Research Article

Robust Exponential Stabilization of Stochastic Delay Interval Recurrent Neural Networks with Distributed Parameters and Markovian Jumping by Using Periodically Intermittent Control

Junhao Hu,¹ Yunjian Peng,² and Yan Li^{3,4}

¹ College of Mathematics and Statistics, South Central University for Nationalities, Wuhan 430074, China

² School of Automation Science and Engineering, South China University of Technology, Guangzhou 510641, China

³ College of Automation, Huazhong University of Science and Technology, Wuhan 430074, China

⁴ College of Science, Huazhong Agriculture University, Wuhan 430070, China

Correspondence should be addressed to Yan Li; ly@mail.hzau.edu.cn

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We consider a class of stochastic delay recurrent neural networks with distributed parameters and Markovian jumping. It is assumed that the coefficients in these neural networks belong to the interval matrices. Several sufficient conditions ensuring robust exponential stabilization are derived by using periodically intermittent control and Lyapunov functional. The obtained results are very easy to verify and implement, and improve the existing results. Finally, an example with numerical simulations is given to illustrate the presented criteria.

1. Introduction

In recent decades, neural network dynamics has been widely studied by many authors due to the fact that neural network dynamics can be applied to associate memory, signal processing, pattern classification, and quadratic optimization. Liao and Mao [1, 2] investigated the stability of stochastic neural network for the first time in 1996. By Razumikhintype theorems, the stability of stochastic neural networks with variable delays was considered [3]. Considering electrons moving in the asymptotic electromagnetic field, the diffusion phenomena could not be ignored. Luo et al. [4] gave several algebra criteria for stochastic Hopfield neural networks with distributed parameters by using average Lyapunov function. The asymptotic stability of stochastic reaction- diffusion systems was also established in [5]. The asymptotic behavior of several classes of neural networks with reaction-diffusion terms has been reported in [6-9]. Hu et al. [10] discussed the exponential stability and synchronization of delay neural networks with reaction-diffusion terms by impulsive control.

However, the parameters in neural networks are always some uncertainty and error. Taking these uncertainty and error into account, Xu et al. [11] investigated stochastic exponential robust stability of interval neural networks with reaction-diffusion terms and mixed delays by applying the vector Lyapunov function method and *M*-matrix theory. Wang and Gao [12] studied global exponential robust stability of reaction-diffusion interval neural networks with timevarying delays by means of the topological degree theory and Lyapunov functional method. And, a sufficient condition was presented for robust global exponential stability of interval reaction-diffusion Hopfield neural networks with distributed delays by constructing Lyapunov functional and utilizing some inequality techniques [13].

The neural networks driven by continuous-time Markov Chains have been also used to model many practical neural networks because they may experience abrupt changes in their structure and parameters caused by phenomena such as component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbances. The exponential stability and stabilization of recurrent neural networks with Markovian jumping were discussed in [14– 20]. Robust stability of stochastic delayed additive neural networks with Markov jumping was investigated in [21]. Mao [22] studied the stability of stochastic delay interval system with Markovian jumping by linear matrix inequality.

Many control approaches have been developed to stable and synchronized system such as impulsive control [23] and intermittent control [24–29]. Gan [24–26] revealed exponential synchronization of three classes of stochastic delay neural networks via periodically intermittent control. Hu et al. [27, 28] investigated exponential stabilization and synchronization of delay neural networks. Huang et al. [29] studied stabilization of delayed chaotic neural networks by periodically intermittent control.

In this paper, we will consider a class of stochastic delay interval recurrent neural networks with distributed parameters and Markovian switching whose active functions are more general than the Lipschitz continuous active function [24–26] and the monotone active function [27–29]. By the average Lyapunov functional and periodically intermittent control, several sufficient conditions ensuring robust exponential stabilization are given. Therefore, the organization of this paper is as follows. Some preliminaries and introduction are given in Section 2. In Section 3, robust exponential stabilization of these stochastic neural networks is proved. An example with numerical simulation is given to illustrate the effectiveness of the obtained results in Section 4.

2. Preliminaries

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right-continuous and \mathcal{F}_0 contains all \mathbb{P} -null sets). Let \mathbb{R}^m be the *m*-dimensional Euclidean space and let $|\cdot|$ be the Euclidean norm in \mathbb{R}^m , $\mathbb{R}_+ = [0, +\infty)$ and $\tau > 0$. Assuming that $\Omega_0 \subset \mathbb{R}^m$ is a bounded compact set with smooth boundary $\partial \Omega_0$ and mes $\Omega_0 > 0$ in space \mathbb{R}^m . Let $C([-\tau, 0] \times$ Ω_0 ; \mathbb{R}^n) denote the family of continuous function $\phi(t, x)$ from $[-\tau, 0] \times \Omega_0$ to \mathbb{R}^n with $\|\phi\| = \sup_{-\tau \le t \le 0, x \in \Omega_0} |\phi(t, x)|$. Denote by $C^b_{\mathscr{F}_0}([-\tau,0] \times \Omega_0,\mathbb{R}^n)$ the family of all bounded, \mathscr{F}_0 measurable, $C([-\tau, 0] \times \Omega_0; \mathbb{R}^n)$ -valued random variables. Let $W(t), t \ge 0$ be *n*-dimension Brownian motion defined on the probability space. Let $r(t), t \ge 0$ be right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, ..., N\}$ with generator $\Gamma = (r_{ij})_{N \times N}$ given by

$$\mathbb{P}\left\{r\left(t+\Delta\right)=j\mid r\left(t\right)=i\right\}$$
$$=\begin{cases} \gamma_{ij}\Delta+o\left(\Delta\right) & \text{if } i\neq j\\ 1+\gamma_{ii}\Delta+o\left(\Delta\right) & \text{if } i=j, \end{cases}$$
(1)

where $\Delta > 0$. Here, $r_{ij} \ge 0$ is the transition rate from *i* to *j* if $i \ne j$ while

$$\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij}.$$
 (2)

We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $W(\cdot)$. It is well known that almost every sample path of r(t) is right-continuous step function with a finite number of simple jumps in any finite subinterval \mathbb{R}_+ .

In this paper, we consider a class of stochastic delay interval recurrent neural networks with distributed parameters and Markovian jumping:

$$du_{i}(t,x) = \left\{ \sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}(r(t)) \frac{\partial u_{i}(t,x)}{\partial x_{k}} \right) - a_{i}(r(t)) u_{i}(t,x) + \sum_{j=1}^{n} b_{ij}(r(t)) f_{j}(u_{j}(t,x)) + \sum_{j=1}^{n} c_{ij}(r(t)) g_{j}(u_{j}(t-\tau_{ij}(t)),x) \right\} dt + \sum_{j=1}^{n} h_{ij}(u_{j}(t,x), u_{j}(t-\tau_{ij}(t),x)) dW_{j}(t),$$
(3)

for $t \ge 0, i = 1, 2, ..., n$, where $n \ge 2$ denotes the number of neurons in neural networks. $x = (x_1, x_2, \dots, x_m)^T \in \Omega_0 \subset$ \mathbb{R}^m is the space variable, $\Omega_0 = \{x = (x_1, x_2, \dots, x_m)^T | |x_k| < 0\}$ $\theta_k, k = 1, 2, \dots, m$ is a bounded compact set with smooth boundary $\partial \Omega_0$, and mes $\Omega_0 > 0$ in space \mathbb{R}^m . $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$ corresponds to the state variable of the *i*th neural in space x and at time t. $D_{ik}(r(t)) \geq 0$ denotes the transmission diffusion operator along the *i*th neuron. $a_i(r(t)) > 0$ denotes the changing time constant or passive decay rate of the *i*th neuron. $b_{ii}(r(t))$ and $c_{ii}(r(t))$ denote the connection weight and the delayed connection weight of the *j*th neuron on the *i*th neuron, respectively. $\tau_{ii}(t)$ corresponds to the transmission delay and satisfies $0 \le \tau_{ij}(t) \le \tau$, $\dot{\tau}_{ij}(t) \le \tau_0 < 1$ for all $t \ge 0$ $(\tau, \tau_0 \text{ is a constant}, i, j = 1, 2, ..., n)$. $h_{ij}(\cdot, \cdot)$ denotes stochastic perturbation function to the neuron.

The boundary condition of system (3),

$$u(t,x)|_{\partial\Omega_0} = 0, \quad (t,x) \in [-\tau,+\infty) \times \partial\Omega_0, \ i = 1,2,\dots,n.$$
(4)

The initial value of system (3),

$$u(t,x) = \phi_i(t,x), \quad (t,x) \in [-\tau,0) \times \Omega_0, \ i = 1, 2, \dots, n.$$
(5)

Moreover, $A(r(t)) = \text{diag}(a_1(r(t)), \dots, a_n(r(t)))$, $B(r(t)) = (b_{ij}(r(t)))_{n \times n}$, and $C(r(t)) = (c_{ij}(r(t)))_{n \times n}$ are the interval connection weight matrix for each value of r(t) in \mathbb{S} with the initial value $r(0) = r_0$; $D(r(t)) = (b_{ij}(r(t)))_{n \times m}$ is interval transmission diffusion operator matrix for each value of r(t) in \mathbb{S} with the initial value $r(0) = r_0$.

For convenience, we give the following notions that for r(t) = l in S:

$$\begin{aligned} A^* &= \left\{ A \left(l \right) = \text{diag} \left(a_1 \left(l \right), \dots, a_n \left(l \right) \right) : \underline{A} \left(l \right) \leq A \left(l \right) \leq \overline{A} \left(l \right), \\ &\text{i.e., } \underline{a}_i \left(l \right) \leq a_i \left(l \right) \leq \overline{a}_i \left(l \right), i = 1, 2, \dots, n \right\}; \\ B^* &= \left\{ B \left(l \right) = \left(b_{ij} \left(l \right) \right)_{n \times n} : \underline{B} \left(l \right) \leq B \left(l \right) \leq \overline{B} \left(l \right), \\ &\text{i.e., } \underline{b}_{ij} \left(l \right) \leq b_{ij} \left(l \right) \leq \overline{b}_{ij} \left(l \right), i, j = 1, 2, \dots, n \right\}; \\ C^* &= \left\{ C \left(l \right) = \left(c_{ij} \left(l \right) \right)_{n \times n} : \underline{C} \left(l \right) \leq C \left(l \right) \leq \overline{C} \left(l \right), \\ &\text{i.e., } \underline{c}_{ij} \left(l \right) \leq c_{ij} \left(l \right) \leq \overline{c}_{ij} \left(l \right), i, j = 1, 2, \dots, n \right\}; \\ D^* &= \left\{ D \left(l \right) = \left(d_{ij} \left(l \right) \right)_{n \times n} : \underline{D} \left(l \right) \leq D \left(l \right) \leq \overline{D} \left(l \right), \\ &\text{i.e., } \underline{d}_{ij} \left(l \right) \leq d_{ij} \left(l \right) \leq \overline{d}_{ij} \left(l \right), \\ &i = 1, 2, \dots, n, j = 1, 2, \dots, m \right\}. \end{aligned}$$

$$(6)$$

Definition 1. The stochastic vector $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$ is called the solution of system (3)–(5), if it satisfies the following conditions:

(i) u(t, x) is adapted to $\{\mathscr{F}_t\}_{t\geq 0}$; (ii) for every $T_0 \in \mathbb{R}_+, u(t, x) \in C^b_{\mathscr{F}_0}([0, T_0] \times \Omega_0; \mathbb{R}^n)$ and

$$\mathbb{E}\left(\max_{x\in\Omega_{0}}\int_{0}^{T_{0}}\left[\left|u\left(t,x\right)\right|^{2}+\left|\nabla u\left(t,x\right)\right|^{2}\right]dt\right)<+\infty;\quad(7)$$

(iii) for every $t \in \mathbb{R}_+$,

ſ

$$\begin{split} \int_{\Omega_{0}} u_{i}(t,x) dx \\ &= \int_{\Omega_{0}} \phi_{i}(0,x) dx + \int_{\Omega_{0}} \int_{0}^{t} \sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \\ &\times \left(D_{ik}(r(s)) \frac{\partial u_{i}(s,x)}{\partial x_{k}} \right) ds dx \\ &+ \int_{\Omega_{0}} \int_{0}^{t} \left[-a_{i}(r(s)) u_{i}(s,x) + \sum_{j=1}^{n} b_{ij}(r(s)) f_{j}(u_{j}(s,x)) \right. \\ &\left. + \sum_{j=1}^{n} c_{ij}(r(s)) g_{j}(u_{j}(s-\tau_{ij}(s),x)) \right] ds dx \\ &+ \int_{\Omega_{0}} \int_{0}^{t} \sum_{j=1}^{n} h_{ij}(u_{j}(s,x), u_{j}(s-\tau_{ij}(s),x)) dW_{j}(s) dx, \\ &(t,x) \in [0,T_{0}] \times \Omega_{0}, \end{split}$$
(8)

so it holds as \mathbb{P} -a.s., $i = 1, 2, \ldots, n$.

Definition 2. System (3)–(5) is called robust exponential stable in *p*th moment for any $A(l) \in A^*$, $B(l) \in B^*$, $C(l) \in C^*$, $D(l) \in D^*$, $l \in S$ if the solution u(t, x) of system (3)–(5) satisfies

$$\lim_{t \to +\infty} \sup_{t} \frac{1}{t} \log \left(\mathbb{E} \left\| u\left(t, x\right) \right\|^{p} \right) < 0, \tag{9}$$

where $||u(t, x)|| = (\int_{\Omega_0} |u(t, x)|^p dx)^{1/p}$, $(t, x) \in \mathbb{R}_+ \times \Omega_0$.

To assure the existence and uniqueness of the solution to system (3)-(5) (see, [30, 31]), we give the following assumptions:

(H1) for $i = 1, 2, ..., n, \forall s_1, s_2 \in \mathbb{R}$, the neuron activation functions f_i, g_i are bounded, $f_i(0) = g_i(0) = 0$, and satisfy

$$L_{i}^{-} \leq \frac{f_{i}(s_{1}) - f_{i}(s_{2})}{s_{1} - s_{2}} \leq L_{i}^{+},$$

$$N_{i}^{-} \leq \frac{g_{i}(s_{1}) - g_{i}(s_{2})}{s_{1} - s_{2}} \leq N_{i}^{+},$$
(10)

where $s_1 \neq s_2$, and $L_i^-, L_i^+, N_i^-, N_i^+$ are constants.

(H2) For i, j = 1, 2, ..., n, $\forall s_1, s_2, \tilde{s}_1, \tilde{s}_2 \in \mathbb{R}$, there exists positive constant σ_{ij} , such that

$$\left|h_{ij}(s_{1},s_{2})-h_{ij}(\tilde{s}_{1},\tilde{s}_{2})\right|^{2} \leq \sigma_{ij}\left(\left|s_{1}-\tilde{s}_{1}\right|^{2}+\left|s_{2}-\tilde{s}_{2}\right|^{2}\right)$$
(11)

and $h_{ij}(0,0) = 0$.

(H3) Time-varying delay function $\tau_{ij}(\cdot)$: $[0, +\infty) \rightarrow [0, +\infty)$ (i, j = 1, 2, ..., n) satisfies

$$0 \le \tau_{ii}(t) \le \tau, \qquad \dot{\tau}_{ii}(t) \le \tau_0 \le 1, \tag{12}$$

for $t \ge 0$, where τ and τ_0 are constants.

It is well known, if the parameters or time-varying delay in neural networks is appropriately chosen, neural networks may lead to some phenomena such as instability, divergence, oscillation, chaos [32, 33].

In order to stabilize the origin of system (3)–(5), we introduce the following periodically intermittent controller:

$$v_{i}(t,x) = \begin{cases} \sum_{j=1}^{n} k_{ij} u_{j}(t,x), & MT \le t < MT + \delta, \\ 0, & MT + \delta \le t < (M+1)T, \end{cases}$$
(13)

where M = 0, 1, 2, ... and k_{ij} is the control gains for i, j = 1, 2, ..., n, T denotes the control period, and $0 < \delta < T$ is called the control width.

Then, system (3) under the periodically intermittent controller (13) is described by the following equations:

$$\begin{aligned} du_i\left(t,x\right) &= \left\{ \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{ik}\left(l\right) \frac{\partial u_i\left(t,x\right)}{\partial x_k} \right) \right. \\ &\quad - a_i\left(r\left(t\right)\right) u_i\left(t,x\right) + \sum_{j=1}^n b_{ij}\left(r\left(t\right)\right) f_j\left(u_j\left(t,x\right)\right) \\ &\quad + \sum_{j=1}^n c_{ij}\left(r\left(t\right)\right) g_j\left(u_j\left(t-\tau_{ij}\left(t\right)\right),x\right) \\ &\quad + \sum_{j=1}^n k_{ij}u_j\left(t,x\right) \right\} dt \\ &\quad + \sum_{j=1}^n h_{ij}\left(u_j\left(t,x\right), u_j\left(t-\tau_{ij}\left(t\right),x\right)\right) dW_j\left(t\right) \\ &\qquad MT \le t < MT + \delta, \end{aligned}$$

$$du_{i}(t,x) = \left\{ \sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}\left(l\right) \frac{\partial u_{i}\left(t,x\right)}{\partial x_{k}} \right) - a_{i}\left(r\left(t\right)\right) u_{i}\left(t,x\right) + \sum_{j=1}^{n} b_{ij}\left(r\left(t\right)\right) f_{j}\left(u_{j}\left(t,x\right)\right) + \sum_{j=1}^{n} c_{ij}\left(r\left(t\right)\right) g_{j}\left(u_{j}\left(t-\tau_{ij}\left(t\right)\right),x\right) \right\} dt + \sum_{j=1}^{n} h_{ij}\left(u_{j}\left(t,x\right), u_{j}\left(t-\tau_{ij}\left(t\right),x\right)\right) dW_{j}\left(t\right)$$

$$MT + \delta \leq t < (M+1)T.$$
(14)

Lemma 3 (see [10]). Let $p \ge 2$ be a positive integer, let θ_k be a positive constant, let Ω_0 be a cube $|x_k| \le \theta_k$ for k = 1, 2, ..., m, and let u(x) be a real-valued function belonging to $C^1(\Omega_0)$ which vanish on the boundary $\partial \Omega_0$; that is, $u(x)|_{\partial \Omega_0} = 0$; then

$$\int_{\Omega_0} |u(x)|^p dx \le \frac{p^2 \theta_k^2}{4} \int_{\Omega_0} |u(x)|^{p-2} \left| \frac{\partial u}{\partial x_k} \right|^2 dx.$$
(15)

3. Robust Exponential Stabilization

In this section, we design suitable *T*, δ , and k_{ij} such that system (3)–(5) under the external controller (13) can realize

robust exponential stability in *p*th moment. For convenience, we give some denotations as follows:

$$\begin{split} \lambda_{i} &= \min_{l \in \mathbb{S}} \mu_{l} \left\{ \sum_{k=1}^{m} \frac{4\left(p-1\right) \underline{D}_{ik}}{p \theta_{k}^{2}} + p a_{i}^{*} \right. \\ &- \sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \left(\tilde{b}_{ij}^{p \alpha_{\ell i j}} \tilde{L}_{j}^{p \beta_{\ell i j}} + \tilde{c}_{ij}^{p \xi_{\ell i j}} \widetilde{N}_{j}^{p \zeta_{\ell i j}} \right) \\ &- \frac{p-1}{2} \sum_{j=1}^{n} \sum_{\ell=1}^{p-2} \left(\sigma_{ij}^{p \epsilon_{\ell i j}} + \sigma_{ij}^{p \omega_{\ell i j}} \right) \\ &- \sum_{j=1}^{n} \left(\tilde{b}_{ji}^{p \alpha_{p j i}} \tilde{L}_{i}^{p \beta_{p j i}} \right. \\ &+ \tilde{c}_{ji}^{p \xi_{p j i}} \widetilde{N}_{i}^{p \zeta_{p j i}} \\ &+ \left. \frac{p-1}{2} \left(\sigma_{ji}^{p \epsilon_{(p-1) j i}} + \sigma_{ji}^{p \epsilon_{p j i}} \right) \right) \right\}, \end{split}$$
(16)

$$\begin{aligned} \kappa_{i} &= \min_{l \in \mathbb{S}} \mu_{l} \left\{ \sum_{k=1}^{m} \frac{4\left(p-1\right) \underline{D}_{ik}}{p \theta_{k}^{2}} + p a_{i}^{*} \right. \\ &- \sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \left(\widetilde{b}_{ij}^{p \overline{\alpha}_{\ell i j}} \widetilde{L}_{j}^{p \overline{\beta}_{\ell i j}} + \widetilde{c}_{ij}^{p \overline{\xi}_{\ell i j}} \widetilde{N}_{j}^{p \overline{\zeta}_{\ell i j}} \right) \\ &- \frac{p-1}{2} \sum_{j=1}^{n} \sum_{\ell=1}^{p-2} \left(\sigma_{ij}^{p \overline{e}_{\ell i j}} + \sigma_{ij}^{p \overline{\omega}_{\ell i j}} \right) \\ &- \sum_{j=1}^{n} \left(\widetilde{b}_{ji}^{p \overline{\alpha}_{p j i}} \widetilde{L}_{i}^{p \overline{\beta}_{p j i}} + \widetilde{c}_{ji}^{p \overline{\xi}_{p j i}} \widetilde{N}_{i}^{p \overline{\zeta}_{p j i}} \right. \\ &+ \frac{p-1}{2} \left(\overline{\sigma}_{ji}^{p \overline{e}_{(p-1) j i}} + \sigma_{ji}^{p \overline{e}_{p j i}} \right) \right\}, \end{aligned}$$

$$(17)$$

$$\nu_{i} = \max_{l \in \mathbb{S}} \mu_{l} \left[pk_{ii} + \sum_{\substack{j=1 \ j \neq i}}^{n} \sum_{\ell=1}^{p-1} |k_{ij}|^{p\eta_{\ell ij}^{*}} + \sum_{\substack{j=1 \ j \neq i}}^{n} |k_{ji}|^{p\eta_{pji}^{*}} \right],$$

$$(18)$$

$$\eta_{i} = \max_{l \in \mathbb{S}} \mu_{l} \sum_{j=1}^{n} \left[\frac{p-1}{2} \left(\sigma_{ji}^{\omega_{(p-1)ji}} + \sigma_{ji}^{\omega_{pji}} \right) + \widetilde{c}_{ji}^{p\xi_{pji}} \widetilde{N}_{i}^{p\zeta_{pji}} \right],$$

$$(19)$$

where $a_i^* = \min_{l \in \mathbb{S}} a_i(l)$, $\tilde{b}_{ij} = \max_{l \in \mathbb{S}} \check{b}_{ij}(l)$, $\check{b}_{ij}(l) = \max\{|\underline{b}_{ij}(l)|, |\overline{b}_{ij}(l)|\}, \tilde{c}_{ij} = \max_{l \in \mathbb{S}} \check{c}_{ij}(l)$,

$$\check{c}_{ij}(l) = \max\left\{ \left| \underline{c}_{ij}(l) \right|, \left| \overline{c}_{ij}(l) \right| \right\}, \qquad (20)$$

$$\widetilde{L}_{j} = \max\left\{ \left| L_{j}^{-} \right|, \left| L_{j}^{+} \right| \right\}, \qquad \widetilde{N}_{j} = \max\left\{ \left| N_{j}^{-} \right|, \left| N_{j}^{+} \right| \right\},$$

 $\mu_l > 0$, and $\alpha_{\ell i j}$, $\beta_{\ell i j}$, $\xi_{\ell i j}$, $\zeta_{\ell i j}$, $\epsilon_{\ell i j}$, $w_{\ell i j}$, $\eta_{\ell i j}^*$, $\overline{\alpha}_{\ell i j}$, $\overline{\beta}_{\ell i j}$, $\overline{\xi}_{\ell i j}$, $\overline{\xi}_{\ell i j}$, $\overline{\epsilon}_{\ell i j}$, and $\overline{w}_{\ell i j}$ are nonnegative constants, satisfying

$$\sum_{\ell=1}^{p} \alpha_{\ell i j} = \sum_{\ell=1}^{p} \beta_{\ell i j} = \sum_{\ell=1}^{p} \xi_{\ell i j} = \sum_{\ell=1}^{p} \zeta_{\ell i j}$$

$$= \sum_{\ell=1}^{p} \epsilon_{\ell i j} = \sum_{\ell=1}^{p} w_{\ell i j} = \sum_{\ell=1}^{p} \overline{\alpha}_{\ell i j} = \sum_{\ell=1}^{p} \overline{\beta}_{\ell i j}$$

$$= \sum_{\ell=1}^{p} \overline{\xi}_{\ell i j} = \sum_{\ell=1}^{p} \overline{\zeta}_{\ell i j} = \sum_{\ell=1}^{p} \overline{\epsilon}_{\ell i j} = \sum_{\ell=1}^{p} \overline{w}_{\ell i j}$$

$$= \sum_{\ell=1}^{p} \eta_{\ell i j}^{*} = 1, \qquad \underline{D}_{i k} = \min_{l \in \mathbb{S}} \underline{D}_{i k} (l).$$

$$(21)$$

In the following, we give an assumption:

(H4) $\lambda_i - \nu_i - \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q - \eta_i / (1 - \tau_0) > 0$ and there exists $\rho_i > 0$ such that

$$\kappa_i + \rho_i - \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q - \frac{\eta_i}{1 - \tau_0} > 0, \quad i = 1, 2, \dots, n.$$
 (22)

We consider the function

$$H_{i}\left(\check{\varepsilon}_{i}\right) = \lambda_{i} - \nu_{i} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \check{\varepsilon}_{i} \max_{l \in \mathbb{S}} \mu_{l} - \frac{\eta_{i} e^{\check{\varepsilon}_{i} \tau}}{1 - \tau_{0}},$$

$$i = 1, 2, \dots, n.$$
(23)

It is easy to see that

$$H_{i}'(\check{e}_{i}) = -\max_{l \in \mathbb{S}} \mu_{l} - \frac{\tau \eta_{i} e^{\check{e}_{i}\tau}}{1 - \tau_{0}} < 0,$$

$$H_{i}(0) = \lambda_{i} - \gamma_{i} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \frac{\eta_{i}}{1 - \tau_{0}} > 0.$$
(24)

On the other hand, $H_i(\check{\varepsilon}_i)$ is continuous on $[0, +\infty)$, and $F_i(\check{\varepsilon}_i) \rightarrow -\infty$ as $\check{\varepsilon}_i \rightarrow +\infty$. Then there exists a positive constant $\check{\varepsilon}_i^*$ such that $H_i(\check{\varepsilon}_i^*) \geq 0$ and $H_i(\check{\varepsilon}_i) > 0$, for $\check{\varepsilon}_i \in (0, \check{\varepsilon}_i^*)$.

Let $\check{\varepsilon} = \min_{1 \le i \le n} \{\check{\varepsilon}_i^*\}$; then we have

$$H_{i}(\check{\varepsilon}) = \lambda_{i} - \nu_{i} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \check{\varepsilon} \max_{l \in \mathbb{S}} \mu_{l} - \frac{\eta_{i}}{1 - \tau_{0}} e^{\check{\varepsilon}\tau} \ge 0.$$

$$(25)$$

In similar, there exists a positive constant $\hat{\varepsilon} > 0$, such that

$$F_{i}\left(\widehat{\varepsilon}\right) = k_{i} + \rho_{i} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \widehat{\varepsilon} \max_{l \in \mathbb{S}} \mu_{l} - \frac{\eta_{i}}{1 - \tau_{0}} e^{\widehat{\varepsilon}\tau} \ge 0.$$

$$(26)$$

Let $\varepsilon = \min{\{\check{\varepsilon}, \widehat{\varepsilon}\}}$; we have

$$H_i(\varepsilon) > 0, \quad F_i(\varepsilon) > 0, \quad i = 1, 2, \dots, n.$$
(27)

We give another assumption:

(H5) $\varepsilon - \rho(T - \delta)/\overline{\mu}T > 0$, where $\rho = \max_{1 \le i \le n} \rho_i, \overline{\mu} = \min_{l \in \mathbb{S}} \{\mu_l\}.$

Theorem 4. Under assumptions (H1)–(H5), the origin of system (3)–(5) under periodically intermittent controller (13) is robust exponentially stable in pth moment.

Proof. Let us define the average Lyapunov-Krasovskii functional (see [4]) $V_1 : C([0, +\infty) \times \Omega_0, \mathbb{R}^n) \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}_+$ by

$$V_{1}(u(t,x),r(t),t) = \int_{\Omega_{0}} V(u(t,x),r(t),t) dx$$
 (28)

with

$$=\mu_{r(t)}\sum_{i=1}^{n}e^{\varepsilon t}|u_{i}(t,x)|^{p}+\frac{e^{\varepsilon \tau}}{1-\tau_{0}}\sum_{i=1}^{n}\eta_{i}\int_{t-\tau_{ij}(t)}^{t}e^{\varepsilon s}|u_{i}(s,x)|^{p}ds,$$
(29)

where $\mu_{r(t)} > 0$.

By the generalized Itô formula (see [31]), we have

$$\mathbb{E}V_{1}(u(t,x),r(t),t)$$

$$=\mathbb{E}V_{1}(\phi,r(0),0) + \mathbb{E}\int_{0}^{t}\int_{\Omega_{0}}\mathscr{L}V(u(s,x),r(s),s)\,dxds.$$
(30)

By Lemma 3.1 in [22] and r(t) = l, we get that for $(t, x) \in [MT, MT + \delta) \times \Omega_0$

$$\begin{aligned} \mathscr{L}V\left(u\left(t,x\right),l,t\right) \\ &= \varepsilon\mu_{l}\sum_{i=1}^{n}e^{\varepsilon t}\left|u_{i}\left(t,x\right)\right|^{p} + p\mu_{l}e^{\varepsilon t}\sum_{i=1}^{n}\left|u_{i}\left(t,x\right)\right|^{p-1} \\ &\times \left\{\sum_{k=1}^{m}\frac{\partial}{\partial x_{k}}\left(D_{ik}\left(l\right)\frac{\partial u_{i}\left(t,x\right)}{\partial x_{k}}\right) - a_{i}\left(l\right)u_{i}\left(t,x\right) \\ &+ \sum_{j=1}^{n}b_{ij}\left(l\right)f_{j}\left(u_{j}\left(t,x\right)\right) \\ &+ \sum_{j=1}^{n}c_{ij}\left(l\right)g_{j}\left(u_{j}\left(t-\tau_{ij}\left(t\right),x\right)\right) \\ &+ \sum_{j=1}^{n}k_{ij}u_{j}\left(t,x\right)\right\} \end{aligned}$$

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$$\begin{aligned} &+ \mu_{l} e^{\varepsilon t} \frac{p\left(p-1\right)}{2} \sum_{i=1}^{n} |u_{i}\left(t,x\right)|^{p-2} \\ &\times \sum_{j=1}^{n} h_{ij}^{2} \left(u_{j}\left(t,x\right), u_{j}\left(t-\tau_{ij}\left(t\right),x\right)\right) \\ &+ \sum_{q=1}^{N} \gamma_{lq} \mu_{q} e^{\varepsilon t} \sum_{i=1}^{n} |u_{i}\left(t,x\right)|^{p} \\ &+ \frac{e^{\varepsilon \tau}}{1-\tau_{0}} \sum_{i=1}^{n} \eta_{i} e^{\varepsilon t} |u_{i}\left(t,x\right)|^{p} \\ &- \frac{e^{\varepsilon \tau}}{1-\tau_{0}} \sum_{i=1}^{n} \eta_{i} e^{\varepsilon (t-\tau_{ij}\left(t\right))} \\ &\times \left|u_{i}\left(t-\tau_{ij}\left(x\right),x\right)\right|^{p} \left(1-\dot{\tau}_{ij}\left(t\right)\right) \\ &+ \sum_{q=1}^{N} \gamma_{lq} \frac{e^{\varepsilon \tau}}{1-\tau_{0}} \sum_{i=1}^{n} \eta_{i} \int_{t-\tau_{ij}\left(t\right)}^{t} e^{\varepsilon s} |u_{i}\left(s,x\right)|^{p} ds. \end{aligned}$$

By the fundamental inequality $|a + b| \le |a| + |b|$, we have

$$\begin{split} \mathscr{L}V(u(t,x),l,t) \\ &\leq \sum_{i=1}^{n} e^{\varepsilon t} \left\{ \varepsilon \mu_{l} |u_{i}(t,x)|^{p} + p\mu_{l} |u_{i}(t,x)|^{p-1} \\ &\times \left(\sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}(l) \right) \frac{\partial u_{i}(t,x)}{\partial x_{k}} \right) \\ &+ \left(k_{ii} - a_{i}(l) \right) p\mu_{l} |u_{i}(t,x)|^{p} \\ &+ p\mu_{l} |u_{i}(t,x)|^{p-1} \left(\sum_{j=1}^{n} \left| b_{ij}(l) \right| \left| f_{j}\left(u_{j}(t,x) \right) \right| \right) \right) \\ &+ p\mu_{l} |u_{i}(t,x)|^{p-1} \\ &\times \left(\sum_{j=1}^{n} \left| c_{ij}(l) \right| \left| g_{j}\left(\mu_{j}\left(t - \tau_{ij}(t) \right), x \right) \right| \right) \\ &+ \sum_{\substack{j=1\\j \neq i}}^{n} p\mu_{l} |u_{i}(t,x)|^{p-1} \left| k_{ij} \right| \left| \mu_{j}(t,x) \right| \\ &+ \mu_{l} \frac{p\left(p - 1 \right)}{2} |u_{i}(t,x)|^{p-2} \\ &\times \sum_{j=1}^{n} \sigma_{ij} \left(\left| u_{j}(t,x) \right|^{2} + \left| u_{j}\left(t - \tau_{ij}(t), x \right) \right|^{2} \right) \\ &+ \sum_{q=1}^{N} \gamma_{lq} \mu_{q} |u_{i}(t,x)|^{p} \end{split}$$

$$+ \frac{e^{\varepsilon\tau}}{1-\tau_0} \eta_i |u_i(t,x)|^p -\eta_i |u_i(t-\tau_{ij}(t),x)|^p \bigg\},$$
(32)

(31)

where we use $\sum_{q=1}^{N} \gamma_{lq} = 0$. By using the fundamental inequality $a_1^p + a_2^p + \dots + a_p^p \ge pa_1a_2, \dots, a_p$ $(a_i \ge 0, i = 1, 2, \dots, p)$, we have

$$\begin{split} p\mu_{l}|u_{i}(t,x)|^{p-1} \left(\sum_{j=1}^{n} \left|b_{ij}(l)\right| \left|f_{j}\left(u_{j}(t,x)\right)\right|\right) \\ &\leq \mu_{l}\sum_{j=1}^{n} p|u_{i}(t,x)|^{p-1} \widetilde{b}_{ij}\widetilde{L}_{j}\left|u_{j}(t,x)\right| \\ &= \mu_{l}\sum_{j=1}^{n} p\left[\prod_{\ell=1}^{p-1} \widetilde{b}_{ij}^{\alpha_{\ell i j}} \widetilde{L}_{j}^{\beta_{\ell i j}}\left|u_{i}(t,x)\right|\right] \\ &\times \left(\widetilde{b}_{ij}^{\alpha_{p i j}} \widetilde{L}_{j}^{\beta_{p i j}}\left|u_{j}(t,x)\right|\right) \\ &\leq \mu_{l}\sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \widetilde{b}_{ij}^{p\alpha_{\ell i j}} \widetilde{L}_{j}^{p\beta_{\ell i j}}\left|u_{i}(t,x)\right|^{p} \\ &+ \mu_{l}\sum_{j=1}^{n} \widetilde{b}_{ij}^{p\alpha_{p i j}} \widetilde{L}_{j}^{p\beta_{p i j}}\left|u_{j}(t,x)\right|^{p}, \\ p\mu_{l}|u_{i}(t,x)|^{p-1} \\ &\times \left(\sum_{j=1}^{n} \left|c_{ij}(l)\right| \left|g_{j}\left(u_{j}\left(t-\tau_{ij}(t),x\right)\right)\right|\right) \end{split}$$

$$\leq p \mu_l \sum_{j=1}^n |u_i(t,x)|^{p-1} \widetilde{c}_{ij} \widetilde{N}_j |u_j(t-\tau_{ij}(t),x)|$$

$$= \mu_l \sum_{j=1}^n p \left[\prod_{\ell=1}^{p-1} \widetilde{c}_{ij}^{\xi_{\ell i j}} \widetilde{N}_j^{\zeta_{\ell i j}} |u_j(t,x)| \right]$$

$$\times \left(\widetilde{c}_{ij}^{\xi_{p i j}} \widetilde{N}_j^{\zeta_{p i j}} |u_j(t-\tau_{ij}(t),x)| \right)$$

$$\leq \mu_l \sum_{j=1}^n \sum_{\ell=1}^{p-1} \widetilde{c}_{ij}^{p \xi_{\ell i j}} \widetilde{N}_j^{p \zeta_{\ell i j}} |u_j(t,x)|^p$$

$$+ \mu_l \sum_{j=1}^n \widetilde{c}_{ij}^{p \xi_{p i j}} \widetilde{N}_j^{p \zeta_{p i j}} |u_j(t-\tau_{ij}(t),x)|^p.$$

(33)

Similarly, we have

Further, we also have

$$\begin{split} \mu_{l} \frac{p(p-1)}{2} |u_{i}(t,x)|^{p-2} \\ \times \sum_{j=1}^{n} \sigma_{ij} |u_{j}(t-\tau_{ij}(t),x)|^{2} \\ = \mu_{l} \frac{p-1}{2} \sum_{j=1}^{n} p \left[\prod_{\ell=1}^{p-2} \sigma_{ij}^{\omega_{\ell ij}} |u_{i}(t,x)| \right] \\ \times \left(\sigma_{ij}^{\omega_{(p-1)ij}} |u_{j}(t-\tau_{ij}(t),x)| \right) \\ \times \left(\sigma_{ij}^{\omega_{pij}} |u_{j}(t-\tau_{ij}(t),x)| \right) \end{split}$$

(34)

$$\leq \mu_{l} \frac{p-1}{2} \sum_{j=1}^{n} \sum_{\ell=1}^{p-2} \left| \sigma_{ij} \right|^{p \omega_{\ell i j}} \left| u_{i}\left(t,x\right) \right|^{p} \\ + \mu_{l} \frac{p-1}{2} \sum_{j=1}^{n} \left(\sigma_{ij}^{\omega_{(p-1)i j}} + \sigma_{ij}^{\omega_{p i j}} \right) \\ \times \left| u_{j} \left(t - \tau_{ij}\left(t\right), x\right) \right|^{p}.$$
(35)

Substituting (33)–(35) into (32), we obtain

$$\begin{split} \mathscr{L}V\left(u\left(t,x\right),l,t\right) \\ &\leq \sum_{i=1}^{n} e^{\varepsilon t} \left\{ \left[\left[\mu_{l} + p\mu_{l}k_{ii} - p\mu_{l}a_{i}^{*} \right. \\ &+ \sum_{q=1}^{N} \gamma_{lq}\mu_{q} + \frac{e^{\varepsilon \tau}\eta_{i}}{1 - \tau_{0}} \right. \\ &+ \mu_{l} \sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \left(\tilde{b}_{ij}^{p\alpha_{\ell ij}} \tilde{L}_{j}^{p\beta_{\ell ij}} + \tilde{c}_{ij}^{p\xi_{\ell ij}} \tilde{N}_{j}^{p\xi_{\ell ij}} \right) \\ &+ \mu_{l} \sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \left| k_{ij} \right|^{p\eta_{\ell ij}^{*}} \\ &+ \mu_{l} \frac{p-1}{2} \sum_{j=1}^{n} \sum_{\ell=1}^{p-2} \left(\sigma_{ij}^{p\varepsilon_{\ell ij}} + \sigma_{ij}^{p\omega_{\ell ij}} \right) \right] \\ &\times \left| u_{i}\left(t,x\right) \right|^{p} \\ &+ \mu_{l} \sum_{j=1}^{n} \left[\tilde{b}_{ij}^{p\alpha_{ij}} \tilde{L}_{j}^{p\beta_{pij}} + \tilde{c}_{ij}^{p\xi_{pij}} \widetilde{N}_{j}^{p\xi_{pij}} \right. \\ &\left. + \frac{p-1}{2} \left(\sigma_{ij}^{p\varepsilon_{(-1)ij}} + \sigma_{ij}^{p\varepsilon_{pij}} \right) \right] \\ &\times \left| u_{i}\left(t,x\right) \right|^{p} + \mu_{l} \sum_{j=1}^{n} \left[\kappa_{ij} \right]^{p\eta_{pij}^{*}} \left| u_{j}\left(t,x\right) \right|^{p} \\ &+ \mu_{l} \frac{p-1}{2} \sum_{j=1}^{n} \left(\sigma_{ij}^{\omega_{(-1)ij}} + \sigma_{ij}^{\omega_{pij}} \right) \right] \\ &\times \left| u_{j}\left(t - \tau_{ij}\left(t\right),x\right) \right|^{p} \\ &+ \mu_{l} \sum_{j=1}^{n} \tilde{c}_{ij}^{p\xi_{pij}} \widetilde{N}_{j}^{p\xi_{pij}} \left| u_{j}\left(t - \tau_{ij}\left(t\right),x\right) \right|^{p} \\ &- \eta_{i} \left| u_{i}\left(t,x\right) \right|^{p-1} \end{split}$$

$$\times \left(\sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}\left(l\right) \frac{\partial u_{i}\left(t,x\right)}{\partial x_{k}} \right) \right)$$

$$\leq \sum_{i=1}^{n} e^{\varepsilon t} \left\{ \varepsilon \max_{l \in \mathbb{S}} \mu_{l} + \nu_{i} - \overline{\lambda}_{i} + \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} + \frac{e^{\varepsilon \tau} \eta_{i}}{1 - \tau_{0}} \right\}$$

$$\times \left| u_{i}\left(t,x\right) \right|^{p} + \sum_{i=1}^{n} e^{\varepsilon t} p \mu_{l} \left| u_{i}\left(t,x\right) \right|^{p-1}$$

$$\times \left(\sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}\left(l\right) \frac{\partial u_{i}\left(t,x\right)}{\partial x_{k}} \right) \right),$$

$$(36)$$

where

$$\begin{split} \overline{\lambda}_{i} &= \min_{l \in \mathbb{S}} \mu_{l} \left\{ pa_{i}^{*} - \sum_{j=1}^{n} \sum_{\ell=1}^{p-1} \left(\widetilde{b}_{ij}^{p\alpha_{\ell ij}} \widetilde{L}_{j}^{p\beta_{\ell ij}} + \widetilde{c}_{ij}^{p\xi_{\ell ij}} \widetilde{N}_{j}^{p\xi_{\ell ij}} \right) \\ &- \frac{p-1}{2} \sum_{j=1}^{n} \sum_{\ell=1}^{p-2} \left(\sigma_{ij}^{p\epsilon_{\ell ij}} + \sigma_{ij}^{p\omega_{\ell ij}} \right) \\ &- \sum_{j=1}^{n} \left(\widetilde{b}_{ji}^{p\alpha_{p ji}} \widetilde{L}_{i}^{p\beta_{p ji}} + \widetilde{c}_{ji}^{p\xi_{p ji}} \widetilde{N}_{i}^{p\xi_{p ji}} \\ &+ \frac{p-1}{2} \left(\sigma_{ji}^{p\epsilon_{(p-1)ji}} + \sigma_{ji}^{p\epsilon_{p ji}} \right) \right) \right\}. \end{split}$$
(37)

Substituting (36) into (30), we obtain

$$\mathbb{E}V_{1}\left(u\left(t,x\right),r\left(t\right),t\right)$$

$$\leq \mathbb{E}V_{1}\left(\phi,r\left(0\right),0\right)$$

$$-\mathbb{E}\int_{0}^{t}\int_{\Omega_{0}}\sum_{i=1}^{n}e^{\varepsilon s}\left[\overline{\lambda}_{i}-\varepsilon\max_{l\in\mathbb{S}}\mu_{l}-\nu_{i}\right]$$

$$-\max_{l\in\mathbb{S}}\sum_{q=1}^{N}\gamma_{lq}\mu_{q}-\frac{e^{\varepsilon \tau}\eta_{i}}{1-\tau_{0}}\right] \quad (38)$$

$$\times\left|u_{i}\left(s,x\right)\right|^{p}dxds$$

$$+ \mathbb{E} \int_{0}^{t} \sum_{i=1}^{n} e^{\varepsilon s} \int_{\Omega_{0}} p \mu_{r(s)} |u_{i}(s,x)|^{p-1} \\ \times \left(\sum_{k=1}^{m} \frac{\partial}{\partial x_{k}} \left(D_{ik}(r(s)) \frac{\partial u_{i}(s,x)}{\partial x_{k}} \right) \right) dx ds.$$

By Lemma 3 and the boundary condition (4), we have

$$\int_{\Omega_0} p |u_i(s, x)|^{p-1} \\ \times \left(\sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{ik}(r(s)) \frac{\partial u_i(s, x)}{\partial x_k} \right) \right) dx$$

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$$\leq -\sum_{k=1}^{m} \frac{4(p-1)D_{ik}(r(s))}{p\theta_{k}^{2}} \int_{\Omega_{0}} |u_{i}(s,x)|^{p} dx$$

$$\leq -\sum_{k=1}^{m} \frac{4(p-1)D_{ik}}{p\theta_{k}^{2}} \int_{\Omega_{0}} |u_{i}(s,x)|^{p} dx.$$
(39)

Substituting these into (38), we get

$$\begin{split} \mathbb{E}V_{1}\left(u\left(t,x\right),r\left(t\right),t\right) \\ &\leq \mathbb{E}V_{1}\left(\phi,r\left(0\right),0\right) \\ &- \mathbb{E}\int_{0}^{t}\int_{\Omega_{0}}\sum_{i=1}^{n}e^{\varepsilon s}\left[\overline{\lambda}_{i}+\min_{l\in\mathbb{S}}\mu_{l}\sum_{k=1}^{m}\frac{4\left(p-1\right)\underline{D}_{ik}}{p\theta_{k}^{2}}\right. \\ &\left.-\varepsilon\max_{l\in\mathbb{S}}\mu_{l}-\nu_{i}-\max_{l\in\mathbb{S}}\sum_{q=1}^{N}\gamma_{lq}\mu_{q}\right. \\ &\left.-\frac{e^{\varepsilon \tau}\eta_{i}}{1-\tau_{0}}\right]\left|u_{i}\left(s,x\right)\right|^{p}dxds \\ &\leq \mathbb{E}V_{1}\left(\phi,r\left(0\right),0\right) \end{split}$$

$$-\mathbb{E}\int_{0}^{t}\int_{\Omega_{0}}\sum_{i=1}^{n}e^{\varepsilon s}\left[\lambda_{i}-\varepsilon\max_{l\in\mathbb{S}}\mu_{l}-\nu_{i}\right]$$
$$-\max_{l\in\mathbb{S}}\sum_{q=1}^{N}\gamma_{lq}\mu_{q}-\frac{e^{\varepsilon \tau}\eta_{i}}{1-\tau_{0}}\right]$$
$$\times\left|u_{i}\left(s,x\right)\right|^{p}dxds$$
$$\leq\mathbb{E}V_{1}\left(\phi,r\left(0\right),0\right)\left(t,x\right)\in\left[MT,MT+\delta\right)\times\Omega_{0}.$$

(40)

Similarly, for $(t, x) \in [MT + \delta, (M + 1)T) \times \Omega_0$, we can obtain

$$\mathbb{E}V_{1}\left(u\left(t,x\right),r\left(t\right),t\right)$$

$$\leq \mathbb{E}V_{1}\left(u\left(MT+\delta,x\right),r\left(MT+\delta\right),MT+\delta\right)$$

$$-\mathbb{E}\int_{0}^{t}\int_{\Omega_{0}}\sum_{i=1}^{n}e^{\varepsilon s}\left[\kappa_{i}+\rho_{i}-\varepsilon\max_{l\in\mathbb{S}}\mu_{l}\right]$$

$$-\max_{l\in\mathbb{S}}\sum_{q=1}^{N}\gamma_{lq}\mu_{q}-\frac{e^{\varepsilon \tau}\eta_{i}}{1-\tau_{0}}\right]$$

$$\times\left|u_{i}\left(s,x\right)\right|^{p}dxds$$
(41)

$$+ \mathbb{E} \int_{0}^{t} \int_{\Omega_{0}} \sum_{i=1}^{n} e^{\varepsilon s} \rho_{i} |u_{i}(s,x)|^{p} dx ds$$

$$\leq \mathbb{E} V_{1} (u (MT + \delta, x), r (MT + \delta), MT + \delta)$$

$$+ \frac{\rho}{\overline{\mu}} \mathbb{E} \int_{0}^{t} V_{1} (u (s,x), r (s), s) ds,$$

where $\rho = \max_{1 \le i \le n} \rho_i$, $\overline{\mu} = \min_{l \in \mathbb{S}} \mu_l$.

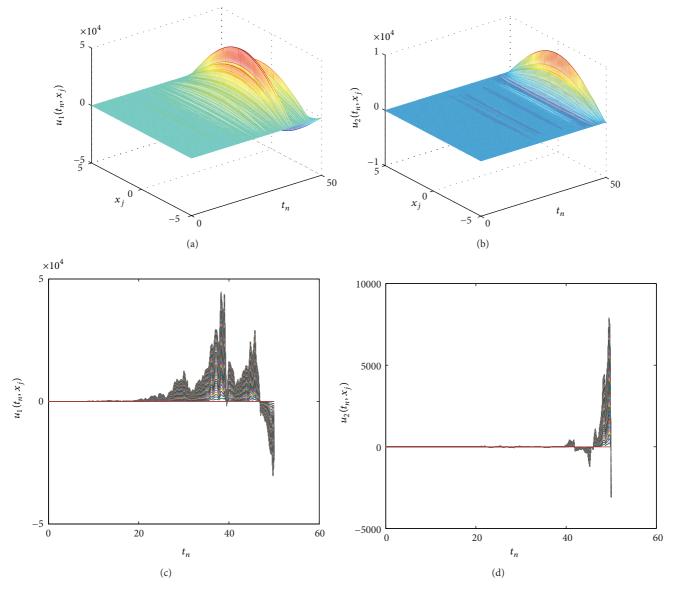


FIGURE 1: Surface curves and state trajectories for system (54) in model 1.

By the Gronwall inequality, we have

$$\mathbb{E}V_{1}\left(u\left(t,x\right),r\left(t\right),t\right)$$

$$\leq \mathbb{E}V_{1}\left(u\left(MT+\delta,x\right),\right.$$

$$r\left(MT+\delta\right),MT+\delta\right)e^{\left(\rho/\overline{\mu}\right)\left(t-MT-\delta\right)}.$$
(42)

Combining (40) and (42), we summarize that,

(I) for $(t, x) \in [0, \delta) \times \Omega_0$, from (40), we have

$$\mathbb{E}V_{1}(u(t,x),r(t),t) \leq \mathbb{E}V_{1}(u(0,x),r(0),0).$$
(43)

(II) For $(t, x) \in [\delta, T) \times \Omega_0$, from (42), we get

$$\mathbb{E}V_{1}(u(t,x), r(t), t) \leq \mathbb{E}V_{1}(u(0,x), r(0), 0) e^{(\rho/\overline{\mu})(t-\delta)}.$$
(44)

(III) For $(t, x) \in [T, T + \delta) \times \Omega_0$, from (40), we have

$$\mathbb{E}V_{1}(u(t,x), r(t), t)$$

$$\leq \mathbb{E}V_{1}(u(T,x), r(T), T) \qquad (45)$$

$$\leq \mathbb{E}V_{1}(u(0,x), r(0), 0) e^{(\rho/\overline{\mu})(T-\delta)}.$$

(IV) For $(t, x) \in [T + \delta, 2T) \times \Omega_0$, from (42), we have

$$\mathbb{E}V_{1}(u(t,x), r(t), t)$$

$$\leq \mathbb{E}V_{1}(u(T+\delta, x), r(T+\delta), T+\delta)e^{(\rho/\overline{\mu})(t-T-\delta)} \quad (46)$$

$$\leq \mathbb{E}V_{1}(u(0, x), r(0), 0)e^{(\rho/\overline{\mu})(t-2\delta)}.$$

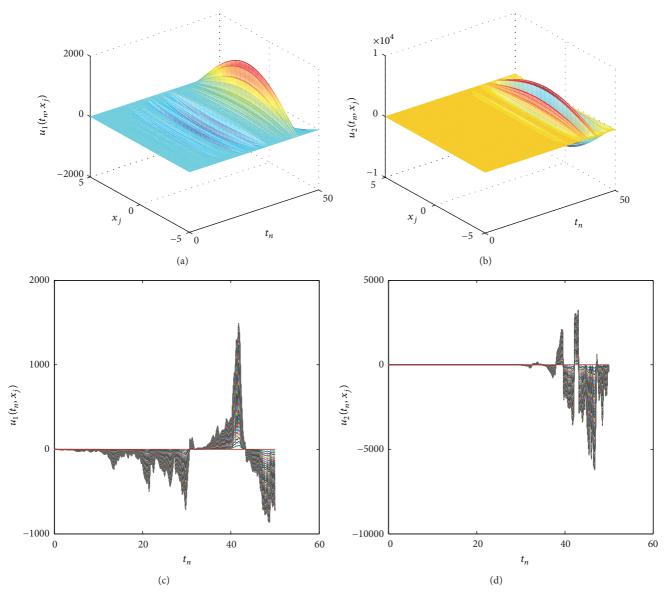


FIGURE 2: Surface curves and state trajectories for system (54) in model 2.

Repeating the above procedure, we obtain that, for $(t, x) \in [MT, MT + \delta), M \leq (t/T)$,

$$\mathbb{E}V_{1}(u(t,x), r(t), t)$$

$$\leq \mathbb{E}V_{1}(u(MT, x), r(MT), MT) \qquad (47)$$

$$\leq \mathbb{E}V_{1}(u(0, x), r(0), 0) e^{(\rho(T-\delta)/\overline{\mu}T)t.}$$

Moreover, for $(t,x) \in [MT+\delta,(M+1)T) \times \Omega_0, (t/T) < M+1,$

$$\mathbb{E}V_1(u(t,x), r(t), t)$$

$$\leq \mathbb{E}V_1(\mu(MT + \delta, x), r(MT + \delta), MT + \delta) e^{(\rho/\overline{\mu})(t - MT - \delta)}$$

$$\leq \mathbb{E}V_1(u(0,x), r(0), 0) e^{(\rho/\overline{\mu})((T - \delta)/T)t}.$$

Hence, for any $(t, x) \in [0, +\infty) \times \Omega_0$, we always have

$$\mathbb{E}V_{1}(u(t,x),r(t),t) \leq \mathbb{E}V_{1}(u(0,x),r(0),0)e^{(\rho/\overline{\mu})((T-\delta)/T)t}.$$
(49)

By (28) and (49), we have

$$e^{\varepsilon t} \overline{\mu} \mathbb{E} \int_{\Omega_0} \sum_{i=1}^n |u_i(t,x)|^p dx$$

$$\leq \mathbb{E} V_1(u(t,x), r(t), t)$$

$$\leq \mathbb{E} V_1(u(0,x), r(0), 0) e^{(\rho/\overline{\mu})((T-\delta)/T)t}.$$
(50)

Note that

$$\begin{split} \mathbb{E}V_{1}\left(u\left(0,x\right),r\left(0\right),0\right) \\ &= \mathbb{E}\int_{\Omega_{0}}\mu_{l}\sum_{i=1}^{n}|u_{i}\left(0,x\right)|^{p}dx \\ &+ \frac{e^{\varepsilon\tau}}{1-\tau_{0}}\mathbb{E}\int_{\Omega_{0}}\sum_{i=1}^{n}\eta_{i}\int_{-\tau_{ij}(0)}^{0}e^{\varepsilon s}|u_{i}\left(s,x\right)|^{p}dsdx \\ &\leq \max_{l\in\mathbb{S}}\mu_{l}\mathbb{E}\int_{\Omega_{0}}\sum_{i=1}^{n}|\phi_{i}\left(0,x\right)|^{p}dx \\ &+ \sup_{-\tau\leq s\leq 0}\left[\left(\max_{1\leq i\leq n}\eta_{i}\right)\frac{\tau e^{\varepsilon\tau}}{1-\tau_{0}}\mathbb{E}\int_{\Omega_{0}}\sum_{i=1}^{n}|\phi_{i}\left(s,x\right)|^{p}dx\right] \\ &= M_{0}. \end{split}$$
(51)

Under assumption (H5), the assertion of Theorem 4 follows from (50) and (51). $\hfill \Box$

Corollary 5. Under assumptions (H1)–(H3), the origin of system (3)–(5) under periodically intermittent control (13) is robust exponentially stable in pth moment if the following conditions hold:

(I)
$$\nu_i < 0, \lambda_i - \nu_i - \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q - (\eta_i / (1 - \tau_0)) > 0, i = 1, 2, \dots, n,$$

(II) $\varepsilon - (\overline{\nu}(T - \delta)/(\overline{\mu}T)) > 0$, where $\overline{\nu} = \max_{1 \le i \le n} \{|\nu_i|\}, \overline{\mu} = \min_{l \in \mathbb{S}} \mu_l$.

Proof. In Theorem 4, let $\alpha_{\ell i j} = \overline{\alpha}_{\ell i j}$, $\beta_{\ell i j} = \overline{\beta}_{\ell i j}$, $\xi_{\ell i j} = \overline{\xi}_{\ell i j}$, $\zeta_{\ell i j} = \overline{\zeta}_{\ell i j}$, $\varepsilon_{\ell i j} = \overline{\varepsilon}_{\ell i j}$, $\omega_{\ell i j} = \overline{\omega}_{\ell i j}$ for all $\ell = 1, 2, ..., p, i, j = 1, 2, ..., n$; then $\lambda_i = \kappa_i$. Under condition (i), select $\rho_i = -\nu_i$, and Corollary 5 holds immediately from Theorem 4.

In Theorem 4, we choose $\alpha_{\ell i j} = \overline{\alpha}_{\ell i j} = \beta_{\ell i j} = \overline{\beta}_{\ell i j} = \xi_{\ell i j} = \overline{\xi}_{\ell i j} = \overline{\xi}_{\ell i j} = \overline{\zeta}_{\ell i j} = \eta^* = \epsilon_{\ell i j} = \overline{\epsilon}_{\ell i j} = \omega_{\ell i j} = \overline{\omega}_{\ell i j} = 1/p$ for $\ell = 1, 2, ..., p$, and i, j = 1, 2, ..., n; then

$$\begin{split} \widetilde{\lambda}_{i} &= \widetilde{\kappa}_{i} \\ &= \min_{l \in \mathbb{S}} \mu_{l} \left\{ \sum_{k=1}^{m} \frac{4\left(p-1\right)\underline{D}_{ik}}{p\theta_{k}^{2}} + pa_{i}^{*} \\ &- \left(p-1\right)\sum_{j=1}^{n} \left(\widetilde{b}_{ij}\widetilde{L}_{j} + \widetilde{c}_{ij}\widetilde{N}_{j}\right) \\ &- \left(p-1\right)\left(p-2\right)\sum_{j=1}^{n} \sigma_{ij} \\ &- \sum_{j=1}^{n} \left(\widetilde{b}_{ji}\widetilde{L}_{j} + \widetilde{c}_{ji}\widetilde{N}_{j} + \left(p-1\right)\sigma_{ji}\right) \right\} \end{split}$$

$$\widetilde{\nu}_{i} = \max_{l \in \mathbb{S}} \mu_{l} \left\{ pk_{ii} + (p-1) \right.$$

$$\times \sum_{\substack{j=1\\j \neq i}}^{n} \left| k_{ij} \right| + \sum_{\substack{j=1\\j \neq i}}^{n} \left| k_{ji} \right| \right\},$$

$$\widetilde{\eta}_{i} = \max_{l \in \mathbb{S}} \mu_{l} \sum_{j=1}^{n} \left[(p-1) \sigma_{ji} + \widetilde{c}_{ji} \widetilde{N}_{j} \right].$$
(52)

Then, as the proof of Theorem 4, we have the following. $\hfill \Box$

Corollary 6. Under assumptions (H1)–(H3), the origin of system (3)–(5) under periodically intermittent control (13) is robust exponentially stable in pth moment if the following conditions hold:

- (I) $\tilde{\lambda}_i \tilde{\nu}_i \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q (\tilde{\eta}_i / (1 \tau_0)) > 0, i = 1, 2, \dots, n,$
- (II) there exists $\tilde{\rho}_i > 0$, such that $\tilde{\lambda}_i + \tilde{\rho}_i \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q \tilde{\eta}_i / (1 \tau_0) > 0$, i = 1, 2, ..., n,
- (III) $\tilde{\varepsilon} (\tilde{\rho}(T \delta)/\overline{\mu}T) > 0$, where $\tilde{\rho} = \max_{1 \le i \le n} \tilde{\rho}_i$, $\overline{\mu} = \min_{l \in \mathbb{S}} \{\mu_l\}$.

Combining Corollary 5 and Corollary 6, we have the following.

Corollary 7. Under assumptions (H1)–(H3), the origin of system(3)–(5) under periodically intermittent control (13) is robust exponentially stable in pth moment if the following conditions hold:

(I)
$$\tilde{\nu}_i < 0, \tilde{\lambda}_i - \tilde{\nu}_i - \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q - (\tilde{\eta}_i / (1 - \tau_0)) > 0,$$

 $i = 1, 2, \dots, n,$

(II) $\tilde{\varepsilon} - (\hat{\nu}(T - \delta)/\overline{\mu}T) > 0$, where $\hat{\nu} = \max_{0 \le i \le n} \{|\tilde{\nu}_i|\}, \overline{\mu} = \min_{l \in \mathbb{N}} \{\mu_l\}.$

Remark 8. By constructing an average Lyapunov function, the stabilization of stochastic Hopfield neural networks with distributed parameters was studied in [4]. The feedback controller in [4] was designed as the compound function of the state and activation function. Therefore, the feedback controller may be the nonlinear functions. However, we see in this paper that the control width is greater than the time delay and the periodically intermittent controller is linear and practical.

Remark 9. In [11–13, 24–26], robust exponential stability and exponential synchronization of some classes of neural networks with reaction-diffusion terms were discussed. The activation function satisfies Lipschitz condition. In fact, the activation function may be not monotone. But, from assumption (H1) in this paper, the activation functions include the monotone functions. So the results of this paper are less conservational and more general.

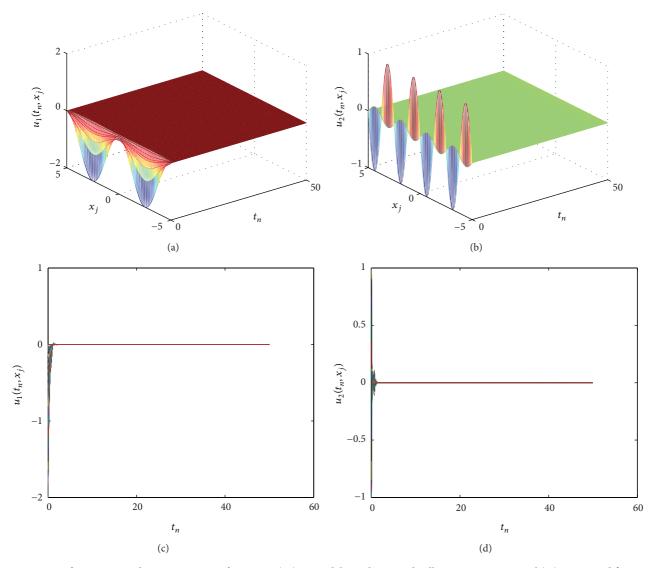


FIGURE 3: Surface curves and state trajectories for system (54) in model 1 under periodically intermittent control (13), T = 5 and $\delta = 4.7$.

Remark 10. In [27, 28], the periodically intermittent controller was designed to stabilization and synchronization of two classes of neural networks, where the activation functions satisfy

$$0 < \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le L_i^+, \qquad 0 < \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le N_i^+.$$
(53)

In fact, they need $L_i^+ \ge L_i^- > 0$, $N_i^+ \ge N_i^- > 0$ in assumption (H1) of this paper. Obviously, the assumption (H1) of this paper is weaker than those of papers [27, 28].

Remark 11. In this paper, if the transmission delay $\tau_{ij}(t)$ is not continuous and differential, we can give new sufficient conditions ensuring robust exponential stabilization and antisynchronization for system (3)–(5) by applying linear matrix inequality (LMI) technique and periodically intermittent control. We will give the topics in future research.

4. Numerical Example

In this section, we give an example with numerical simulations to illustrate our result in the preceding section.

Example 1. Consider the 2-dimensional stochastic interval recurrent neural networks with two models as follows:

$$du_{1}(t,x) = \left\{ D_{1}(r(t)) \frac{\partial^{2} u_{1}(t,x)}{\partial x^{2}} - a_{1}(r(t)) u_{1}(t,x) + \sum_{j=1}^{2} b_{1j}(r(t)) f_{j}(u_{j}(t,x)) + \sum_{j=1}^{2} c_{1j}(r(t)) g_{j}(u_{j}(t-\tau_{1j}(t),x)) \right\} dt$$

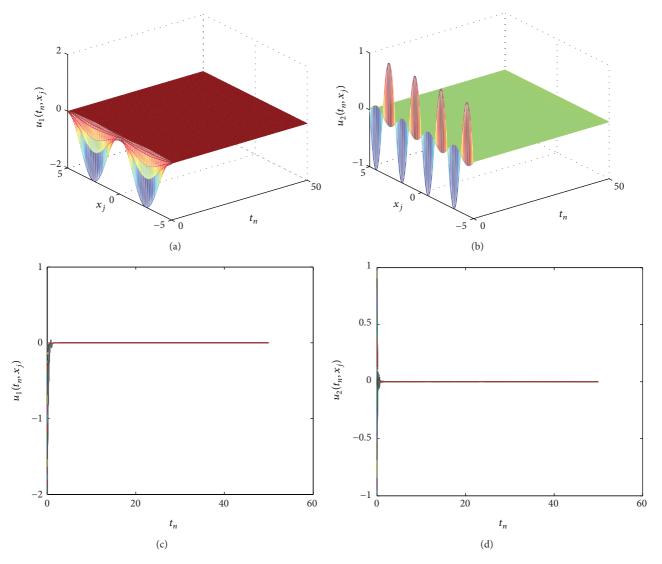


FIGURE 4: Surface curves and state trajectories for system (54) in model 2 under periodically intermittent control (13), T = 5 and $\delta = 4.7$.

$$+\sum_{j=1}^{2}h_{1j}\left(u_{j}(t,x),u_{j}\left(t-\tau_{1j}(t),x\right)\right)dW_{j}(t),$$

$$du_{2}(t,x) = \left\{D_{2}(r(t))\frac{\partial^{2}u_{2}(t,x)}{\partial x^{2}} - a_{2}(r(t))u_{2}(t,x) + \sum_{j=1}^{2}b_{2j}(r(t))f_{j}\left(u_{j}(t,x)\right) + \sum_{j=1}^{2}c_{2j}(r(t))\right)$$

$$\times g_{j}\left(u_{j}\left(t-\tau_{2j}(t),x\right)\right)\right\}dt$$

$$+\sum_{j=1}^{2}h_{2j}\left(u_{j}(t,x),u_{j}\left(t-\tau_{2j}(t),x\right)\right)dW_{j}(t),$$
(54)

with the boundary conditions $u_1(t,0) = u_2(t,0) = u_1(t,2) = u_2(t,2) = 0$, $t \ge -1$ and the initial value

 $u_1(t, x) = e^t(\cos(2\pi x) - 1), \ u_2(t, x) = e^{2t}\sin(4\pi x), \ t \in [-1, 0] \times \Omega_0$, where $\tau_{ij}(t) = (e^t/(1 + e^t)), \ \Omega_0 = [-5, 5] \in \mathbb{R}$, and the generator of the Markov chain

$$\begin{split} \Gamma &= \begin{pmatrix} -1 & 1 \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}, \\ f_i \left(u_i \right) &= \frac{3}{4} \sin \left(u_i \right) + \frac{1}{4} u_i, \\ g_i \left(u_i \right) &= \frac{1}{2} \left(\left| u_i + 1 \right| - \left| u_i - 1 \right| \right), \\ h_{11} \left(u_1 \left(t, x \right), u_1 \left(t - \tau_{11} \left(t \right), x \right) \right) \\ &= 0.1 u_1 \left(t, x \right) + 0.2 u_1 \left(t - \tau_{11} \left(t \right), x \right), \\ h_{12} \left(u_2 \left(t, x \right), u_2 \left(t - \tau_{12} \left(t \right), x \right) \right) \\ &= h_{21} \left(u_1 \left(t, x \right), u_1 \left(t - \tau_{21} \left(t \right) \right), x \right) = 0, \end{split}$$

$$h_{22}\left(u_{2}(t, x), u_{2}(t - \tau_{22}(t), x)\right)$$

= $0.3u_{2}(t, x) + 0.4u_{2}(t - \tau_{22}(t), x).$ (55)

We assume that the interval matrices are the same in every model; let

$$D(r(t)) = \begin{pmatrix} D_{1}(r(t)) \\ D_{2}(r(t)) \end{pmatrix} = \begin{pmatrix} [1,2] \\ [2,3] \end{pmatrix},$$

$$A(r(t)) = \begin{pmatrix} a_{1}(r(t)) \\ a_{2}(r(t)) \end{pmatrix}$$

$$= \begin{pmatrix} [0.28, 0.42] \\ [0.18, 0.35] \end{pmatrix},$$

$$B(r(t)) = \begin{pmatrix} B_{11}(r(t)) & B_{12}(r(t)) \\ B_{21}(r(t)) & B_{22}(r(t)) \end{pmatrix} = \begin{pmatrix} [4,5] \\ [3,4] \end{pmatrix},$$

$$C(r(t)) = \begin{pmatrix} C_{11}(r(t)) & C_{12}(r(t)) \\ C_{21}(r(t)) & C_{22}(r(t)) \end{pmatrix}$$

$$= \begin{pmatrix} [0.2, 0.3] \\ [0.3, 0.4] \end{pmatrix}.$$
(56)

The surface curves and state trajectories of system (54) in model (1) and model (2) are given, respectively, as shown in Figures 1 and 2. They exhibit instability behavior.

Let $p = 2, \mu_1 = 2, \mu_2 = 4$. By simple calculation, we obtain

$$L_{i}^{-} = -\frac{1}{2}, \qquad L_{i}^{+} = 1, \qquad N_{i}^{-} = 0,$$

$$N_{i}^{+} = 1, \qquad \tau = 1, \qquad \tau_{0} = \frac{1}{4} \qquad (57)$$

$$\tilde{\lambda}_{1} = \tilde{\kappa}_{1} = -40.16, \qquad \tilde{\eta}_{1} = 1.52$$

$$\tilde{\lambda}_{2} = \tilde{\kappa}_{2} = -34.4, \qquad \tilde{\eta}_{2} = 2.88.$$

Now, we consider the periodically intermittent control (13), where the parameters are given as follows:

$$k_{11} = -10, \quad k_{22} = -10, \quad k_{12} = 0, \quad k_{21} = 0.$$
 (58)

Then $\tilde{\nu}_1 = -80$ and $\tilde{\nu}_2 = -80$,

$$\begin{split} \widetilde{\lambda}_{1} &- \widetilde{\nu}_{1} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \frac{\widetilde{\eta}_{1}}{1 - \tau_{0}} > 0, \\ \widetilde{\lambda}_{2} &- \widetilde{\nu}_{2} - \max_{l \in \mathbb{S}} \sum_{q=1}^{N} \gamma_{lq} \mu_{q} - \frac{\widetilde{\eta}_{2}}{1 - \tau_{0}} > 0. \end{split}$$
(59)

From $H_i(\tilde{\varepsilon}_i) = \tilde{\lambda}_i - \tilde{\gamma}_i - \max_{l \in \mathbb{S}} \sum_{q=1}^N \gamma_{lq} \mu_q - \tilde{\varepsilon}_i \max_{l \in \mathbb{S}} \mu_l - (\tilde{\eta}_i/(1 - \tau_0)) e^{\tilde{\varepsilon}_i \tau} = 0, i = 1, 2$, we have

$$\tilde{\varepsilon} = \min{\{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\}} = \min{\{2.605, 2.413\}} = 2.413.$$
 (60)

Choose T = 5, from Corollary 7, $\delta = 4.7$. Then the origin of system (54) under the periodically intermittent controller (13) is robust exponentially stable in mean square. The surface curves and state trajectories in model (1) and model (2) are given, respectively, as shown in Figures 3 and 4.

Abstract and Applied Analysis

Conflict of Interests

The authors declare no competing financial interests.

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