2T(A^{m+n+1}) = A^m T(A) A^n + A^n T(A) A^m,$$

for all $A \in A(X)$ and some nonnegative integers m, n such that $m + n \neq 0$. In this case T is of the form $T(A) = \lambda A$, for all $A \in A(X)$ and some $\lambda \in F$.

In the proof of Theorem 2 we shall use Theorem A.

Proof of Theorem 2. We have the relation

$$2T(A^{m+n+1}) = A^m T(A) A^n + A^n T(A) A^m. \tag{1}$$

Let us first consider the restriction of T on $F(X)$. Let A be from $F(X)$ and let $P \in F(X)$, be a projection with $AP = PA = A$. From the above relation one obtains $T(P) = PT(P)P$, which gives

$$T(P)P = PT(P) = PT(P)P. \tag{2}$$

Putting $A + P$ for A in the relation (1), we obtain

$$\begin{aligned} &2\sum_{i=0}^{m+n+1} \binom{m+n+1}{i} T\left(A^{m+n+1-i} P^i\right) \\ &= \left(\sum_{i=0}^{m} \binom{m}{i} A^{m-i} P^i\right) \left(T\left(A\right) + B\right) \left(\sum_{i=0}^{n} \binom{n}{i} A^{n-i} P^i\right) \\ &\quad + \left(\sum_{i=0}^{n} \binom{n}{i} A^{n-i} P^i\right) \left(T\left(A\right) + B\right) \left(\sum_{i=0}^{m} \binom{m}{i} A^{m-i} P^i\right), \end{aligned} \tag{3}$$

where B stands for $T\left(P\right)$. Using (1) and rearranging the equation (3) in sense of collecting together terms involving equal number of factors of P we obtain:

$$\sum_{i=1}^{m+n} f_i\left(A, P\right) = 0, \tag{4}$$

where $f_i(A,P)$ stands for the expression of terms involving i factors of P. Replacing A by $A+2P$, $A+3P$, ..., $A+(m+n)P$ in turn in the equation (1), and expressing the resulting system of $m+n$ homogeneous equations of variables $f_i(A,P)$, $i=1,2,...,m+n$, we see that the coefficient matrix of the system is a van der Monde matrix

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 2 & 2^2 & \cdots & 2^{m+n} \\ \vdots & \vdots & \vdots & \vdots \\ m+n & (m+n)^2 & \cdots & (m+n)^{m+n} \end{bmatrix}.$$

Since the determinant of the matrix is different from zero, it follows that the system has only a trivial solution.

In particular,

$$\begin{aligned} f_{m+n-1}(A,P) &= 2\tbinom{m+n+1}{m+n-1}T\left(A^2\right) - \left[\tbinom{m}{m-2}\tbinom{n}{n} + \tbinom{m}{m}\tbinom{n}{n-2}\right]A^2B \\ &- \left[\tbinom{m}{m}\tbinom{n}{n-2} + \tbinom{m}{m-2}\tbinom{n}{n}\right]BA^2 - \left[\tbinom{m}{m-1}\tbinom{n}{n} + \tbinom{m}{m}\tbinom{n}{n-1}\right]AT(A)P \\ &- \left[\tbinom{m}{m}\tbinom{n}{n-1} + \tbinom{m}{m-1}\tbinom{n}{n}\right]PT(A)A - 2\tbinom{m}{m-1}\tbinom{n}{n-1}ABA = 0, \end{aligned}$$

and

$$\begin{aligned} f_{m+n}(A,P) &= 2\tbinom{m+n+1}{m+n}T(A) - \left[\tbinom{m}{m-1}\tbinom{n}{n} + \tbinom{m}{m}\tbinom{n}{n-1}\right]AB \\ &- \left[\tbinom{m}{m}\tbinom{n}{n-1} + \tbinom{m}{m-1}\tbinom{n}{n}\right]BA - 2\tbinom{m}{m}\tbinom{n}{n}PT(A)P = 0. \end{aligned}$$

The above equations reduce to

$$\begin{aligned} &2(m+n+1)(m+n)T\left(A^2\right) \\ &= [m(m-1)+n(n-1)]A^2B + [m(m-1)+n(n-1)]BA^2 \\ &+4mnABA + 2(m+n)AT(A)P + 2(m+n)PT(A)A, \end{aligned} \tag{5}$$

and

$$2(m+n+1)T(A) = (m+n)AB + (m+n)BA + 2PT(A)P. \tag{6}$$

Right multiplication of the relation (6) by P gives

$$2(m+n+1)T(A)P = (m+n)AB + (m+n)BA + 2PT(A)P. \tag{7}$$

Similarly one obtains

$$2\left(m+n+1\right)PT\left(A\right)=\left(m+n\right)AB+\left(m+n\right)BA+2PT\left(A\right)P. \tag{8}$$

Combining (7) with (8) we arrive at

$$T(A)P=PT(A),$$

which reduces the relations (5) to

$$\begin{aligned}&2\left(m+n+1\right)\left(m+n\right)T\left(A^{2}\right)\\&\quad=\left[m\left(m-1\right)+n\left(n-1\right)\right]A^{2}B+\left[m\left(m-1\right)+n\left(n-1\right)\right]BA^{2}\\&\quad+4mnABA+2\left(m+n\right)AT\left(A\right)+2\left(m+n\right)T\left(A\right)A,\end{aligned} \tag{9}$$

and the relation (7) to

$$2T\left(A\right)P=AB+BA. \tag{10}$$

Combining (10) with (6) we arrive at

$$T(A)=T(A)P. \tag{11}$$

From the above relation one can conclude that T maps $F(X)$ into itself. According to the above relation the relation (10) reduces to

$$2T\left(A\right)=AB+BA. \tag{12}$$

From the above relation we can conclude that T is linear on $F(X)$. Now applying the relation above we obtain

$$\begin{aligned}2mnABA&=mn(AB)A+mnA(BA)=mn(2T(A)-BA)A\\&+mnA(2T(A)-AB)=2mn(T(A)A+AT(A))-mn\left(A^{2}B+BA^{2}\right).\end{aligned}$$

We have therefore

$$2ABA=2(T(A)A+AT(A))-A^{2}B-BA^{2}.$$

Applying the relation (12) and the relation above in the relation (9) we obtain

$$2T(A^{2})=T(A)A+AT(A). \tag{13}$$

Therefore we have a linear mapping $T:F(X)\rightarrow F(X)$ satisfying the relation (13) for all $A\in F(X)$. Since $F(X)$ is prime one can conclude according to Theorem A that T is a left and also a right centralizer on $F(X)$. We intend to prove that there exists an operator $C \in L(X)$, such that

$$T(A) = CA, A \in F(X) \tag{14}$$

For any fixed $x \in X$ and $f \in X^*$ we denote by $x \otimes f$ an operator from $F(X)$ defined by $(x \otimes f)y = f(y)x$, $y \in X$. For any $A \in L(X)$ we have $A(x \otimes f) = ((Ax) \otimes f)$. Let us choose f and y such that $f(y) = 1$ and define $Cx = T(x \otimes f)y$. Obviously, C is linear. Using the fact that T is a left centralizer on $F(X)$ we obtain

$$(CA)x = C(Ax) = T((Ax) \otimes f)y = T(A(x \otimes f))y = T(A)(x \otimes f)y = T(A)x, x \in X.$$

We have therefore $T(A) = CA$ for any $A \in F(X)$. Since T is a right centralizer on $F(X)$ we obtain $C(AB) = T(AB) = AT(B) = ACB$. We have therefore $[A, C]B = 0$ for any $A, B \in F(X)$ whence it follows that $[A, C] = 0$ for any $A \in F(X)$. Using closed graph theorem one can easily prove that C is continuous. Since C commutes with all operators from $F(X)$ one can conclude that $Cx = \lambda x$ holds for any $x \in X$ and some $\lambda \in F$, which gives together with the relation (14) that T is of the form

$$T(A) = \lambda A \tag{15}$$

any $A \in F(X)$ and some $\lambda \in F$. It remains to prove that the above relation holds on $A(X)$ as well. Let us introduce $T_1 : A(X) \to L(X)$ by $T_1(A) = \lambda A$and consider $T_0 = T - T_1$. The mapping T_0 is, obviously, additive and satisfies the relation (1). Besides T_0 vanishes on $F(X)$. Let $A \in A(X)$, let $P \in F(X)$, be a projection and $S = A + PAP - (AP + PA)$. Note that S can be written in the form $S = (I - P)A(I - P)$, where I denotes the identity operator on X. Since, obviously, $S - A \in F(X)$, we have $T_0(S) = T_0(A)$. Besides $SP = PS = 0$. We have therefore the relation

$$2T_0(S^{m+n+1}) = S^m T_0(S) S^n + S^n T_0(S) S^m, \tag{16}$$

Applying the above relation and the fact that $T_0(P) = 0$, $SP = PS = 0$, we obtain

$$\begin{aligned}
&S^m T_0(S) S^n + S^n T_0(S) S^m = 2T_0(S^{m+n+1}) = 2T_0(S^{m+n+1} + P) \\
&= 2T_0((S+P)^{m+n+1}) = (S+P)^m T_0(S+P)(S+P)^n \\
&\quad +(S+P)^n T_0(S)(S+P)^m = (S^m + P) T_0(S)(S^n + P) \\
&\quad +(S^n + P) T_0(S)(S^m + P) = S^m T_0(S) S^n + P T_0(S) S^n + S^m T_0(S) P \\
&\quad +P T_0(S) P + S^n T_0(S) S^m + P T_0(S) S^m + S^n T_0(S) P + P T_0(S) P.
\end{aligned}$$

We have therefore

$$P T_0(A)(S^m + S^n) + (S^m + S^n) T_0(A) P + 2P T_0(A) P = 0. \tag{17}$$

Multiplying the above relation from both sides by P we arrive at

(18) $$PT_0(A)P = 0,$$

which reduces the relation (17) to

$$PT_0(A)(S^m + S^n) + (S^m + S^n)T_0(A)P = 0.$$

Right multiplication of the above relation by P gives

(19) $$(S^m + S^n)T_0(A)P = 0.$$

We intend to prove that

(20) $$S^m T_0(A)P = 0.$$

In case $m = n$ there is nothing to prove according to (19). Let us therefore assume that $m \neq n$. Putting in the relation (19) $2A$ for A we obtain

$$(2^{m+1}S^m + 2^{n+1}S^n)T_0(A)P = 0.$$

Multiplying the relation (19) by 2^{n+1} and subtracting the relation so obtained from the above relation we obtain $(2^{m+1} - 2^{n+1})S^m T_0(A)P = 0$ whence it follows the relation (20). Let us prove that

(21) $$S^{m-1}T_0(A)P = 0.$$

Putting $A + B$ for A,where $B \in F(X)$, in (20) and using the fact that T_0 vanishes on $F(X)$, we obtain

$$(S_1 S^{m-1} + SS_1 S^{m-2} + ... + S^{m-1}S_1)T_0(A)P = 0,$$

where S_1 stands for $(I - P)B(I - P)$ (see [5]). The substitution $T_0(A)PB$ for B in the above relation gives because of (18)

$$(T_0(A)PBS^{m-1} + ST_0(A)PBS^{m-2} + ... + S^{m-1}T_0(A)PB)T_0(A)P = 0.$$

Left multiplication of the above relation by S^{m-1} and applying the relation (20) we obtain

$$(S^{m-1}T_0(A)P)B(S^{m-1}T_0(A)P) = 0,$$

for all $B \in F(X)$. Now it follows $S^{m-1}T_0(A)P = 0$ by primeness of $F(X)$, which proves (21). Now, since (20) implies (21), one can conclude by induction that $ST_0(A)P = 0$, which gives

$$AT_0(A)P - PAT_0(A)P = 0,$$

because of (18). Putting $A+B$ for A, where $B \in F(X)$, we obtain $0=(A+B)T_0(A)P - P(A+B)T_0(A)P = BT_0(A)P - PBT_0(A)P$. We have therefore proved that $BT_0(A)P - PBT_0(A)P = 0$ holds for all $A \in A(X)$ and all $B \in F(X)$. The substitution $T_0(A)PB$ for B in the above relation gives, because of (18), $(T_0(A)P)B(T_0(A)P) = 0$, for all $B \in F(X)$. Thus it follows $T_0(A)P = 0$ by primeness of $F(X)$. Since P is an arbitrary one-dimensional projection, one can conclude that $T_0(A) = 0$, for any $A \in A(X)$. In other words, we have proved that T is of the form $T(A) = \lambda A$, for all $A \in A(X)$ and some $\lambda \in F$. Obviously, T is linear and bounded. The proof of the theorem is complete.

Let us point out that in Theorem 2 we obtain continuity of T under purely algebraic conditions concerning the mapping T.

It should be mentioned that in the proof of Theorem 2 we used some methods similar to those used by Molnár in [16].

Proof of Theorem 1. The proof goes through using the same arguments as in the proof of Theorem in [16] with the exception that one has to use Theorem 2 instead of Lemma in [16].

Since in the formulation of the results presented in this paper we have used only algebraic concepts, it would be interesting to study the problem in a purely ring theoretical context. We conclude with the following conjecture.

Conjecture. Let $T : R \rightarrow R$ be an additive mapping, where R is a semiprime ring, and let $T : R \rightarrow R$ be an additive mapping satisfying the relation

$$2T(x^{m+n+1}) = x^m T(x) x^n + x^n T(x) x^m$$

for all $x \in R$and some nonnegative integers m, n such that $m+n \neq 0$. In this case T is a left and a right centralizer.

In case $m = 0$, $n = 1$ the conjecture above has been proved by Vukman (Theorem A). Since semisimple H^*-algebras are semiprime Theorem 1 proves the conjecture above in a special case. We are going to prove the conjecture above in case a semiprime ring R has the identity element.

Theorem 3. *Let m, n be nonnegative integers such that $m+n \neq 0$ and let R be a 2, $m+n$ and $m+n+2mn$ -torsion free semiprime ring with the identity element. Suppose that there exists an additive mapping $T : R \rightarrow R$ satisfying the relation*

$$2T(x^{m+n+1}) = x^m T(x) x^n + x^n T(x) x^m$$

for all $x \in R$. *In this case* T *is of the form* $T(x) = ax$ *for all* $x \in R$ *where* a *is a fixed element from* $Z(R)$.

Proof. We have the relation

$$2T(x^{m+n+1}) = x^m T(x) x^n + x^n T(x) x^m, x \in R. \tag{22}$$

With the same approach as in the proof of Theorem 2 we obtain from the above relation

$$\begin{aligned} &2(m+n+1)(m+n)T\left(x^2\right) = (m(m-1) + n(n-1)x^2 a \\ &\quad + m(m-1) + n(n-1))ax^2 + 4mnxax \\ &\quad + 2(m+n)xT(x) + 2(m+n)T(x)x, x \in R \end{aligned} \tag{23}$$

and

$$2T(x) = xa + ax, x \in R, \tag{24}$$

where a stands for $T(e)$. In the procedure mentioned above we used the fact that R is $m+n$ -torsion free.

According to (24) one obtains the relation

$$2T\left(x^2\right) = x^2 a + ax^2, x \in R. \tag{25}$$

Multiplying the relation (24) by x from both sides we obtain

$$2T(x)x = xax + ax^2, x \in R \tag{26}$$

and

$$2xT(x) = x^2 a + xax, x \in R. \tag{27}$$

Using (25), (26) and (27) in the relation (22) we obtainafter some calculation

$$x^2 a + ax^2 - 2xax = 0, x \in R,$$

since R is $m+n+2mn$ -torsion free. The above relation can be written in the form

$$[[a,x],x] = 0, x \in R. \tag{28}$$

Putting $x+y$ for x in the above relation we obtain

$$[[a,x],y] + [[a,y],x] = 0, x, y \in R. \tag{29}$$

Putting xy for y in relation (29) we obtain because of (28) and (29):

$$\begin{aligned} 0 &= [[a,x],xy] + [[a,xy],x] \\ &= [[a,x],x]\,y + x\,[[a,x],y] \quad + [[a,x]\,y + x\,[a,y],x] \\ &= x\,[[a,x],y] + [[a,x],x]\,y + [a,x]\,[y,x] + x\,[[a,y],x] \\ &= [a,x]\,[y,x],\, x,y \in R. \end{aligned}$$

Thus we have

$$[a,x]\,[y,x] = 0, x, y \in R.$$

The substitution ya for y in the above relation gives $[a,x]\,y\,[a,x] = 0$, for all pairs $x, y \in R$. Let us point out that so far we have not used the assumption that R is semiprime. Since R is semiprime, it follows from the last relation that $[a,x] = 0$, for all $x \in R$. In other words, $a \in Z(R)$, which reduces the relation (24) to $T(x) = ax$, $x \in R$, since R is 2 -torsion free. The proof of the theorem is complete.

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