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Research Article

Progress and Regress of Time Dependent Data and Application in Bank Branch

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To evaluate each decision making unit having time dependent inputs and outputs data, a new method has been developed and reported here. This method uses the Malmquist productivity index, and is a very simple function based on Cubic Spline function to determine the progress and regress of that unit. To show the capability of this developed method, the data of 9 branches of a commercial bank has been used, evaluated, and reported.

1. Introduction

In the modern managements, several applied scientific methods have been developed to improve the quality of systems. Data Envelopment Analysis (DEA) is one of the most suitable and applicable methods to evaluate and compare several units or a unit in various times [1–3]. By using this scientific technique, the system manager is in the position to find and apply the best decision for improving that system [4, 5].

At the beginning, the efficiency change was the only possible criteria to find the progress and regress of a unit. Via research developments, it has been shown that the technical change has effect in productivity, too, and the Malmquist Productivity Index (MPI) was introduced by Malmquist in 1953 [6]. Later the researchers recommended productivity evaluation and defined MPI for each unit based on inputs disposal and outputs products [7–10]. Hereafter several researches focused on calculating this index and several applications were procured [11–13].

Those evaluations used fixed and certain values of the units and excluded some application cases, which have used the time dependent available or foresight data [14–16]. To determine the progress and regress of a Decision Making Unit (DMU) is necessary to use models with time dependent data analysis capability.

In this paper, the definition of MPI will be illustrated [17–19], then the efficiency of time dependent data will be defined, and a new method to determine progress and regress of units will be presented and applied for a commercial bank and the results will be discussed.

2. Malmquist Productivity Index

Farell (1957) determined a suitable method to evaluate experimental production function for several inputs and outputs with using linear programming technique and Data Envelopment Analyses (DEA) [20]. By applying DEA, the best efficiency frontier will be calculated with a set of DMUs and omitting of any priority for inputs and outputs. The DMUs of efficiency frontier are the units with the maximum output and/or the minimum input levels. Using the efficient units and efficiency frontier, the analysis of other inefficiency units is possible.

MPI is defined with assimilation efficiency changes of each unit and technology changes. MPI can be calculated via several functions, such as distance function

$$D\left(X_{\circ}, Y_{\circ}\right) = \inf \left\{ \frac{\theta}{\left(\theta X_{\circ}, Y_{\circ}\right)} \in \text{PPS} \right\}. \tag{1}$$

This equation shows in special conditions only the efficiency frontier change at time $t+1$ related to t that could not be a suitable criterion to calculate the technology change. If $D^k(X^k, Y^k) = 1$, then kth unit is hypothesized as efficient.

This distance function does not define the inefficiency values. Fare decomposed MPI into two components, using linear inefficiency of technology frontier. The efficiency frontier will be specified for each DMU with DEA. Production function is hypothesized instant t and $t+1$. Calculation of the MPI requires four linear programming problems as follows:

$$\begin{gathered} \circ \in Q = \{1, 2, \ldots, n\}, \\ D_\circ^t\left(X_\circ^t, Y_\circ^t\right) = \operatorname{Min}\theta, \\ \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij}^t \leq \theta x_{io}^t, \quad i = 1, \ldots, m, \\ \sum_{j=1}^n \lambda_j y_{rj}^t \geq y_{ro}^t, \quad r = 1, \ldots, s, \\ \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{gathered} \tag{2}$$

x_{io}^t is the ith input and y_{ro}^t is the rth output of $\text{DMU}_\circ$ at time t. The efficiency ($D_\circ^t(X_\circ^t, Y_\circ^t) = \theta_\circ^*$) shoes the highest possible decrease of $\text{DMU}_\circ$ input for related output.

Instead t, CCR problem (2), is calculated at time $t+1$ and is equal to $D^{t+1}(X_\circ^{t+1}, Y_\circ^{t+1})$ and is the technical efficiency for $\text{DMU}_\circ$ at time $t+1$. The value of $D^t(X_\circ^{t+1}, Y_\circ^{t+1})$ for $\text{DMU}_\circ$ is the distance of $\text{DMU}_\circ$ at $t+1$ with the frontier of time t, calculated by the following problem:

$$\begin{gathered} D^t\left(X_\circ^{t+1}, Y_\circ^{t+1}\right) = \operatorname{Min}\theta, \\ \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij}^t \leq \theta x_{io}^{t+1}, \quad i = 1, \ldots, m, \\ \sum_{j=1}^n \lambda_j y_{rj}^t \geq y_{ro}^{t+1}, \quad r = 1, \ldots, s, \\ \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{gathered} \tag{3}$$

The same model $D^{t+1}(X_\circ^t, Y_\circ^t)$ is calculated:

$$\begin{gathered} D^{t+1}\left(X_\circ^t, Y_\circ^t\right) = \operatorname{Min}\theta, \\ \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij}^{t+1} \leq \theta x_{io}^t, \quad i = 1, \ldots, m, \\ \sum_{j=1}^n \lambda_j y_{rj}^{t+1} \geq y_{ro}^t, \quad r = 1, \ldots, s, \\ \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{gathered} \tag{4}$$

Fare hypotheses $D_\circ^{t+1}(X_\circ^{t+1}, Y_\circ^{t+1})$, $D_\circ^t(X_\circ^t, Y_\circ^t)$ must be equal to 1 to be efficient. Therefore he defined relative efficiency change as

$$\text{TEC}_\circ = \frac{D_\circ^{t+1}\left(X_\circ^{t+1}, Y_\circ^{t+1}\right)}{D_\circ^t\left(X_\circ^t, Y_\circ^t\right)}. \tag{5}$$

He described one geometric compotation to determine technology change between t and $t+1$:

$$\text{FS}_\circ = \left[\frac{D_\circ^t(X_\circ^{t+1}, Y_\circ^{t+1})}{D_\circ^{t+1}(X_\circ^{t+1}, Y_\circ^{t+1})} \cdot \frac{D_\circ^t(X_\circ^t, Y_\circ^t)}{D_\circ^{t+1}(X_\circ^t, Y_\circ^t)}\right]^{1/2}. \tag{6}$$

MPI will be calculated from multiplication efficiency change and technology change for each input oriented $\text{DMU}_\circ$ at time t and $t+1$:

$$\begin{aligned} M_\circ = {} & \frac{D_\circ^{t+1}\left(X_\circ^{t+1}, Y_\circ^{t+1}\right)}{D_\circ^t\left(X_\circ^t, Y_\circ^t\right)} \\ & \times \left[\frac{D_\circ^t\left(X_\circ^{t+1}, Y_\circ^{t+1}\right)}{D_\circ^{t+1}\left(X_\circ^{t+1}, Y_\circ^{t+1}\right)} \cdot \frac{D_\circ^t\left(X_\circ^t, Y_\circ^t\right)}{D_\circ^{t+1}\left(X_\circ^t, Y_\circ^t\right)}\right]^{1/2}. \end{aligned} \tag{7}$$

The simple form of relation (9) is

$$M_\circ = \left[\frac{D_\circ^t(X_\circ^{t+1}, Y_\circ^{t+1})}{D_\circ^t(X_\circ^t, Y_\circ^t)} \cdot \frac{D_\circ^{t+1}(X_\circ^{t+1}, Y_\circ^{t+1})}{D_\circ^{t+1}(X_\circ^t, Y_\circ^t)}\right]^{1/2}. \tag{8}$$

This value defines geometric convex compotation, because it specified the smallest decrease of efficiencies and any small change in each efficiency effect in MPI. Three conditions are available.

(1) $M_\circ > 1$; increase productivity and observe progress.

(2) $M_\circ < 1$; decrease productivity and observe regress.

(3) $M_\circ = 1$; no change is seen in productivity at time $t+1$ in comparison to t.

3. Efficiency of Time Dependent Units

Usually, study cases have time dependent inputs and outputs, and their efficiency is managers' interest. In this relation a time dependent function is necessary to evaluate the efficiency of this kind of data in virtual intervals.

A system that includes n DMUs which include m inputs and s outputs is supposed. Inputs and outputs may be a function dependent on time; then assume that $f_{ip}(t)$ and $g_{op}(t)$ are ith inputs and oth output of DMU_p. As this, each DMU is represented as

$$\begin{aligned} & \text{DMU}_p \\ & = \left(f_{1p}(t), f_{2p}(t), \ldots, f_{mp}(t), g_{1p}(t), g_{2p}(t), \ldots, g_{sp}(t)\right)^t. \end{aligned} \tag{9}$$

These functions can contemplate linear or nonlinear and may be constant. To calculate the efficiency of DMU_p, in case of known function, for some t the inputs and outputs will be in hand. In some interval $[a, b]$, this parameter may assume random numbers. By using uniform distribution it may be supposed that

$$t_i = a + (b-a)\, d_i, \quad i = 1, \ldots, r, \tag{10}$$

where r is the number of numeral used in this interval and d_i are random numbers (without losing the generality, we assume that t_i are distinct). At first, the time points t_i will be sorted in an increasing order and then named w_i and indeed $a = w_0$ and $b = w_{r+1}$. Because of this, in the fixed amount of r, all inputs and outputs for any DMUs are fixed also. For each t, all DMUs are fixed. Because of this, in the fixed amount of r, all inputs and outputs for any DMUs are fixed, also. For each t, all DMUs are fixed; and the efficiency of DMU_p will be determined by using CCR model. To find the efficiency, it is necessary to solve n linear programs (LPs). Therefore, always the number of LPs is more. In supposed model, only distinguished number models are solved and, by estimating the function of efficiency, efficiency of ideal t is known.

The efficiency of time dependent data will be as follows according to the Cubic Spline function for pth DMU:

$$\theta(t) = \operatorname{Min}\{\theta_i(t), 1\} \quad \text{for } t \in [w_{i-1}, w_i],\ i = 1, \ldots, r+1. \tag{11}$$

That $\theta_i(t)$ is determined by

$$\begin{aligned}\theta_i(t) &= \alpha_i + \beta_i(t - w_{i-1}) + \gamma_i(t - w_{i-1})^2 \\ &\quad + \delta_i(t - w_{i-1})^3, \quad i = 1, \ldots, r+1.\end{aligned} \tag{12}$$

By calculating the efficiency values in some times (k) of this interval Cubic Spline function can be predicted. This data is very close to efficiency function and has less calculating error.

The goal of this study was to determine the progress and regress of time dependent DMUs. Following a simple method will be illustrated in this relation.

4. The Progress and Regress of Time Dependent Units

Here the value of progress and regress of time dependent data will be introduced and the developed method will be schemed and reported.

4.1. Assumed. As the inputs and outputs of each DMU are functions dependent on time, the centers of gravity of each subinterval are estimated.

In supposed method, at first the interval $[a, b]$ will be divided into n subintervals, showed by $I_k = [a_k, b_k]$. Here we hypothesize that the DMUs have m inputs and s outputs that are functions of time. Then $\mathrm{DMU}_j = (X_j, Y_j)$, $j = 1, \ldots, n$ are defined as follow:

$$\begin{aligned}X_j &= \left(f_{1j}(t), f_{2j}(t), \ldots, f_{mj}(t)\right), \\ Y_j &= \left(g_{1j}(t), g_{2j}(t), \ldots, g_{sj}(t)\right).\end{aligned} \tag{13}$$

In each subinterval I_k, an indicator can be set. To compute the mention coordinates, the centers of gravity of inputs and outputs are determined. These are contemplating as follows.

P_{ik}^j : the center of gravity of DMU_j for ith input in I_k.

Q_{rk}^j : the center of gravity of DMU_j for rth output in I_k.

The center of gravity coordinate is showed as

$$P_{ik}^j = (\bar{t}_{ik}, \bar{x}_{ik}), \qquad Q_{rk}^j = (\bar{t}_{rk}, \bar{y}_{rk}), \tag{14}$$

$\bar{t}_{ik}$ and $\bar{x}_{ik}$ are calculated by

$$\bar{t}_{ik}^j = \frac{\int_{a_k}^{b_k} t \cdot f_{ij}(t)\,dt}{\int_{a_k}^{b_k} f_{ij}(t)\,dt} \qquad \bar{x}_{ik}^j = \frac{(1/2)\int_{a_k}^{b_k} (f_{ij}(t))^2\,dt}{\int_{a_k}^{b_k} f_{ij}(t)\,dt}, \quad i = 1, \ldots, m. \tag{15}$$

And the $\bar{t}_{rk}$ and $\bar{y}_{rk}$ are computed:

$$\bar{t}_{rk}^j = \frac{\int_{a_k}^{b_k} t \cdot g_{rj}(t)\,dt}{\int_{a_k}^{b_k} g_{rj}(t)\,dt}, \qquad \bar{y}_{rk}^j = \frac{(1/2)\int_{a_k}^{b_k} \left(g_{rj}(t)\right)^2\,dt}{\int_{a_k}^{b_k} g_{rj}(t)\,dt}, \quad r = 1, \ldots, s. \tag{16}$$

Because of the above formula, the indicators of each subinterval are determined by

$$m_k^j = \left(P_{ik}^j, Q_{rk}^j\right), \quad i = 1, \ldots, m,\ r = 1, \ldots, s, \quad k = 1, \ldots, n-1. \tag{17}$$

These indicators are distinct; then the MPI should be exploited by using (8).

4.2. The Malmquist Productivity Index for Time Dependent Data. At first the centers of gravity of each subinterval are taken into consideration. As the functions are dependent on one variable, the first quantities are not used to calculate. Therefore the center of gravity is

$$P_{ik}^j = (\bar{x}_{ik}), \quad Q_{rk}^j = (\bar{x}_{rk}), \quad i = 1, \ldots, m,\ r = 1, \ldots, s, \tag{18}$$

If m inputs and s outputs are available, then $ns + nm$ notes should be computed; they are remembered by (17).

The MPI index for $m_1^j, \ldots, m_k^j$ is considered by M_k^j in subinterval I_k. All subintervals may be compared with each other. These values show the scope of progress and regress between two consecutive subintervals. If the calculative value is more than 1, this unit has progress, and for less than one the unit demonstrates regress. Otherwise it does not show any changes.

The managers understand DMU behavior in subintervals by using this comparison and have the possibility to apply this data for scientific evaluations.

The goal of this study is to define and set a time dependent function of progress and regress in $[a, b]$. By applying this function, man can estimate the unit change in the particular important time.

In Section 2, the Cubic Spline function was offered as a suitable approximation. The one variable function $M_j(t)$ is

Table 1: Inputs and outputs.

DMU	Branch	Input	Output 1	Output 2
1	Tabriz	$-0.7049t^3 + 8.3799t^2 - 29.811t + 53.557$	$-7E+09t^5+1E+11t^4-9E+11t^3+3E+12t^2-4E+12t+2E+12$	$3E+08t^4 - 3E+09t^3 + 1E+10t^2 - 2E+10t + 3E+10$
2	Ahvaz	$-0.7049t^3 + 8.3799t^2 - 29.811t + 53.557$	$1E+10t^4 - 2E+11t^3 + 8E+11t^2 - 1E+12t + 1E+12$	$2E+08t^5-3E+09t^4+2E+10t^3-5E+10t^2+7E+10t-2E+10$
3	Shiraz	$-0.3245t^3 + 3.6285t^2 - 11.966t + 40.723$	$-1E+09t^3 + 1E+10t^2 - 4E+10t + 3E+11$	$6E+08t^3 - 6E+09t^2 + 2E+10t + 4E+09$
4	Kish	$-0.352t^3 + 3.9853t^2 - 12.666t + 33.953$	$-2E+10t^5+4E+11t^4-2E+12t^3+7E+12t^2-1E+13t+5E+12$	$2E+08t^5-4E+09t^4+3E+10t^3-9E+10t^2+1E+11t-7E+10$
5	Mashhad	$-0.9613t^3 + 11.56t^2 - 41.79t + 90.297$	$-6E+09t^5+1E+11t^4-7E+11t^3+2E+12t^2-3E+12t+2E+12$	$2E+10e^{0.1409t}$
6	Semnan	$-0.3832t^3 + 4.3138t^2 - 13.503t + 42.087$	$1E+09t^4 - 2E+10t^3 + 9E+10t^2 - 2E+11t + 5E+11$	$2E+08t^5-3E+09t^4+1E+10t^3-3E+10t^2+4E+10t-7E+09$
7	Karaj	$-0.659t^3 + 7.9091t^2 - 28.539t + 58.553$	$3E+10t^4 - 3E+11t^3 + 1E+12t^2 - 2E+12t + 1E+12$	$2E+09t^5-3E+10t^4+2E+11t^3-7E+11t^2+1E+12t-5E+11$
8	Esfahan	$-0.7724t^3 + 9.2403t^2 - 32.373t + 60.097$	$6E+10t^4 - 7E+11t^3 + 3E+12t^2 - 6E+12t + 6E+12$	$2E+09t^5-2E+10t^4+1E+11t^3-3E+11t^2+4E+11t-1E+11$
9	Yazd	$16.935e^{0.0359t}$	$4E+09t^5-6E+10t^4+3E+11t^3-7E+11t^2+7E+11t+2E+11$	$4E+08t^5-7E+09t^4+4E+10t^3-1E+11t^2+1E+11t-4E+10$

Table 2: The input and outputs center of gravity for DMU_1.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	13.0252	10.3282	10.8812	12.3981	12.7758
$Y_1(t)$	$8.63E+10$	$3.59E+11$	$1.62E+12$	$5.14E+12$	$1.22E+13$
$Y_2(t)$	$7.09E+09$	$3.95E+09$	$3.46E+09$	$3.73E+09$	$1.68E+10$

Table 3: The input and outputs center of gravity for DMU_2.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	13.0252	10.3282	10.8812	12.3981	12.7758
$Y_1(t)$	$3.41E+11$	$3.71E+11$	$3.48E+11$	$8.91E+11$	$2.37E+12$
$Y_2(t)$	$1.41E+10$	$3.10E+10$	$6.62E+10$	$1.27E+11$	$2.30E+11$

supposed for the evaluation of MPI of DMU_j. To determine this function, the values M_k^j calculated by center of gravity are used. These are the evaluation between (P_{il}^j, Q_{rl}^j) and (P_{il+1}^j, Q_{rl+1}^j) for $l = 1, \ldots, n$. Therefore $n-2$ values of MPI are in hand. By using these values, the Malmquist function can be calculated. This function should be a piecewise linear function. This function should be a piecewise linear function. It will be computed in interval I_k by Cubic Spline function as

$$M_k^j(t) = \alpha_j + \beta_j\left(t - M_k^j\right) + \gamma_j\left(t - M_k^j\right)^2 + \delta_j\left(t - M_k^j\right)^3, \quad k = 1, \ldots, n, \quad j = 1, \ldots, n. \tag{19}$$

In the vole of hypotheses interval the MPI is computed with (20) for DMU_j:

$$M_{\mathrm{DMU}_j}(t) = \begin{cases} M_1^j(t); & t \in [a_1, b_1] \\ M_2^j(t); & t \in [a_2, b_2] \\ \quad\vdots & \\ M_n^j(t); & t \in [a_n, b_n]. \end{cases} \tag{20}$$

Function 20 determines the value of MPI at the defined special time. The following cases are possible.

(A) If the function value is more than one during consideration time, the DMU has progress.

(B) If the function value is less than one during consideration time, the DMU has regress.

(C) Otherwise, the intersection of this function and line one is necessary to calculate. The sums of function integral of regions above and under mentioned line 1 should be subtracted. If this difference is more than zero the DMU has progress, and if it is less than zero, the evaluated DMU has regress.

To explain case (C), suppose the above region functions are $M_l(t)$, $M_m(t)$, and $M_n(t)$ and the below functions are $M_r(t)$, $M_s(t)$, $M_t(t)$, and $M_u(t)$; then,

$$\begin{aligned} S_1 &= \int_{a_1}^{b_1} M_l(t)\,dt + \int_{a_2}^{b_2} M_m(t)\,dt + \int_{a_3}^{b_3} M_n(t)\,dt, \\ S_2 &= \int_{a_4}^{b_4} M_r(t)\,dt + \int_{a_5}^{b_5} M_s(t)\,dt \\ &\quad + \int_{a_6}^{b_6} M_t(t)\,dt + \int_{a_7}^{b_7} M_u(t)\,dt. \end{aligned} \tag{21}$$

And $[a_i, b_i]$ are the intersections point; according to this,

$$S = S_1 - S_2 \tag{22}$$

is the criterion of progress and regress.

(1) If $S < 0$ then the DMU has progress in the evaluated interval.

(2) If $S > 0$ then the DMU has regress in the evaluated interval.

(3) Otherwise the DMU does not have any changes in this evaluation interval.

The proposed method of this study should be assigned with simple calculation and solving $n-2$ linear programing. Therefore this method is very useful and when the Malmquist function is set, for each time, the system programmer can determine the progress and regress with replacing the considered time of this function.

5. Application

In this section, the application of the mentioned method will be reported for studying the progress and regress of nine commercial bank branches in Iran.

These branches have similar time dependent data. The input is the personal value and the relevant outputs are the sum of four deposits and interest, which are linear or nonlinear function of time. The units are as in Table 1.

The study interval was $[1, 6]$. By using (18), the center of the gravity for DMUs is calculated. Each month is supposed to be a subinterval. These centers are shown in Tables 2, 3, 4, 5, 6, 7, 8, 9 and 10.

Table 4: The input and outputs center of gravity for DMU_3.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	15.0275	14.2578	14.7013	15.3614	15.2734
$Y_1(t)$	$1.30E+11$	$1.24E+11$	$1.20E+11$	$1.16E+11$	$1.08E+11$
$Y_2(t)$	$1.12E+10$	$1.29E+10$	$1.31E+10$	$1.37E+10$	$1.64E+10$

Table 5: The input and outputs center of gravity for DMU_4.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	11.4851	14.2578	11.696	12.7644	13.0662
$Y_1(t)$	$7.86E+11$	$3.74E+12$	$1.08E+13$	$2.49E+13$	$4.93E+13$
$Y_2(t)$	$2.03E+10$	$2.46E+10$	$1.69E+10$	$1.87E+10$	$5.12E+10$

Table 6: Input and outputs center of gravity for DMU_5.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	25.6474	21.7128	22.2869	24.3196	24.928
$Y_1(t)$	$1.83E+11$	$3.99E+11$	$1.19E+12$	$2.56E+12$	$4.77E+12$
$Y_2(t)$	$8.71E+08$	$1.00E+09$	$1.15E+09$	$1.33E+09$	$1.53E+09$

Table 7: The input and outputs center of gravity for DMU_6.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	15.2452	14.7128	15.6412	16.8485	17.1966
$Y_1(t)$	$1.71E+11$	$1.44E+11$	$9.96E+10$	$4.69E+11$	$1.66E+11$
$Y_2(t)$	$5.80E+09$	$2.51E+10$	$8.55E+10$	$2.05E+11$	$3.99E+11$

Table 8: The input and outputs center of gravity for DMU_7.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	15.9811	13.2981	13.7001	15.0854	15.4789
$Y_1(t)$	$3.31E+11$	$6.56E+11$	$1.07E+12$	$1.37E+12$	$1.11E+12$
$Y_2(t)$	$3.30E+10$	$1.24E+11$	$2.27E+11$	$2.73E+11$	$1.99E+11$

Between each two subintervals, the MPI is rated by using formula (8). These values are reported in Table 11.

Because of 6-month study duration, four indexes are reported for each branch and the Cubic Spline functions of these amounts are introduced as follow:

$$M_1(t) = 0.5837t^3 - 7.3715t^2 + 29.3121 - t33.0331,$$
$$M_2(t) = 0.2368t^3 - 2.6753t^2 + 1.2343t - 9.0667,$$
$$M_3(t) = -0.0108t^3 + 0.2168t^2 - 9.6318t - 3.1143,$$
$$M_4(t) = 0.3868t^3 - 4.6083t^2 + 16.7943t - 15.3944,$$
$$M_5(t) = 0.3408t^3 - 4.2112t^2 + 16.3108t - 17.2072,$$
$$M_6(t) = 0.3202t^3 - 3.9928t^2 + 15.516t - 15.9178,$$
$$M_7(t) = -0.2917t^3 + 4.089t^2 - 31.8263t + 19.3241,$$
$$M_8(t) = -2.1667t^3 + 26.333t^2 - 101.153t + 124.5783,$$
$$M_9(t) = 0.9095t^3 - 11.0407t^2 + 43.0116t - 51.6083. \tag{23}$$

With drawing these functions plotted in Figures 1, 2, 3, 4, 5, 6, 7, 8, and 9, researchers can consider and analyze the functions easily.

As shown in Figures 1 to 9, DMU_1 and DMU_2 have progress because their relative diagram values are more than one for all points in supposed interval; the diagram value of DMU_2 is higher than DMU_1; it demonstrates that this unit has more progress than the other.

The DMU_1, DMU_4, DMU_5, and DMU_6 have progress with decreasing in their values. DMU_2 fluctuates at first but increases gradually. DMU_9 changes in interval $[1, 6]$. DMU_6 progress is higher than DMU_5 and less than DMU_1 and DMU_4.

Table 9: The input and outputs center of gravity for DMU_8.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	15.2414	12.5698	13.5039	15.5444	16.3844
$Y_1(t)$	$8.96E+11$	$5.84E+11$	$3.83E+11$	$3.32E+11$	$1.41E+12$
$Y_2(t)$	$3.71E+10$	$1.03E+11$	$2.02E+10$	$1.92E+11$	$8.76E+11$

Table 10: Input and outputs center of gravity for DMU_9.

Month	First	Second	Third	Fourth	Fifth
$X(t)$	0.1604	0.1663	0.1723	0.1786	0.1852
$Y_1(t)$	$2.06E+11$	$1.55E+11$	$6.64E+11$	$4.29E+11$	$1.03E+12$
$Y_2(t)$	$6.86E+09$	$1.24E+10$	$2.22E+10$	$5.72E+10$	$1.34E+11$

Table 11: The Malmquist productivity index.

DMU	M_1	M_2	M_3	M_4
DMU_1	3.295	4.283	2.785	2.303
DMU_2	1.993	2.027	1.684	2.385
DMU_3	1.214	0.985	0.962	1.08
DMU_4	3.834	3.52	2.113	1.934
DMU_5	2.575	2.906	1.972	1.818
DMU_6	2.92	3.204	2.226	1.907
DMU_7	4.515	1.777	1.092	0.71
DMU_8	2.422	0.225	5.194	4.329
DMU_9	1.127	2.678	1.247	2.291

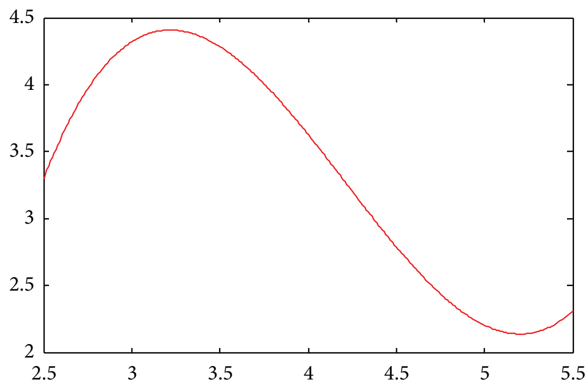

Figure 1: The diagram of Malmquist product index DMU_1.

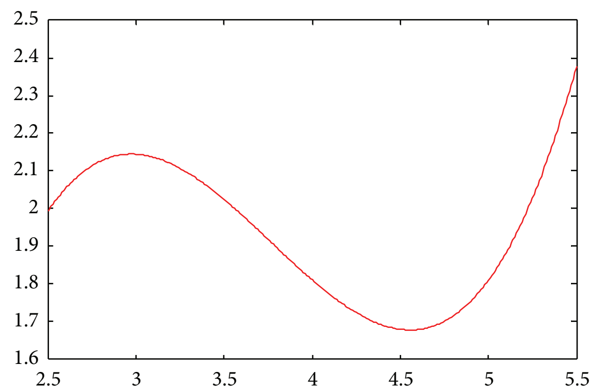

Figure 2: The diagram of Malmquist product index DMU_2.

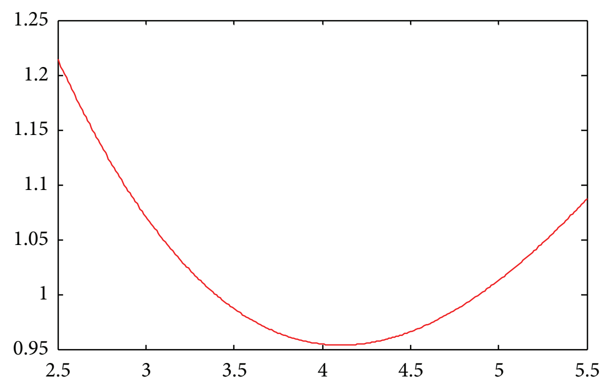

Figure 3: The diagram of Malmquist product index DMU_3.

Because DMU_3, DMU_7, and DMU_8 cut the line 1, so it is necessary to calculate the intersection points and related regions.

DMU_3 cuts the line at two points in the mentioned interval. The amount of function will be set zero, when the function value is less than zero. The values of the areas are

$$\begin{aligned} S_1 &= \int_{2.5}^{3.3956} \left(-0.0108t^3 + 0.2168t^2 - 9.6318t - 3.1143\right) dt \\ &\quad + \int_{4.8912}^{5.5} \left(-0.0108t^3 + 0.2168t^2 - 9.6318t - 3.1143\right) dt \\ &= 1.6107, \\ S_2 &= \int_{3.3956}^{4.8912} \left(-0.0108t^3 + 0.2168t^2 - 9.6318t - 3.1143\right) dt \\ &= 1.4496. \end{aligned} \tag{24}$$

This unit has progress in the interval. It observes decreasing and, after point 4.8912, increases. This is to see in Table 11, too. DMU_7 also cuts the line in one point. These spaces are

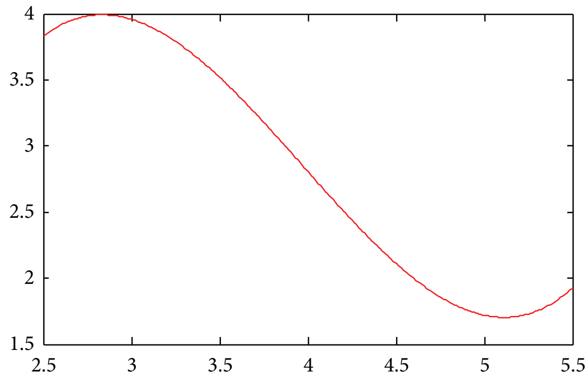

FIGURE 4: The diagram of Malmquist product index DMU_4.

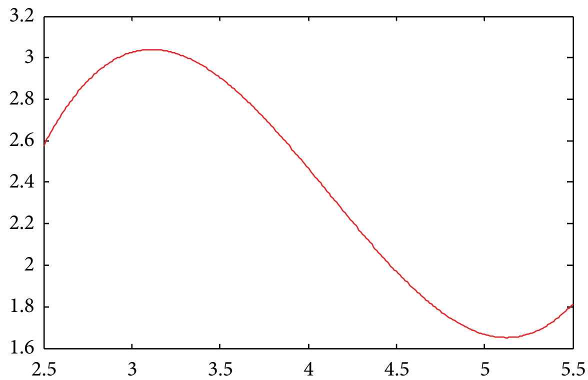

FIGURE 5: The diagram of Malmquist product index DMU_5.

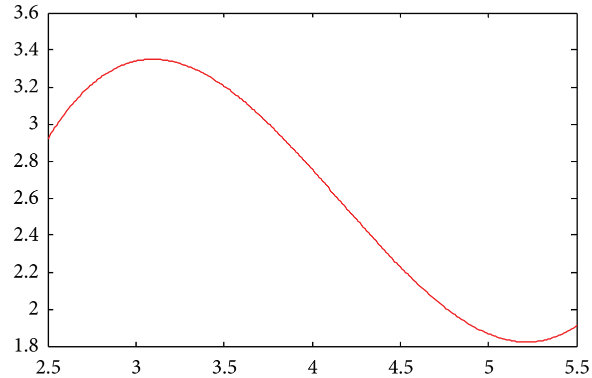

FIGURE 6: The diagram of Malmquist product index DMU_6.

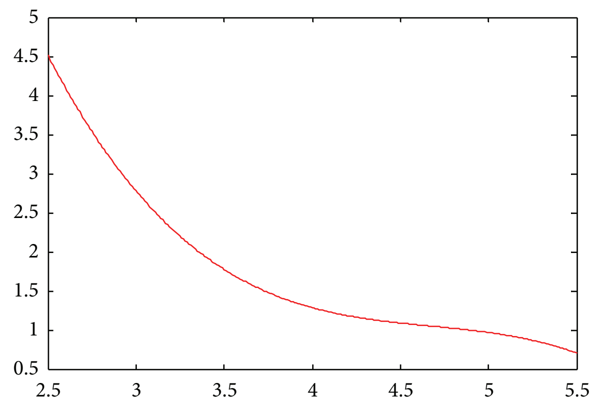

FIGURE 7: The diagram of Malmquist product index DMU_7.

determined as (25), so this units has progress, too. Consider the following:

$$\begin{aligned} S_1 &= \int_{2.5}^{4.8881} \left(-0.2917t^3 + 4.089t^2 - 31.8263t + 19.3241\right) dt \\ &= 4.6408, \\ S_2 &= \int_{4.8881}^{5.5} \left(-0.2917t^3 + 4.089t^2 - 31.8263t + 19.3241\right) dt \\ &= 0.5383. \end{aligned} \tag{25}$$

For the range of less than zero value of the efficiency function value related to DMU_8, those values will be set equal zero. Consider the following:

$$\begin{aligned} S_1 &= \int_{2.5}^{2.6675} \left(-2.1667t^3 + 26.333t^2 - 101.153t + 124.5783\right) dt \\ &\quad + \int_{3.6872}^{5.5} \left(-2.1667t^3 + 26.333t^2 - 101.153t + 124.5783\right) dt \\ &= 8.6052, \\ S_2 &= \int_{2.6675}^{3.6872} \left(-2.1667t^3 + 26.333t^2 - 101.153t + 124.5783\right) dt \\ &= 0.2037. \end{aligned} \tag{26}$$

By this application, all units have progress in the study time interval. It is obvious that, by interval increasing and studying the bank branches for a year, the results may be changed. Some units have progress in supposed interval but show regress in some subintervals, and vice versa. These results and study helped the bank managers to render the units at short and long terms.

6. Conclusion

The proposed and study method reported here is a valuable scientific method. This method is applicable for examining and analyzing time dependent inputs and outputs. This introduced function is a calculating function and solves less linear programing. This simple function could be used to evaluate the progress and regress of each or several units with the time. It is very suitable for application programing. It

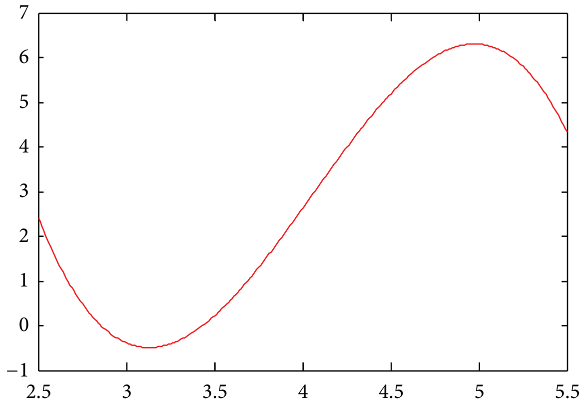

Figure 8: The diagram of Malmquist product index DMU_8.

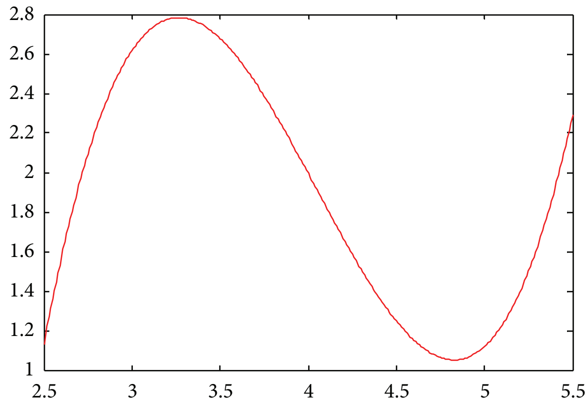

Figure 9: The diagram of Malmquist product index DMU_9.

foresees the other data out of the supposed interval and offers the initial evaluation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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