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A PERTURBATION THEOREM OF MIYADERA TYPE FOR LOCAL C-REGULARIZED SEMIGROUPS

Ti-Jun Xiao, Jin Liang and Fang Li

Abstract. In this paper, we investigate the perturbation problem for local C-regularized semigroups on a Banach space and establish a Miyadera type perturbation theorem.

1. Introduction and Preliminaries

The Miyadera perturbation theorem for C_0 semgroups was established in 1966 ([4]). Since then, there have been some generalizations (cf., e.g., [1, 3, 5, 9] and references therein). The aim of this paper is to extend this theorem to local C-regularized semigroups (introduced in [8]) and present a Miyadera type perturbation theorem. This result contains the classical Miyadera perturbation theorem as a special case. Moreover, it is also suitable for non-exponentially-bounded regularized semigroups, while the C_0 semigroup and the other operator families concerned in [1, 3, 5, 9] are all exponentially bounded on $[0, \infty)$. For more information on local regularized semigroups and regularized semigroups, we refer the reader to [2, 6, 7, 8, 10] and references cited there.

Throughout this paper, all operators are linear; X is a Banach space; $\mathcal{L}(X, Y)$ denotes the space of all continuous linear operators from X to a space Y, and $\mathcal{L}(X, X)$ will be abbreviated to $\mathcal{L}(X)$; $\mathcal{L}_s(X)$ is the space of all continuous linear operators from X to X with the strong operator topology; C is an injective operator in $\mathcal{L}(X)$; $C([0, t], \mathcal{L}_s(X))$ is the space of all strongly continuous $\mathcal{L}(X)$-valued functions, equipped with the norm

$$\|F\|_\infty = \sup_{r\in[0,t]} \|F(r)\|.$$

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Moreover, for an operator A, we write $\mathcal{D}(A)$, $\mathcal{R}(A)$, $\rho(A)$, respectively, for the domain, the range, the resolvent set of A, and we denote by $[\mathcal{D}(A)]$ the space $\mathcal{D}(A)$ with the graph norm.

Definition 1.1. ([8]) Assume $\tau > 0$. A one-parameter family $\{T(t)\}_{t\in[0,\tau]} \subset \mathcal{L}(X)$ is called a local C-regularized semigroup on X if

(i) $T(0) = C$ and $T(t+s)C = T(t)T(s) \quad (\forall s,\ t,\ s+t \in [0,\tau])$,

(ii) $T(\cdot)x : [0,\tau] \to X$ is continuous for every $x \in X$.

The operator A defined by

$$\mathcal{D}(A) = \{x \in X : \lim_{t\to 0^+} \frac{1}{t}(T(t)x - Cx) \text{ exists and is in } \mathcal{R}(C)\}$$

and

$$Ax = C^{-1} \lim_{t\to 0^+} \frac{1}{t}(T(t)x - Cx), \quad \forall x \in \mathcal{D}(A),$$

is called the generator of $\{T(t)\}_{t\in[0,\tau]}$. It is also called that A generates $\{T(t)\}_{t\in[0,\tau]}$.

Remark 1.2. When $C = I$, $\{T(t)\}_{t\in[0,\tau]}$ can be extended uniquely (in an obvious way) to a C_0 semigroup $\{T(t)\}_{t\geq 0}$ with A as its generator.

The following two lemmas will be used freely in the proofs of our results below. Lemma 1.3 comes from [8] and Lemma 1.4 is implied in [2].

Lemma 1.3. *Let A generate a local C-regularized semigroup $\{T(t)\}_{t\in[0,\tau]}$ on X. Then*

(i) *For $x \in \mathcal{D}(A)$, $t \in [0,\tau]$, $T(t)x \in \mathcal{D}(A)$ and $AT(t)x = T(t)Ax$.*

(ii) *For $x \in X$, $t\in[0,\tau]$, $\int_0^t T(s)xds \in \mathcal{D}(A)$ and $A\int_0^t T(s)xds = T(t)x - Cx$.*

(iii) *For $x \in \mathcal{D}(A)$, $t \in [0,\tau]$, $\int_0^t T(s)Axds = A\int_0^t T(s)xds = T(t)x - Cx$.*

Lemma 1.4. *Suppose an extension of A, $\widetilde{A}$, generates a local C-regularized semigroup. Then $C(\mathcal{D}(\widetilde{A})) \subset \mathcal{D}(A)$ is equivalent to $C^{-1}AC = \widetilde{A}$.*

2. Results and Proofs

Theorem 2.1. *Assume that a densely defined linear operator A in X generates a local C-regularized semigroup $\{T(t)\}_{t\in[0,\tau]}$ on X. If $P \in \mathcal{L}(X)$ satisfying*

$(\mathbf{H1})$ $\rho((I+P)A) \neq \emptyset$,

$(\mathbf{H2})$ *for all* $x \in \mathcal{D}(A)$, *and* $\Psi \in C([0,\tau], \mathcal{L}_s(X))$,

$$\left\| \int_0^t \Psi(s) C^{-1} PAT(t-s) x ds \right\| \le \beta(t) \|\Psi\|_\infty \|x\|, \quad t \in [0,\tau],$$

where $\beta(\cdot)$ *is a function with* $\limsup_{t\to 0^+} \beta(t) < 1$,

$(\mathbf{H3})$ *there exists an injective operator* $C_1 \in \mathcal{L}(X)$ *such that* $\mathcal{R}(P) \subset \mathcal{R}(C_1) \subset \mathcal{R}(C)$, $C_1(I+P)A \subset (I+P)AC_1$, *and* $C^{-1}C_1(\mathcal{D}(A))$ *is a dense subspace in* $\mathcal{D}(A)$,

then $(I+P)A$ *generates a local* C_1*-regularized semigroup on* X.

Proof. Let $\tau > \tau_1 > 0$, such that $\beta(t) \le \kappa < 1$, for all $t \in [0,\tau_1]$. Define

$$(\mathcal{H}\mathcal{U})(t)x = \int_0^t \mathcal{U}(s) C^{-1} PAT(t-s) x ds, \quad t \in [0,\tau_1],\, x \in \mathcal{D}(A),$$

for any strongly continuous operator function $\mathcal{U} : [0,\tau_1] \to \mathcal{L}(X)$.

Clearly, $(\mathcal{H}\mathcal{U})(t)x$ is continuous in t on $[0,\tau_1]$ and depends linearly on $x \in \mathcal{D}(A)$. Since

$$\begin{aligned} \|(\mathcal{H}\mathcal{U})(t)x\| &= \left\| \int_0^t \mathcal{U}(s) C^{-1} PAT(t-s) x ds \right\| \\ &\le \beta(t) \|\mathcal{U}\|_\infty \|x\| \end{aligned}$$

for every $t \in [0,\tau_1]$, and $\mathcal{D}(A)$ is dense in X, we can extend the operator $(\mathcal{H}\mathcal{U})(t)$ to a continuous operator on X, and the extended operator function $(\overline{\mathcal{H}}\mathcal{U})(\cdot)$ is strongly continuous on $[0,\tau_1]$. Hence $\mathcal{H}$ maps $C([0,\tau_1], \mathcal{L}_s(X))$ into itself. Since

$$\|(\overline{\mathcal{H}}\mathcal{U}_1 - \overline{\mathcal{H}}\mathcal{U}_2)(t)\| \le \beta(t) \|\mathcal{U}_1 - \mathcal{U}_2\|_\infty \le \kappa \|\mathcal{U}_1 - \mathcal{U}_2\|_\infty,$$

there exists a unique $\mathcal{U} \in C([0,\tau_1], \mathcal{L}_s(X))$ satisfying

$$\mathcal{U}(t)x = T(t)x + \int_0^t \mathcal{U}(s) C^{-1} PAT(t-s) x ds, \quad t \in [0,\tau_1],\, x \in \mathcal{D}(A). \tag{2.1}$$

Setting

$$\mathcal{V}(t) = \mathcal{U}(t) C^{-1} C_1, \quad t \in [0,\tau_1],$$

we have, from (2.1),

$$\mathcal{V}(t)x = T(t)C^{-1}C_1 x + \int_0^t \mathcal{V}(s) C_1^{-1} PAT(t-s) C^{-1} C_1 x ds, \quad x \in \mathcal{D}(A),\ t \in [0,\tau_1].$$

Hence, for $x \in \mathcal{D}(A)$,

$$\int_0^t \mathcal{V}(s)xds = \int_0^t T(s)C^{-1}C_1xds + \int_0^t \int_0^s \mathcal{V}(\sigma)C_1^{-1}PAT(s-\sigma)C^{-1}C_1xd\sigma ds, \quad t \in [0, \tau_1].$$

It follows that for $x \in X$,

$$\int_0^t \mathcal{V}(s)xds = \int_0^t T(s)C^{-1}C_1xds + \int_0^t \mathcal{V}(\sigma)C_1^{-1}P[T(t-\sigma)C^{-1}C_1x - C^{-1}C_1x]d\sigma, \ t \in [0, \tau_1], \tag{2.2}$$

due to the density of $\mathcal{D}(A)$. Note $\mathcal{D}(A) \subset \mathcal{D}(C_1^{-1}PAC_1)$, since $AC_1 = (AC)(C^{-1}C_1)$ and $C^{-1}C_1$ maps $\mathcal{D}(A)$ into $\mathcal{D}(A)$. So, for $x \in \mathcal{D}(A)$,

$$\int_0^t \mathcal{V}(s)C_1^{-1}PAC_1xds = \int_0^t T(s)C^{-1}PAC_1xds + \int_0^t \mathcal{V}(\sigma)C_1^{-1}P[T(t-\sigma)C^{-1}PAC_1x - PAC_1x]d\sigma,$$

by (2.2). Thus, we see that for $x \in \mathcal{D}(A)$,

$$\begin{aligned} &\int_0^t \mathcal{V}(s)(I+P)Axds \\ &= \int_0^t T(s)C^{-1}(I+P)AC_1xds \\ &\quad + \int_0^t \mathcal{V}(\sigma)C_1^{-1}P[T(t-\sigma)C^{-1}(I+P)AC_1x - (I+P)AC_1x]d\sigma \\ &= T(t)C^{-1}C_1x - C_1x + \int_0^t T(s)C^{-1}PAC_1xds \\ &\quad + \int_0^t \mathcal{V}(\sigma)C_1^{-1}PAT(t-\sigma)C^{-1}C_1xds - \int_0^t \mathcal{V}(\sigma)C_1^{-1}PAC_1xds \\ &\quad + \int_0^t \mathcal{V}(\sigma)C_1^{-1}P[T(t-\sigma)C^{-1}PAC_1x - PAC_1x]d\sigma \\ &= \mathcal{V}(t)x - C_1x. \end{aligned} \tag{2.3}$$

Now we consider the integral equation

$$h(t)x = C_1x + \int_0^t h(s)(I+P)Axds, \quad x \in \mathcal{D}(A), \ t \in [0, \tau_1], \tag{2.4}$$

for $h(t) \in C([0,\tau_1], \mathcal{L}_s(X))$. Let $h(t)$ be a solution of (2.4). Then from (2.4) it follows that for $x \in \mathcal{D}(A)$,

$$\begin{aligned}
&\int_0^t h(s)(I+P)A\int_0^{t-s} T(\sigma)C^{-1}C_1xd\sigma ds \\
=&\int_0^t\int_0^{t-\sigma} h(s)(I+P)AT(\sigma)C^{-1}C_1xdsd\sigma \\
=&\int_0^t h(s)T(t-s)C^{-1}C_1xds - C_1\int_0^t T(s)C^{-1}C_1xds.
\end{aligned}$$

On the other hand, for $x \in \mathcal{D}(A)$,

$$\begin{aligned}
&\int_0^t h(s)(I+P)A\int_0^{t-s} T(\sigma)C^{-1}C_1xd\sigma ds \\
=&\int_0^t h(s)[T(t-s)C^{-1}C_1x - C_1x]ds + \int_0^t h(s)PA\int_0^{t-s} T(\sigma)C^{-1}C_1xd\sigma ds.
\end{aligned}$$

Hence, for $x \in \mathcal{D}(A)$,

$$\int_0^t h(s)C_1xds = C_1\int_0^t T(s)C^{-1}C_1xds + \int_0^t h(s)PA\int_0^{t-s} T(\sigma)C^{-1}C_1xd\sigma ds,$$

that is,

$$(h(t)C)C^{-1}C_1x = C_1T(t)C^{-1}C_1x + \int_0^t (h(s)C)C^{-1}PAT(t-s)C^{-1}C_1xds.$$

Noting that $C^{-1}C_1(\mathcal{D}(A)) \subset \mathcal{D}(A)$ is dense in X, and the solution $\overline{h}(t)$ of the equation

$$\overline{h}(t)y = C_1T(t)y + \int_0^t \overline{h}(s)C^{-1}PAT(t-s)yds, \quad y \in C^{-1}C_1(\mathcal{D}(A)),\ t \in [0,\tau_1]$$

in $C([0,\tau_1], \mathcal{L}_s(X))$ is unique, we see the solution of (2.4) is also unique.

By the equality (2.3), (H3), the uniqueness of solution of (2.4) and the density of $\mathcal{D}(A)$, we obtain

$$(\lambda_0-(I+P)A)^{-1}\mathcal{V}(t) = \mathcal{V}(t)(\lambda_0-(I+P)A)^{-1}, \quad t \in [0,\tau_1],\ \lambda_0 \in \rho((I+P)A),$$

and therefore

$$(I+P)A\mathcal{V}(t)x = \mathcal{V}(t)(I+P)Ax, \quad x \in \mathcal{D}(A),\ t \in [0,\tau_1].$$

Since $\rho((I+P)A) \neq \emptyset$, $(I+P)A$ is a closed operator. Thus from (2.3), the denseness of $\mathcal{D}(A)$ and the closedness of $(I+P)A$, it follows that $\int_0^t \mathcal{V}(s)xds \in \mathcal{D}(A)$ and

$$\mathcal{V}(t)x = C_1x + (I+P)A\int_0^t \mathcal{V}(s)xds, \quad x \in X,\ t \in [0,\tau_1]. \tag{2.5}$$

Let $x \in \mathcal{D}(A)$. Then for $t,\ h \in [0,\tau_1]$,

$$\begin{aligned}
&\mathcal{V}(h)\mathcal{V}(t)x \\
&= \mathcal{V}(h)\int_0^t \mathcal{V}(\sigma)(I+P)Axd\sigma + \mathcal{V}(h)C_1x \\
&= \int_0^t \mathcal{V}(h)\mathcal{V}(\sigma)(I+P)Axd\sigma + C_1^2x + \int_0^h \mathcal{V}(s)(I+P)AC_1xds,
\end{aligned}$$

and that for $t,\ t+h \in [0,\tau_1]$,

$$\begin{aligned}
&\mathcal{V}(t+h)C_1x \\
&= \int_0^{t+h} \mathcal{V}(s)(I+P)AC_1xds + C_1^2x \\
&= \int_h^{t+h} \mathcal{V}(s)(I+P)AC_1xds + \int_0^h \mathcal{V}(s)(I+P)AC_1xds + C_1^2x \\
&= \int_0^t \mathcal{V}(s+h)C_1(I+P)Axds + C_1^2x + \int_0^h \mathcal{V}(s)(I+P)AC_1xds.
\end{aligned}$$

As a consequence,

$$\mathcal{V}(h)\mathcal{V}(t)x - \mathcal{V}(h+t)C_1x = \int_0^t [\mathcal{V}(h)\mathcal{V}(\sigma) - \mathcal{V}(\sigma+h)C_1](I+P)Axd\sigma.$$

It follows from the uniqueness of the solution of (2.4) that

$$\mathcal{V}(t)\mathcal{V}(h) = \mathcal{V}(t+h)C_1, \quad t,\ h,\ t+h \in [0,\tau_1].$$

Hence $\{\mathcal{V}(t)\}_{t\in[0,\tau_1]}$ is a local C_1-regularized semigroup on X. Denote by A_0 the generator of $\{\mathcal{V}(t)\}_{t\in[0,\tau_1]}$. We see easily from (2.3) that $\mathcal{D}((I+P)A) = \mathcal{D}(A) \subset \mathcal{D}(A_0)$. On the other hand, for any $x \in \mathcal{D}(A_0)$, we have

$$\lim_{m\to\infty} m\int_0^{\frac{1}{m}} \mathcal{V}(s)xds = C_1x,$$

$$\lim_{m\to\infty}(I+P)A\left[m\int_0^{\frac{1}{m}} \mathcal{V}(s)xds\right] = \lim_{m\to\infty} m\left[\mathcal{V}\left(\frac{1}{m}\right)x - C_1x\right] = C_1A_0x,$$

by (2.5). It follows that $C_1(\mathcal{D}(A_0)) \subset \mathcal{D}((I+P)A)$. Consequently, $A_0 = C_1^{-1}(I+P)AC_1$ by Lemma 1.4. But

$$C_1^{-1}(I+P)AC_1 = (I+P)A,$$

since $\rho((I+P)A) \neq \emptyset$. This ends the proof.

Corollary 2.2. *Suppose that a densely defined linear operator A in X generates a local C-regularized semigroup $\{T(t)\}_{t\in[0,\tau]}$ on X. If $B \in \mathcal{L}([\mathcal{D}(A)], X)$ satisfying*

(H1′) $\rho(A) \neq \emptyset$ *and* $\rho(A+B) \neq \emptyset$,

(H2′) *there exist* $\tau_1 \in (0,\tau]$, $\gamma \in (0,1)$ *such that*

$$\int_0^{\tau_1} \left\|C^{-1}BT(s)x\right\| ds \leq \gamma\|x\|, \quad x \in \mathcal{D}(A),$$

(H3′) *there exists an injective operator $C_1 \in \mathcal{L}(X)$ such that $\mathcal{R}(B) \subset \mathcal{R}(C_1) \subset \mathcal{R}(C)$, $C_1(A+B) \subset (A+B)C_1$, and $C^{-1}C_1(\mathcal{D}(A))$ is a dense subspace in $\mathcal{D}(A)$,*

then $A+B$ generates a local C_1-regularized semigroup.

Proof. Take $\lambda_0 \in \rho(A)$. Then $A-\lambda_0$ generates a local C-regularized semigroup $\{e^{-\lambda_0 t}T(t)\}_{t\in[0,\tau]}$ on X. Setting $P = B(A-\lambda_0)^{-1}$, we have $P \in \mathcal{L}(X)$. It's clear from (H_2') that for $x \in \mathcal{D}(A)$, and $\Psi \in C([0,\tau_1], \mathcal{L}_s(X))$,

$$\left\|\int_0^t \Psi(s)C^{-1}P(A-\lambda_0)e^{-\lambda_0(t-s)}T(t-s)xds\right\| \leq \gamma_1\|\Psi\|_\infty\|x\|, \quad t \in [0,\tau_1],$$

for some $\tau_1 \in (0,\tau]$, $\gamma_1 \in (\gamma, 1)$. Thus making use of Theorem 2.1, we infer that $(I+P)(A-\lambda_0)$ generates a local C_1-regularized semigroup $\{\mathcal{V}(t)\}_{t\in[0,\tau_1]}$, and therefore $A+B = (I+P)(A-\lambda_0)+\lambda_0$ is the generator of the local C_1-regularized semigroup $\{e^{\lambda_0 t}\mathcal{V}(t)\}_{t\in[0,\tau_1]}$. This completes the proof. ■

Remark 2.3. Corollary 2.2 is a generalization of the Miyadera perturbation theorem ([4]). Actually, when A generates a C_0 semigroup on X, and $C = C_1 = I$, Corollary 2.2 is just the Miyadera perturbation theorem (see also Remark 1.2).

Finally, we present a concrete example to show how our results can be used.

Example 2.4. Let $X_1 = L^2(\Omega)$, $X_2 = C_0(\gamma)$, where Ω is a bounded domain in R^n with smooth boundary, and

$$\gamma := \{s + ie^{s^2};\ s \geq 0\}.$$

Define

$$A_1 := i\Delta, \quad \mathcal{D}(A_1) = H^2(\Omega) \cap H_0^1(\Omega),$$

$$(A_2\varphi)(\xi) = \xi\varphi(\xi), \quad \text{with } \varphi \in D(A_2) := \{\varphi \in C_0(\gamma);\ \xi \mapsto \xi\varphi(\xi) \in C_0(\gamma)\}.$$

Then, A_1 generates a strongly continuous group $\{T_1(t)\}_{t\in R}$ on X_1, $\overline{\mathcal{D}(A_2)} = X_2$ and A_2 generates (cf. [2, p. 110, Ex. 18.2]) an A_2^{-1}-regularized semigroup $\{T_2(t)\}_{t\geq 0}$ on X_2 given by

$$T_2(t)\varphi(\xi) = \frac{1}{\xi}e^{t\xi}\varphi(\xi).$$

Let $q_1,\ q_2 \in C_c(\Omega)$, $r_1 \in \mathcal{D}(A_1)$, $r_2 \in \mathcal{D}(A_2)$. Define $P_1 : X_2 \to X_1$, $P_2 : X_1 \to X_2$ by

$$(P_1\varphi)(\xi) = r_1(\xi)\int_\Omega q_1(\sigma)\varphi(\sigma)d\sigma,$$

$$(P_2\varphi)(\xi) = r_2(\xi)\int_\Omega q_2(\sigma)\varphi(\sigma)d\sigma.$$

Set

$$X := X_1 \times X_2;$$

$$A := \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}, \quad \text{with } \mathcal{D}(A) = \mathcal{D}(A_1) \times \mathcal{D}(A_2);$$

$$P := \begin{pmatrix} 0 & P_1 \\ P_2 & 0 \end{pmatrix}, \quad \text{with } \mathcal{D}(P) = X.$$

Then $\mathcal{R}(P) \subset \mathcal{D}(A)$. Writing $C = A^{-1}$, we see that A generates a C-regularized semigroup $\{T(t)\}_{t\geq 0}$ on X given by

$$T(t) := \begin{pmatrix} T_1(t)A_1^{-1} & 0 \\ 0 & T_2(t) \end{pmatrix},$$

and for $x := \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathcal{D}(A)$, $0 \leq s \leq t < 1$,

$$C^{-1}PAT(t-s)x := \begin{pmatrix} A_1P_1A_2T_2(t-s)x_2 \\ A_2P_2T_1(t-s)x_1 \end{pmatrix}.$$

It is not hard to see that the operators $A_1P_1A_2$ and A_2P_2 have bounded extensions, and therefore there exists $M > 0$ such that

$$\|C^{-1}PAT(t-s)\| \leq M, \quad 0 \leq s \leq t \leq 1.$$

Put $\tau = \min\{1,\ (2M)^{-1}\}$, we get

$$\left\| \int_0^t \Psi(s) C^{-1} PAT(t-s) x ds \right\| \leq \frac{1}{2} \|\Psi\|_\infty \|x\|, \quad t \in [0, \tau],$$

for $x \in \mathcal{D}(A)$, $\Psi \in C([0, \tau], \mathcal{L}_s(X))$, which means (H2) holds. Next, we let $P_2 = 0$ for simplicity. Then

$$(I + P)^{-1} = \begin{pmatrix} I & -P_1 \\ 0 & I \end{pmatrix} \in \mathcal{L}(X).$$

Therefore, $0 \in \rho((I + P)A)$. Set $C_1 = A^{-1}(I + P)^{-1}$. Then

$$\mathcal{R}(C_1) = \mathcal{D}(A), \quad C^{-1}C_1 = (I + P)^{-1}.$$

Thus, we see that Theorem 2.1 is applicable to this situation, and yields that $\begin{pmatrix} A_1 & P_1 A_2 \\ 0 & A_2 \end{pmatrix}$ generates a local C_1-regularized semigroup on X.

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Ti-Jun Xiao, Jin Liang and Fang Li
Department of Mathematics,
University of Science and Technology of China,
Hefei, Anhui 230026,
People's Republic of China.
E-mail: xiaotj@ustc.edu.cn
E-mail: jliang@ustc.edu.cn
E-mail: fangli@ustc.edu.cn