

Research Article

Limit Cycles in a Cubic Kolmogorov System with Harvest and Two Positive Equilibrium Points

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A class of planar cubic Kolmogorov systems with harvest and two positive equilibrium points is investigated. With the help of computer algebra system MATHEMATICA, we prove that five limit cycles can be bifurcated simultaneously from the two critical points (1, 1) and (2, 2), respectively, in the first quadrant. Moreover, the necessary conditions of centers are obtained.

1. Introduction

In mathematical ecology, a class of systems of the form

$$\frac{dx_i}{dt} = x_i F(x_1, \dots, x_n), \quad i = 1, \dots, n, \quad (1)$$

are frequently used to model the interaction of species occupying the same ecological niche. The differential equations modeling the interaction of two species

$$\begin{aligned} \frac{dx}{dt} &= xF(x, y), \\ \frac{dy}{dt} &= yG(x, y), \end{aligned} \quad (2)$$

being known as Kolmogorov systems have been studied extensively. It is well known that there is no limit cycles in the classical Lotka-Volterra-Gause model, where F and G are linear. There can of course only be one critical point in the interior of the realistic quadrant ($x > 0, y > 0$) in this case, but this can be a centre; however, there are no isolated periodic solutions.

If F and G are quadratic, one might think by analogy that the behavior within the first quadrant is similar to that of a quadratic system. The examples we give show that this is

not the case, even when F and G factories. There are many contributions about this system (see [1, 2]). The latter poses the question whether a predator-prey system can have two or more ecologically stable cycles. If F and G are cubic, there are also many works to consider its limit cycles and dynamics behaviors, see [3, 4]. In [5], the authors discussed a class of cubic Kolmogorov systems with three invariant algebraic curves.

Recently, a system with three positive equilibrium points has been investigated, the authors have investigated the center-focus problems and limit cycles bifurcations. They have proved that each of the two points (1, 2) and (2, 1) can bifurcate 1 small limit cycle under a certain condition, and 3 limit cycles can occur near point (1, 1) at the same step [6]. Other Kolmogorov systems were also investigated recently in [7, 8]. In this paper, we will consider limit cycles which bifurcate from a class of systems of the form

$$\begin{aligned} \frac{dx}{dt} &= -2(2\beta + \theta) + (8\beta + 5\theta)x - \frac{7}{2}(2\beta + \theta)x^2 \\ &+ \frac{1}{2}(4\beta - \theta)xy + a_{20}x^3 + \frac{1}{2}(6\beta + 3\theta - 4a_{20})x^2y \\ &+ \frac{1}{2}(-4\beta - \theta + 2a_{20})x^2y, \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= 2(2\beta + \theta) - (8\beta + 5\theta)y + \frac{7}{2}(2\beta + \theta)y^2 \\ &+ \frac{1}{2}(-4\beta + \theta)xy + \frac{1}{2}(2b_{20} - 4\beta - \theta)x^3 + b_{20}x^2y \\ &+ \frac{1}{2}(2\beta - \theta - 4b_{20})xy^2, \end{aligned} \quad (3)$$

which have two positive equilibrium points (1, 1) and (2, 2). We use our Computer Algebra procedure Mathematic (described in [9]) to compute the focal values at the critical points (1, 1) and (2, 2). We will show that five limit cycles can bifurcate from points (1, 1) and (2, 2) simultaneously. Furthermore, some necessary and sufficient conditions for two positive equilibrium points to be centers are also to be given.

This paper is divided into three sections. In Section 2, we use the recursive algorithm to obtain that 5 limit circles could be bifurcated from points (1, 1) and (2, 2). In Section 3, necessary and sufficient conditions for two positive equilibrium points to be centers are proved.

2. Bifurcations of Limit Circles at Two Positive Equilibrium Points

First of all, it is easy to testify that points (1, 1) and (2, 2) are two positive equilibriums of system (3) and they are all center or focus. So we need to compute the Lyapunov constants to determine its kind of singular. Now, we consider the point (1, 1).

Through the transformations $\bar{x} = x - 1$, $\bar{y} = y - 1$, $\tau = \beta t$, we still denote \bar{x}, \bar{y} by x, y for convenience. The point (1, 1) will be moved into (0, 0) of the new system, and the system will be changed into the following system:

$$\begin{aligned} \frac{dx}{d\tau} &= 4x^2 - y - 4xy - 3x^2y + 2y^2 + 2xy^2 \\ &- \frac{(a_{20}x^2)}{\beta} - \frac{(a_{20}x^3)}{\beta} + \frac{(2a_{20}xy)}{\beta} \\ &+ \frac{(2a_{20}x^2y)}{\beta} - \frac{(a_{20}y^2)}{\beta} - \frac{(a_{20}xy^2)}{\beta} + \frac{(2x^2\theta)}{\beta} \\ &- \frac{(3xy\theta)}{(2\beta)} - \frac{(3x^2y\theta)}{(2\beta)} + \frac{(y^2\theta)}{(2\beta)} + \frac{(xy^2\theta)}{(2\beta)}, \quad (4) \\ \frac{dy}{d\tau} &= x - 2y^2 - xy^2 + 2y^3 - \frac{(b_{20}x^2)}{\beta} + \frac{(2b_{20}xy)}{\beta} \\ &- \frac{(b_{20}x^2y)}{\beta} - \frac{(b_{20}y^2)}{\beta} + \frac{(2b_{20}xy^2)}{\beta} - \frac{(b_{20}y^3)}{\beta} \\ &+ \frac{(xy\theta)}{(2\beta)} - \frac{(3y^2\theta)}{(2\beta)} + \frac{(xy^2\theta)}{(2\beta)} + \frac{(y^3\theta)}{(2\beta)}. \end{aligned}$$

Furthermore, by the transformations

$$\begin{aligned} z &= x + iy, & w &= x - iy, \\ T &= i\tau, & i &= \sqrt{-1}, \end{aligned} \quad (5)$$

system (4) can be transformed into the following system:

$$\begin{aligned} \frac{dz}{dT} &= z + \frac{1}{8\beta}(-2(2a_{20} + 2ib_{20} - (6 - 2i)\beta - (3 - 2i)\theta)z^2 \\ &- 2(-4ia_{20} + 4b_{20} + (4 + 12i)\beta + (3 + 5i)\theta)zw \\ &+ 2(2a_{20} + 2ib_{20} - (2 + 2i)\beta - i\theta)w^2 \\ &+ (-2a_{20} - 2b_{20} + (4 + 4i)\beta + (1 + i)\theta)z^3 \\ &+ 2((-1 + 2i)a_{20} + (1 + 2i)b_{20} \\ &+ (1 - 4i)\beta + (1 - i)\theta)z^2w \\ &+ ((2 + 4i)a_{20} + (2 - 4i)b_{20} \\ &- (1 - i)(4\beta + \theta))zw^2 \\ &- 2(-a_{20} + b_{20} + \beta + \theta)w^3; \\ \frac{dw}{dT} &= -w - \frac{1}{8\beta}(-2(2a_{20} - 2ib_{20} - (6 + 2i)\beta - (3 + 2i)\theta)w^2 \\ &- 2(4ia_{20} + 4b_{20} + (4 - 12i)\beta + (3 - 5i)\theta)zw \\ &+ 2(2a_{20} - 2ib_{20} - (2 - 2i)\beta + i\theta)z^2 \\ &+ (-2a_{20} - 2b_{20} + (4 - 4i)\beta + (1 - i)\theta)w^3 \\ &+ 2((-1 - 2i)a_{20} + (1 - 2i)b_{20} \\ &+ (1 + 4i)\beta + (1 + i)\theta)w^2z \\ &+ ((2 - 4i)a_{20} + (2 + 4i)b_{20} \\ &- (1 + i)(4\beta + \theta))wz^2 \\ &- 2(-a_{20} + b_{20} + \beta + \theta)z^3; \end{aligned} \quad (6)$$

applying the recursive formulae in Theorem 2.5 in [9], we compute singular point quantities and simplify them; then, we have the following theorem.

Theorem 1. *The first four singular point quantities at the origin of system (6) are as follows:*

$$\begin{aligned}
 u_1 &= \frac{i}{8\beta^2} (-2a_{20} - 2b_{20} + 4\beta + \theta) \\
 &\quad \times (-4a_{20} + 4b_{20} + 12\beta + 9\theta), \\
 u_2 &= \frac{i}{192\beta^3} (-2a_{20} - 2b_{20} + 4\beta + \theta) \\
 &\quad \times (344\beta^2 + 750\beta\theta + 325\theta^2), \\
 u_3 &= -\frac{i}{12288\beta^5} (-2a_{20} - 2b_{20} + 4\beta + \theta) f_1, \\
 u_4 &= -\frac{i}{5898240\beta^7} (-2a_{20} - 2b_{20} + 4\beta + \theta) f_2,
 \end{aligned} \tag{7}$$

where

$$\begin{aligned}
 f_1 &= -213760b_{20}^2\beta^2 - 213760b_{20}\beta^3 + 147392\beta^4 \\
 &\quad - 323520b_{20}^2\beta\theta - 697600b_{20}\beta^2\theta + 84104\beta^3\theta \\
 &\quad - 116000b_{20}^2\theta^2 - 682160b_{20}\beta\theta^2 - 554530\beta^2\theta^2 \\
 &\quad - 203000b_{20}\theta^3 - 544505\beta\theta^3 - 128125\theta^4; \\
 f_2 &= 14520162123939840\beta^{10} + 77088521496444928\beta^9\theta \\
 &\quad + 146627049158313472\beta^8\theta^2 + 98147703728382592\beta^7\theta^3 \\
 &\quad - 45397952769129120\beta^6\theta^4 - 116390817537863400\beta^5\theta^5 \\
 &\quad - 79417641637292700\beta^4\theta^6 - 27193429543260550\beta^3\theta^7 \\
 &\quad - 5139731868326875\beta^2\theta^8 - 582232780375000\beta\theta^9 \\
 &\quad - 38940972656250\theta^{10}.
 \end{aligned} \tag{8}$$

While $u_1 = u_2 = u_3 = 0, u_4 \neq 0$, the following theorem holds.

Theorem 2. *The origin of system (6) is a 4-order weak focus if and only if*

$$\begin{aligned}
 a_{20} &= \frac{1}{4} (4b_{20} + 12\beta + 9\theta), \\
 344\beta^2 + 750\beta\theta + 325\theta^2 &= 0, \\
 f_1 &= 0.
 \end{aligned} \tag{9}$$

Proof. $u_1 = u_2 = u_3 = 0$ implies that the relations above among parameters hold. Further, when $a_{20} = (1/4)(4b_{20} + 12\beta + 9\theta)$, $344\beta^2 + 750\beta\theta + 325\theta^2 = 0$,

$$\begin{aligned}
 f_1 &= -3164304640b_{20}^2 + 94432128\sqrt{1153}b_{20}^2 \\
 &\quad - 505511440b_{20}\theta + 16321424\sqrt{1153}b_{20}\theta \\
 &\quad - 1443860355\theta^2 + 42746637\sqrt{1153}\theta^2; \\
 f_2 &= -748902307758080b_{20}^4 + 22349440598016\sqrt{1153}b_{20}^4 \\
 &\quad - 239280807055360b_{20}^3\theta + 7725648121856\sqrt{1153}b_{20}^3\theta \\
 &\quad + 5835163629391040b_{20}^2\theta^2 - 171639395144768\sqrt{1153}b_{20}^2\theta^2 \\
 &\quad + 969876044134880b_{20}\theta^3 - 28776579351136\sqrt{1153}b_{20}\theta^3 \\
 &\quad + 3030081181205925\theta^4 - 89269930142715\sqrt{1153}\theta^4,
 \end{aligned} \tag{10}$$

we have

$$\text{Resultant}[f_1, f_2, b_{20}] = 2.87343 \times 10^{53}\theta^8 \neq 0, \tag{11}$$

where $\text{Resultant}[g(x), f(x), x]$ denotes the resultant of $f(x), g(x)$ with respect to x . So $f_2 \neq 0$ when $f_1 = 0$. Namely, point $(1, 1)$ is a fourth-order weak focus. \square

We next study bifurcation of limit cycles of the perturbed system of (4). When condition (9) holds, we can obtain

$$J = \begin{vmatrix} D[u_1, a_{20}] & D[u_1, \beta] & D[u_1, b_{20}] \\ D[u_2, a_{20}] & D[u_2, \beta] & D[u_2, b_{20}] \\ D[u_3, a_{20}] & D[u_3, \beta] & D[u_3, b_{20}] \end{vmatrix} \neq 0. \tag{12}$$

In fact,

$$\begin{aligned}
 J &= \frac{7001704448i}{234375(-75 + \sqrt{1153})^{11}\theta^7} \\
 &\quad \times (688b_{20} + 227\theta + 5\sqrt{1153}\theta)^2 \\
 &\quad \times (-17721692278080b_{20}^2 + 519304637376\sqrt{1153}b_{20}^2 \\
 &\quad - 2991374746080b_{20}\theta + 85097939808\sqrt{1153}b_{20}\theta \\
 &\quad - 2771947690805\theta^2 + 80900776387\sqrt{1153}\theta^2).
 \end{aligned} \tag{13}$$

So $\text{Resultant}[J, f_3, b_{20}] \neq 0$, namely, $J \neq 0$, when $a_{20} = (1/4)(4b_{20} + 12\beta + 9\theta)$, $344\beta^2 + 750\beta\theta + 325\theta^2 = 0$.

The statement mentioned above yields that the following theorem holds.

Theorem 3. *If the origin of system (4) is a 4-order weak focus, making a small perturbation to the coefficients of system (4),*

then, for perturbed system (4), in a small neighborhood of the origin, there exist exactly 5 small amplitude limit cycles enclosing the point (1, 1).

With a similar method, we could discuss the bifurcation of limit cycles from point (2, 2). Through the transformations $\bar{x} = x - 2, \bar{y} = y - 2, \tau = \theta t$, we still denote \bar{x}, \bar{y} by x, y for convenience. Then point (1, 1) will be moved into point (0, 0) of the new system, and the system will be changed into the following system:

$$\begin{aligned} \frac{dx}{d\tau} &= \frac{x^2}{2} - y - \frac{(7xy)}{2} - \frac{(3x^2y)}{2} \\ &+ y^2 + \frac{(xy^2)}{2} - \frac{(2a_{20}x^2)}{\theta} - \frac{(a_{20}x^3)}{\theta} \\ &+ \frac{(4a_{20}xy)}{\theta} + \frac{(2a_{20}x^2y)}{\theta} - \frac{(2a_{20}y^2)}{\theta} \\ &- \frac{(a_{20}xy^2)}{\theta} + \frac{(x^2\beta)}{\theta} - \frac{(6xy\beta)}{\theta} - \frac{(3x^2y\beta)}{\theta} \\ &+ \frac{(4y^2\beta)}{\theta} + \frac{(2xy^2\beta)}{\theta}, \tag{14} \\ \frac{dy}{d\tau} &= x + \frac{(3xy)}{2} + \frac{y^2}{2} + \frac{(xy^2)}{2} + \frac{y^3}{2} \\ &- \frac{(2b_{20}x^2)}{\theta} + \frac{(4b_{20}xy)}{\theta} - \frac{(b_{20}x^2y)}{\theta} \\ &- \frac{(2b_{20}y^2)}{\theta} + \frac{(2b_{20}xy^2)}{\theta} - \frac{(b_{20}y^3)}{\theta} \\ &- \frac{(2xy\beta)}{\theta} + \frac{(3y^2\beta)}{\theta} - \frac{(xy^2\beta)}{\theta} + \frac{(2y^3\beta)}{\theta}. \end{aligned}$$

Furthermore, by the transformations

$$z = x + iy, \quad w = x - iy, \quad T = i\tau, \quad i = \sqrt{-1}, \tag{15}$$

system (14) can be transformed into the following system:

$$\begin{aligned} \frac{dz}{dT} &= z + \frac{1}{8\theta} \left(2(-4a_{20} - 4ib_{20} + (3 + 5i)\beta + (3 - i)\theta) z^2 \right. \\ &+ 2(8ia_{20} - 8b_{20} + (6 - 10i)\beta + (1 - 3i)\theta) zw \\ &- 2(4a_{20} + 4ib_{20} - (9 - i)\beta - (4 - 2i)\theta) w^2 \\ &+ (-2a_{20} - 2b_{20} + 4(1 + 4i)\beta + (1 + i)\theta) z^3 \\ &+ 2((-1 + 2i)a_{20} + (1 + 2i)b_{20} \\ &\quad \left. + (1 - 4i)\beta + (1 - i)\theta) z^2 w \right. \\ &+ (2(1 + 2i)a_{20} + 2(1 - 2i)b_{20} - (1 - i)(4\beta + \theta)) zw^2 \\ &\left. - 2(-a_{20} + b_{20} + \beta + \theta) w^3 \right); \end{aligned}$$

$$\begin{aligned} \frac{dw}{dT} &= -w - \frac{1}{8\theta} \left(2(-4a_{20} + 4ib_{20} + (3 - 5i)\beta + (3 + i)\theta) w^2 \right. \\ &+ 2(-8ia_{20} - 8b_{20} + (6 + 10i)\beta + (1 + 3i)\theta) zw \\ &- 2(4a_{20} - 4ib_{20} - (9 + i)\beta - (4 + 2i)\theta) z^2 \\ &+ (-2a_{20} - 2b_{20} + 4(1 - 4i)\beta + (1 - i)\theta) w^3 \\ &+ 2((-1 - 2i)a_{20} + (1 - 2i)b_{20} \\ &\quad \left. + (1 + 4i)\beta + (1 + i)\theta) w^2 z \right. \\ &+ (2(1 - 2i)a_{20} + 2(1 + 2i)b_{20} - (1 + i)(4\beta + \theta)) wz^2 \\ &\left. - 2(-a_{20} + b_{20} + \beta + \theta) z^3 \right). \tag{16} \end{aligned}$$

Direct computation can give the following theorem.

Theorem 4. *The first four singular point quantities at the origin of system (16) are as follows:*

$$\begin{aligned} u_1 &= \frac{i}{4\theta^2} (-2a_{20} - 2b_{20} + 4\beta + \theta) (8a_{20} - 8b_{20} - 3\theta), \\ u_2 &= -\frac{i}{48\theta^3} (-2a_{20} - 2b_{20} + 4\beta + \theta) (56\beta^2 + 57\beta\theta + 10\theta^2), \\ u_3 &= \frac{i}{6144\theta^5} (-2a_{20} - 2b_{20} + 4\beta + \theta) f_3, \\ u_4 &= -\frac{i}{5898240\beta^7} (-2a_{20} - 2b_{20} + 4\beta + \theta) f_4, \tag{17} \end{aligned}$$

where

$$\begin{aligned} f_3 &= -143360b_{20}^2\beta^2 + 286720b_{20}\beta^3 - 206080\beta^4 \\ &- 168960b_{20}^2\beta\theta + 355840b_{20}\beta^2\theta - 264320\beta^3\theta \\ &- 40960b_{20}^2\theta^2 + 103040b_{20}\beta\theta^2 - 82336\beta^2\theta^2 \\ &+ 5120b_{20}\theta^3 - 1939\beta\theta^3 + 980\theta^4, \\ f_4 &= 2967207936000\beta^{10} - 9714651627520\beta^9\theta \\ &- 65013770321920\beta^8\theta^2 - 124569363533824\beta^7\theta^3 \\ &- 124914242976768\beta^6\theta^4 - 75962577303936\beta^5\theta^5 \\ &- 29760585599448\beta^4\theta^6 - 7669544070125\beta^3\theta^7 \\ &- 1275738647714\beta^2\theta^8 - 125233021760\beta\theta^9 \\ &- 5502919200\theta^{10}. \tag{18} \end{aligned}$$

While solving $u_1 = u_2 = u_3 = 0$ and $u_4 \neq 0$, we can immediately obtain the following theorem.

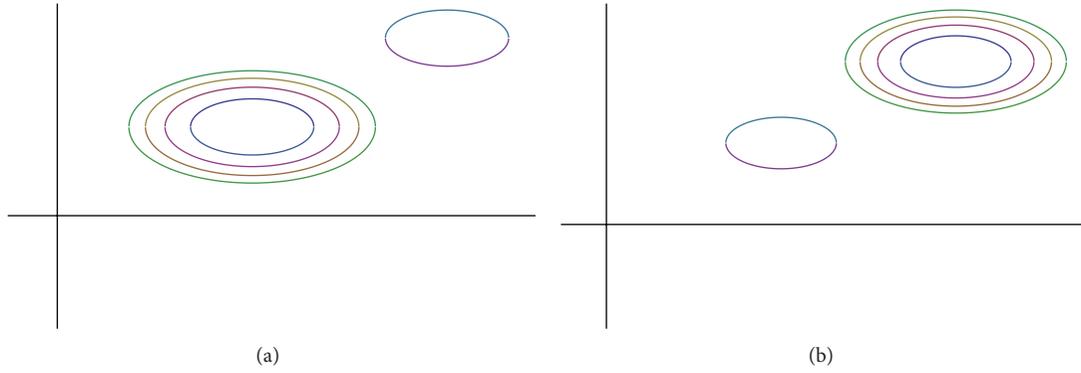


FIGURE 1: Distribution of limit cycles.

Theorem 5. *The origin of system (16) is a 4-order weak focus if and only if*

$$a_{20} = \frac{1}{8}(8b_{20} + 3\theta), \quad 56\beta^2 + 57\beta\theta + 10\theta^2 = 0, \quad (19)$$

$$f_3 = 0.$$

Furthermore, the following theorem also holds for system (14).

Theorem 6. *If the origin of system (14) is a 4-order weak focus, making a small perturbation to the coefficients of system (14), then, for perturbed system (14), in a small neighborhood of the origin, there exist exactly 5 small amplitude limit cycles enclosing the point (2, 2).*

The results also yield that when point (1, 1) is a fourth-order weak focus, point (2, 2) of system is a first-order weak focus at the same time. Point (1, 1) is a first-order weak focus when point (2, 2) is a fourth-order weak focus. Namely, by a simultaneously perturbation, five limit cycles could be bifurcated with two different distribution; see Figure 1.

3. Center Conditions of Two Positive Equilibrium Points

If all Lyapunov constants are equal to zero, it will be a center condition of a critical point. So Theorems 1 and 4 imply the following theorem.

Theorem 7. *If the origins of systems (4) and (14) are centers if and only if*

$$a_{20} = \frac{1}{2}(-2b_{20} + 4\beta + \theta). \quad (20)$$

Proof. When $a_{20} = (1/2)(-2b_{20} + 4\beta + \theta)$, system (4) could be rewritten as

$$\frac{dx}{d\tau} = -y - \frac{1}{2\beta}(-2b_{20}x^2 - 2b_{20}x^3 + 4b_{20}xy + 4b_{20}x^2y - 2b_{20}y^2 - 2b_{20}xy^2 - 4x^2\beta + 4x^3\beta$$

$$-2x^2y\beta - 3x^2\theta + x^3\theta + xy\theta + x^2y\theta),$$

$$\frac{dy}{d\tau} = x - \frac{1}{2\beta}(2b_{20}x^2 - 4b_{20}xy + 2b_{20}x^2y + 2b_{20}y^2 - 4b_{20}xy^2 + 2b_{20}y^3 - 2x\beta + 4y^2\beta + 2xy^2\beta - 4y^3\beta - xy\theta + 3y^2\theta - xy^2\theta - y^3\theta). \quad (21)$$

By transformation

$$x = -\frac{u+v}{2(v-1)}, \quad y = -\frac{-u+v}{2(v-1)}, \quad \tau = (v-1)^2t, \quad (22)$$

system (21) becomes

$$\frac{du}{d\tau} = -v - \frac{1}{4\beta}(-8b_{20}u^2 + 4b_{20}u^4 + 4b_{20}u^2v - 3u^4\beta + 4v\beta + 11u^2v\beta - 12v^2\beta - 9u^2v^2\beta + 9v^3\beta - 4u^2\beta + 11u^2v\theta - 2v^2\theta - 8u^2v^2\theta + 3v^3\theta),$$

$$\frac{dv}{d\tau} = (u(-1+v)(-4b_{20}u^2 - 4\beta + 3u^2\beta + 9v^2\beta - 6v\theta + 8v^2\theta)) \times (4\beta)^{-1} \quad (23)$$

which is symmetric with v -axis.

When $a_{20} = (1/2)(-2b_{20} + 4\beta + \theta)$, system (14) could be rewritten as

$$\frac{dx}{d\tau} = -y - \frac{1}{2\theta}(-4b_{20}x^2 - 2b_{20}x^3 + 8b_{20}xy + 4b_{20}x^2y$$

$$\begin{aligned}
& -4b_{20}y^2 - 2b_{20}xy^2 + 6x^2\beta + 4x^3\beta - 4xy\beta \\
& -2x^2y\beta + x^2\theta + x^3\theta + 3xy\theta + x^2y\theta), \\
\frac{dy}{d\tau} & = x + \frac{1}{2\theta} \left(-4b_{20}x^2 + 8b_{20}xy - 2b_{20}x^2y - 4b_{20}y^2 \right. \\
& \quad + 4b_{20}xy^2 - 2b_{20}y^3 - 4xy\beta + 6y^2\beta \\
& \quad \left. - 2xy^2\beta + 4y^3\beta + 3xy\theta + y^2\theta + xy^2\theta + y^3\theta \right). \tag{24}
\end{aligned}$$

Transformations

$$x = -\frac{u+v}{2(v-1)}, \quad y = -\frac{-u+v}{2(v-1)}, \quad \tau = (v-1)^2t, \tag{25}$$

bring system (24) into

$$\begin{aligned}
\frac{du}{d\tau} & = -v - \frac{1}{4\theta} \left(-16b_{20}u^2 + 4b_{20}u^4 + 12b_{20}u^2v \right. \\
& \quad + 10u^2\beta - 3u^4\beta - 15u^2v\beta + 2v^2\beta + 7u^2v^2\beta \\
& \quad \left. - v^3\beta + 2u^2\theta - 7u^2v\theta - 4v^2\theta + 4u^2v^2\theta + v^3\theta \right), \\
\frac{dv}{d\tau} & = \left(u(-1+v) \left(4b_{20}u^2 - 3u^2\beta - 12v\beta \right. \right. \\
& \quad \left. \left. + 7v^2\beta + 4\theta - 10v\theta + 4v^2\theta \right) \right) \times (4\theta)^{-1} \tag{26}
\end{aligned}$$

which is symmetric with v -axis. So the origin is a center of system (14). \square

Conflict of Interests

The authors declare that they have no conflict of interests.

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