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Research Article

Some New Exact Solutions of (1+2)-Dimensional Sine-Gordon Equation

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We use a generalized tanh function expansion method and a direct method to study the analytical solutions of the (1+2)-dimensional sine Gordon (2DsG) equation. We obtain some new interaction solutions among solitary waves and periodic waves, such as the kink-periodic wave interaction solution, two-periodic solitoff solution, and two-toothed-solitoff solution. We also investigate the propagation properties of these solutions.

1. Introduction

Sine-Gordon (sG) equation is one of the most famous partial differential equations that have been investigated by many physicists for decades years. The sG equation has played a central role in lots of different scientific fields, such as in differential geometry [1], plasma physics [2], nonlinear optics [3], condensed matter physics [4], quantum field theory [5, 6], and so forth. Researchers have been spending a great deal of effort to generalize (1+1)-dimensional soliton equations to (2+1)-dimensional equations. Remarkable of these equations, in the 1980s, the Nizhnik-Novikov-Veselov (NNV) equation [7–9] and the Davey-Stewartson (DS) equation [10–12] were found. The NNV equation and the DS equation are (2+1) dimensional generalizations of the Korteweg-de Vries (KdV) equation and nonlinear Schrödinger (NLS) equation, respectively. After that, in 1991, Konopelchenko and Rogers [13, 14] proposed a significant symmetry to generalize the (1+1)-dimensional sG equation to (2+1)-dimensional sG equation through a reinterpretation and generalization of a class of infinitesimal Bäcklund transformation. The well-known non-integrable (2+1)-dimensional sine-Gordon (2DsG) equation is as follows:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial t^2} = m \sin \Phi. \quad (1)$$

Various methods have been used to study this equation because of its rich symmetrical structure. The brief and effective methods for solving the 2DsG equation include the binary Darboux transformation [15, 16], the extensive symmetry group analysis [17, 18], Hirota's method [19], Lamb's method [20, 21], the Painlevé transcendents [22], and the Bäcklund transformation [23]. And researchers have found abundant types of solutions of 2DsG equation, such as the multisoliton solutions and vortex-like solution [24], line and ring solitons [25, 26], curve soliton, point instanton soliton and doubly periodic wave solutions [27–29], Solitoff structure solution, and snake-shape solitary wave solution [30].

Recently, some new useful and powerful methods have been proposed to search for the accurate solutions of nonlinear partial differential equations, such as the general algebra method for the coupled Schrödinger-Boussinesq equations [31], the general mapping deformation method for the generalized variable-coefficient Gardner equation with forcing term [32], the generalized tanh function expansion method for the Abowitz-Kaup-Nwell-Segur system [33], the bosonized supersymmetric KdV model [34], and the Broer-Kaup system [35]. Significantly, the generalized tanh function expansion method is an effective new technique for us to obtain some new interaction solutions of 2DsG equation. Also, we can solve the 2DsG equation by a direct method based on the mapping relations between 2DsG equation and the cubic nonlinear Klein-Gordon (CNKG) equation. This method can be also applied to solve the double sine-Gordon equation, the triple sine-Gordon equation, and the Ginzburg-Landau equation [36], and so forth. In this paper, we want to seek more interaction solutions of new types among solitary waves and periodic waves of the 2DsG equation by the generalized tanh function expansion method and the direct method.

This paper is organized as follows. In Section 2, a kink-periodic wave interaction solution of 2DsG equation is obtained by using of the generalized tanh function expansion method. In Section 3, two-periodic solitoff solution, periodic soliton-periodic travelling wave interaction solution, two-toothed-solitoff solution, and periodic solitoff-kink interaction solution of 2DsG equation are obtained by using the direct method. In Section 4, a short summary and discussions are given.

2. Kink-Periodic Wave Interaction Solutions

The 2DsG equation (1) cannot be solved directly by the generalized tanh function expansion method [33–35], and to find some soliton-periodic wave interaction solutions of 2DsG equation, we suppose

$$\Phi = -i \ln [W(X,T)], \tag{2}$$

and take the following coordinates transformation:

$$X = x + \alpha_1 y + \beta_1 t, \qquad T = \alpha_2 x + y + \beta_2 t. \tag{3}$$

Then, we substitute (2) with (3) into (1) and arrive at

$$W_X W_T - W W_{XT} + W - W^3 = 0, \tag{4}$$

with the constants α_1, α_2, β_1, and β_2 satisfying

$$\alpha_2^2 = \beta_2^2 - 1, \qquad \beta_1^2 = 1 + \alpha_1^2, \qquad \alpha_1 = \beta_1\beta_2 - \alpha_2 + \frac{m}{4}. \tag{5}$$

It is worth noting that (4) can be solved by using the generalized tanh function expansion method. Firstly, we set

$$W = u_2 \tanh^2(\Psi) + u_1 \tanh(\Psi) + u_0, \tag{6}$$

where u_2, u_1, u_0, and Ψ are functions of variables (X,T). In order to obtain some soliton-periodic wave interaction solutions, let

$$\Psi = \xi_1 + \psi_1(\xi), \tag{7}$$

where $\xi_1 = k_1 X + \omega_1 T$, $\xi = kX + \omega T$, in which k, ω, k_1, and ω_1 are undetermined constants. Then, we substitute (6) and (7) into (4) and analyse the coefficients of function $\tanh(\Psi)$ order by order we get the expression of W

$$\begin{aligned} W &= 2[\omega_1 + \psi_2(\xi)\omega][k_1 + \psi_2(\xi)k]\tanh^2(\Psi) \\ &\quad - 2\psi_3(\xi)k\omega\tanh(\Psi) \\ &\quad + \frac{\psi_3^2(\xi)k^2\omega^2}{2[\omega_1 + \psi_2(\xi)\omega][k_1 + \psi_2(\xi)k]}, \end{aligned} \tag{8}$$

where the functions $\psi_2(\xi)$ and $\psi_3(\xi)$ satisfy

$$\psi_2(\xi) = \psi_{1\xi}(\xi), \qquad \psi_3(\xi) = \psi_{2\xi}(\xi). \tag{9}$$

Furthermore, $\psi_2(\xi)$ is a solution of the following Jacobi elliptic function equation:

$$\psi_{2\xi}^2(\xi) = a_0 + a_1\psi_2(\xi) + a_2\psi_2^2(\xi) + a_3\psi_2^3(\xi) + 4\psi_2^4(\xi), \tag{10}$$

with these parameters a_0, a_1, a_2, and a_3 satisfying

$$\begin{aligned} a_0 &= \frac{2\omega_1 k_1 (2\gamma k_1\omega_1 g + \gamma^2 k_1 k - \omega_1\omega)}{k^2\omega^2\gamma g}, \\ a_1 &= \frac{2(4\gamma k_1\omega_1 g h + \gamma^2 k_1 k q - \omega_1\omega p)}{k^2\omega^2\gamma g}, \\ a_2 &= \frac{2(4\gamma k_1 k\omega_1\omega g + \gamma^2 k^2 p + 2\gamma g h^2 - \omega^2 q)}{k^2\omega^2\gamma g}, \\ a_3 &= \frac{2(\gamma^2 k^2 + 4\gamma g h - \omega^2)}{k\omega\gamma g}, \end{aligned} \tag{11}$$

where $g = k\omega_1 - k_1\omega$, $h = k\omega_1 + k_1\omega$, $p = (3h - g)/2$, and $q = (3h + g)/2$, in which $k\omega_1 \neq k_1\omega$ and γ is a constant.

Now we choose the sine Jacobi elliptic function as a solution of (10),

$$\psi_2(\xi) = a\,\mathrm{sn}(b\xi), \tag{12}$$

and the functions $\psi_1(\xi)$ and $\psi_3(\xi)$ are easy to be obtained

$$\begin{aligned} \psi_1(\xi) &= \frac{a \ln[\mathrm{dn}(b\xi) - n\,\mathrm{cn}(b\xi)]}{bn}, \\ \psi_3(\xi) &= ab\,\mathrm{cn}(b\xi)\,\mathrm{dn}(b\xi). \end{aligned} \tag{13}$$

Then substituting (12), (13), and (11) into (10), relationships of these parameters are written as

$$\begin{aligned} b &= \frac{2a}{n}, \\ \omega_1 &= -\frac{k_1\omega(\gamma k^2[\gamma n^2 - 4a^2\omega^2(n^2+1)] + \omega^2 n^2(8\gamma k_1^2 + 1))}{k(\gamma k^2[\gamma n^2 + 4a^2\omega^2(n^2+1)] - \omega^2 n^2(8\gamma k_1^2 + 3))}, \\ \omega &= \frac{nk\gamma}{\sqrt{-2(1+8\gamma k_1^2)[2a^2k^2\gamma(n^2+1) - n^2(4\gamma k_1^2+1)] + n^2}}, \\ \gamma &= -\frac{n\left[-k_1 n + \sqrt{a^2k^2(n^2+1) - k_1^2 n^2}\right]}{4k_1[a^2k^2(n^2+1) - 2k_1^2 n^2]}, \\ k_1 &= \frac{\sqrt{2}k\sqrt{a^2(n^2+1) \pm a\sqrt{a^2(n^2+1)^2 - b^2n^4}}}{2n}, \end{aligned} \tag{14}$$

where n is the modulus of the Jacobi elliptic function $\mathrm{sn}(z) = \mathrm{sn}(z, n)$.

Finally, the accurate expression of Φ is gained:

$$\Phi = -i\ln\Bigg[\Bigg[2(\omega_1 + a\omega S)(k_1 + akS) \times \tanh\left[\xi_1 + \frac{a\ln(D - nC)}{bn}\right] - abk\omega CD\Bigg]^2 \times \left(2(\omega_1 + a\omega S)(k_1 + akS)\right)^{-1}\Bigg], \quad (15)$$

where $S = \mathrm{sn}(b\xi)$, $C = \mathrm{cn}(b\xi)$, and $D = \mathrm{dn}(b\xi)$. The solution of (15) denotes a kink-periodic wave interaction solution of 2DsG equation. Velocities of these two travelling waves are $v_1 = (k_1\beta_1 + \omega_1\beta_2)/[(k_1 + \omega_1\alpha_2)^2 + (k_1\alpha_1 + \omega_1)^2]^{1/2}$ and $v_2 = (k\beta_1 + \omega\beta_2)/[(k + \omega\alpha_2)^2 + (k\alpha_1 + \omega)^2]^{1/2}$, respectively.

Figure 1 shows the density distribution of a kink-periodic wave interaction solution on the x-y plane given by $[-\exp(i\Phi)]$ and (15) with these parameters

$$a = \frac{4}{5}, \quad n = \frac{9}{10}, \quad k = \frac{7}{2}, \quad b = \frac{16}{9}, \quad k_1 = \frac{28}{9},$$
$$\omega = -\frac{135\sqrt{10}}{8512}, \quad \omega_1 = -\frac{27\sqrt{10}}{2128}, \quad m = \frac{1}{4}, \quad \alpha_1 = -2,$$
$$\alpha_2 = -\frac{33 + 5\sqrt{13}}{64}, \quad \beta_1 = \sqrt{5}, \quad \beta_2 = -\frac{33\sqrt{5} + \sqrt{65}}{64}. \quad (16)$$

at time $t = 1$. This figure exhibits a special interaction structure of a kink and a periodic wave. Figure 2 shows the propagation of the kink-periodic wave solution at $y = 0$ and $t = 1$. In this figure, the soliton propagates along the negative direction of the x-axis, and its velocity is quicker than the one of the periodic wave, which also propagates along the negative x direction.

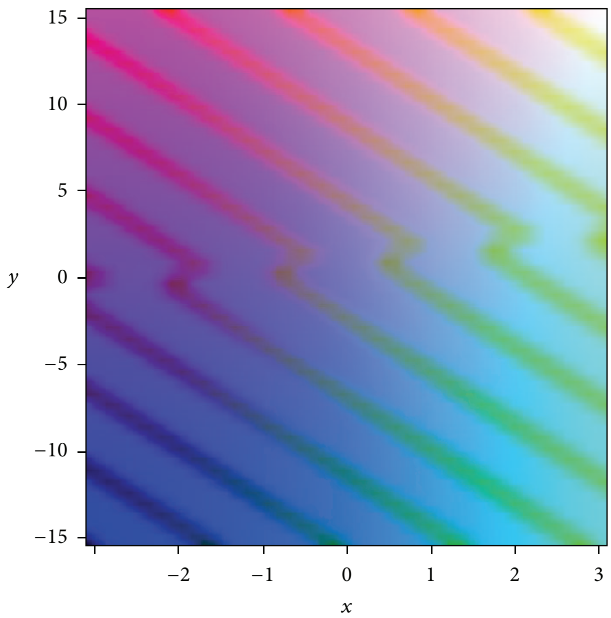


Figure 1: The density distribution of a kink-periodic wave interaction solution $[-\exp(i\Phi)]$ and (15) with (16) on the x-y plane at time $t = 1$.

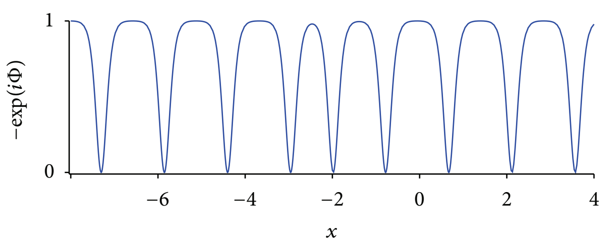


Figure 2: The propagation of the kink-periodic interaction solution $[-\exp(i\Phi)]$ and (15) with (16) at $y = 0$ and $t = 1$.

3. Solitoff, Periodic Soliton-Periodic Travelling Wave, and Periodic Solitoff-Kink Interaction Solutions

In this section, we use the direct method to study the 2DsG equation. Based on the Lamb substitution [20, 21], the solution of (1) can be set to the following form:

$$\Phi(x, y, t) = 4\arctan[M(x, y, t)], \quad (17)$$

in which the function $M(x, y, t)$ is the solution of the CNKG equation [30, 36],

$$M_{xx} + M_{yy} - M_{tt} = \lambda M + \mu M^3, \quad (18)$$

under the constrained condition

$$M_x^2 + M_y^2 - M_t^2 = \lambda M^2 + \frac{\mu}{2}M^4 + \frac{\mu}{2}, \quad (19)$$

with $m = \lambda - \mu$. Function $M(x, y, t)$ can be various styles, such as exp, tanh, sn, and dn [36]. Here we take

$$M = \sqrt{n}\,\mathrm{sn}(\widetilde{V}), \quad (20)$$

where function $\widetilde{V} = (\sqrt{|m|}V)/(n + 1)$, in which V is a function of variables (x, y, t), and the constant n is the modulus of the Jacobi elliptic function. Then, we substitute (17) and (20) into 2DsG equation and get

$$\begin{aligned} &|m|\left(\left(\tilde{\nabla}V\right)^2 + \frac{m}{|m|}\right)\left(n^2 + n\right)\mathrm{sn}^3\left(\widetilde{V}\right) \\ &+ n\sqrt{|m|}\Box V\,\mathrm{dn}\left(\widetilde{V}\right)\mathrm{cn}\left(\widetilde{V}\right)\mathrm{sn}^2\left(\widetilde{V}\right) \\ &- |m|\left(\left(\tilde{\nabla}V\right)^2 + \frac{m}{|m|}\right)(n + 1)\,\mathrm{sn}\left(\widetilde{V}\right) \\ &+ \sqrt{n}\sqrt{|m|}\Box V\,\mathrm{dn}\left(\widetilde{V}\right)\mathrm{cn}\left(\widetilde{V}\right) = 0, \end{aligned} \quad (21)$$

with the constrained conditions

$$\Box V = 0, \quad \left(\tilde{\nabla}V\right)^2 = \pm 1. \quad (22)$$

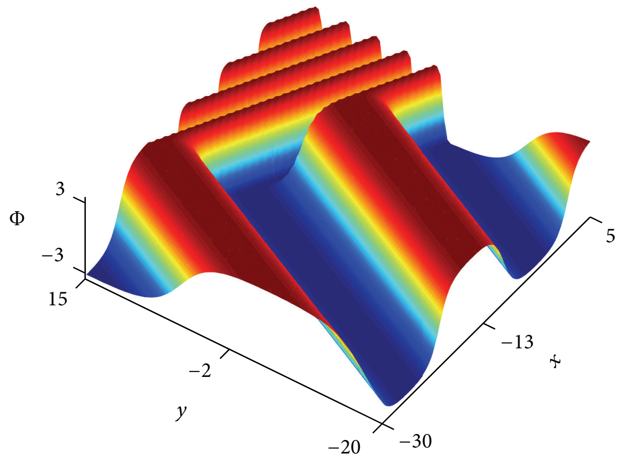


FIGURE 3: A two-periodic solitoff solution of 2DsG equation (27) with (28) at time $t = 0$.

Here we define

$$\square = \partial_x^2 + \partial_y^2 - \partial_t^2, \qquad (\tilde{\nabla})^2 = (\partial_x)^2 + (\partial_y)^2 - (\partial_t)^2; \tag{23}$$

then an arbitrary function $v(\xi)$ can be included in the function V by solving (22), namely,

$$V = v(\xi) + \xi_0 = v(k_{11}x + k_{12}y + \omega_1 t) + k_{01}x + k_{02}y + \omega_0 t, \tag{24}$$

and parameters k_{01}, k_{02}, k_{11}, k_{12}, ω_0, and ω_1 satisfy

$$k_{01}^2 + k_{02}^2 - \omega_0^2 = \pm 1, \qquad k_{11}^2 + k_{12}^2 - \omega_1^2 = 0, \qquad k_{01}k_{11} + k_{02}k_{12} - \omega_0\omega_1 = 0, \tag{25}$$

where the sign "±" in (22) and (25) takes "−" when $m > 0$ and takes "+" when $m < 0$. Due to the existence of the arbitrary functions, abundant exact solutions of (1) will be obtained as long as the function $v(\xi)$ is properly selected.

When we take

$$v(\xi) = \xi \arctan(\xi), \tag{26}$$

a (2+1)-dimensional two-periodic solitoff solution of 2DsG equation can be obtained:

$$\Phi = 4\arctan\left(\sqrt{n}\,\mathrm{sn}\left(\frac{\sqrt{|m|}}{1+n}\left((k_{11}x + k_{12}y + \omega_1 t) \times \arctan(k_{11}x + k_{12}y + \omega_1 t) + k_{01}x + k_{02}y + \omega_0 t\right)\right)\right). \tag{27}$$

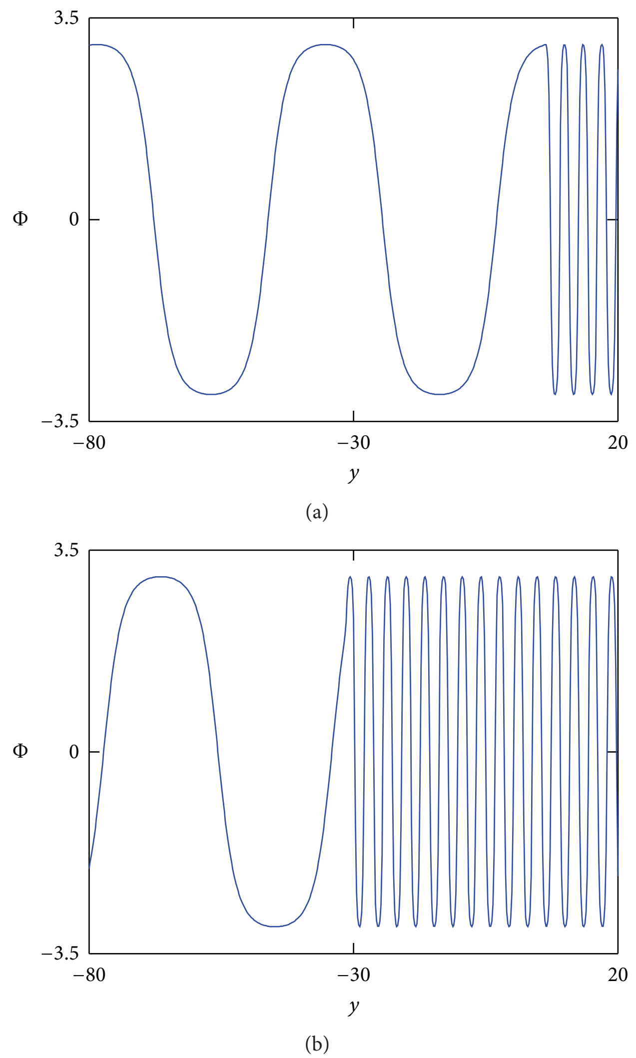


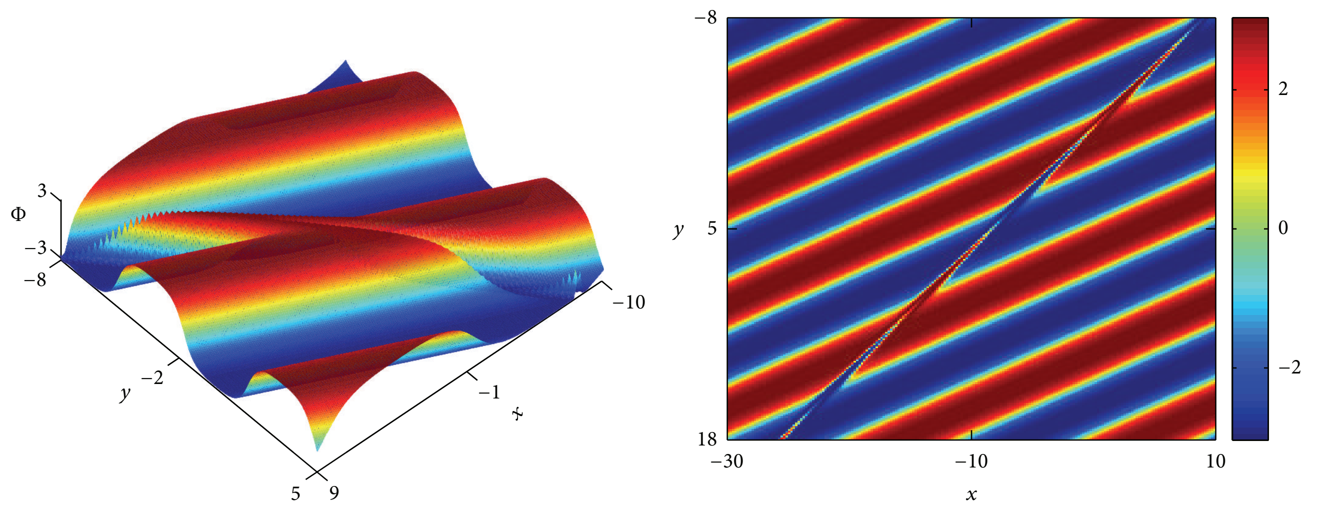
FIGURE 4: The propagation of two-periodic solitoff solution (27) with (28) along the y-axis when $x = 0$ at (a) $t = -5$ and (b) $t = 25$.

We know that a solitoff is defined as a half line soliton. The solution of (27) indicates a solitoff type solution constructed by two travelling waves that propagate in different directions. Velocities of these two travelling waves are $v_1 = \omega_0/\sqrt{k_{01}^2 + k_{02}^2} \neq 1$ and $v_2 = \omega_1/\sqrt{k_{11}^2 + k_{12}^2} = 1$, respectively.

Figure 3 shows a two-periodic solitoff solution (27) with these parameters

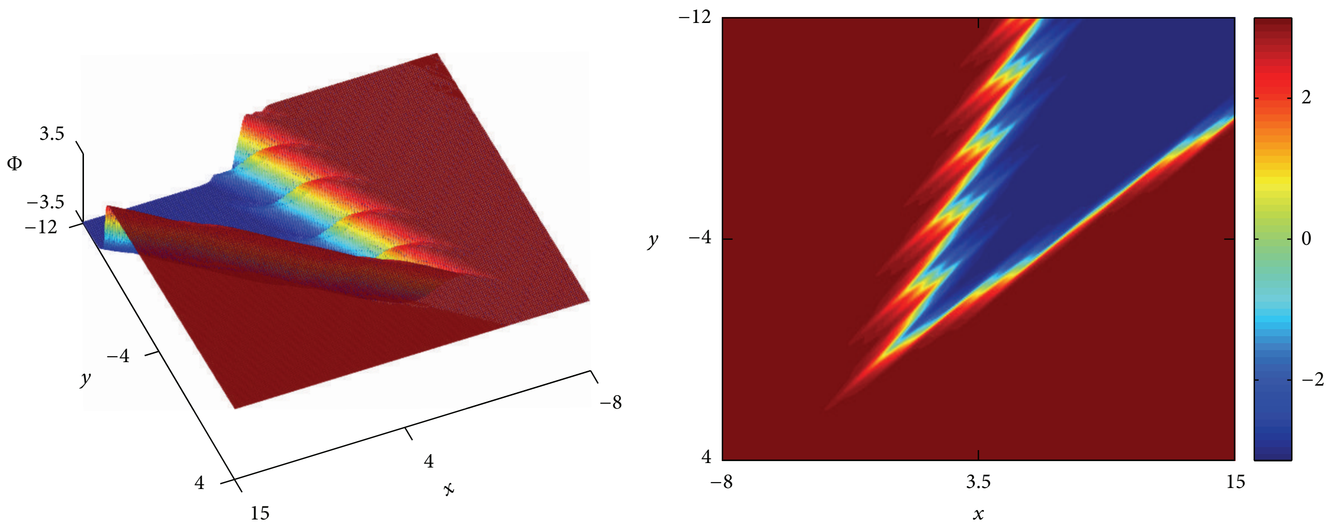
$$k_{01} = 1, \quad k_{02} = 3, \quad k_{11} = 1.2, \quad k_{12} = 1.6, \quad \omega_0 = 3, \quad \omega_1 = 2, \quad m = -1, \quad n = 0.9 \tag{28}$$

at time $t = 0$. The angle of the two-periodic solitoff in this figure is actually an obtuse angle although it seems to be orthogonal. It is because $\vec{k}_1 \cdot \vec{k}_2 = k_{01}k_{11} + k_{02}k_{12} = \omega_0\omega_1 \neq 0$. Figure 4 shows more details of the two-periodic solitoff solution (27) with (28). The two-periodic solitoff solution with different wavelength has the same amplitude and keeps



(a) (b)

Figure 5: (a) A periodic soliton-periodic travelling wave interaction solution (29) with (31) at time $t = 0$. (b) The density of Φ on the x-y plane.



(a) (b)

Figure 6: (a) A two-toothed-solitoff solution (33) with (31) at time $t = 1$ except for the modulus $n = 1$. (b) The density of Φ on the x-y plane.

the peak unchanged during the propagation process. Their phase velocities are different, but their travelling directions are same; they propagate along the negative y-axis.

A periodic soliton-periodic travelling wave interaction solution of 2DsG equation can be obtained:

$$\Phi = 4\arctan\left(\sqrt{n}\,\mathrm{sn}\left(\frac{\sqrt{|m|}}{1+n}\left(k_{01}x + k_{02}y + \omega_0 t + 7\mathrm{sech}^2\left(k_{11}x + k_{12}y + \omega_1 t\right) + 3\right)\right)\right), \tag{29}$$

by choosing

$$v(\xi) = 7\mathrm{sech}^2(\xi) + 3. \tag{30}$$

Figure 5(a) shows the periodic soliton-periodic travelling wave interaction solution (29) with these parameters

$$\begin{gathered} k_{01} = 1, \quad k_{02} = 3, \quad k_{11} = 3, \quad k_{12} = 4, \\ \omega_0 = 3, \quad \omega_1 = 5, \quad m = -1, \quad n = 0.9 \end{gathered} \tag{31}$$

at time $t = 0$. The solitoff-type structure solution does not appear, whereas these two travelling waves propagate in the different directions. The graph is similar to the soliton-periodic interaction wave in [33], but the soliton really has

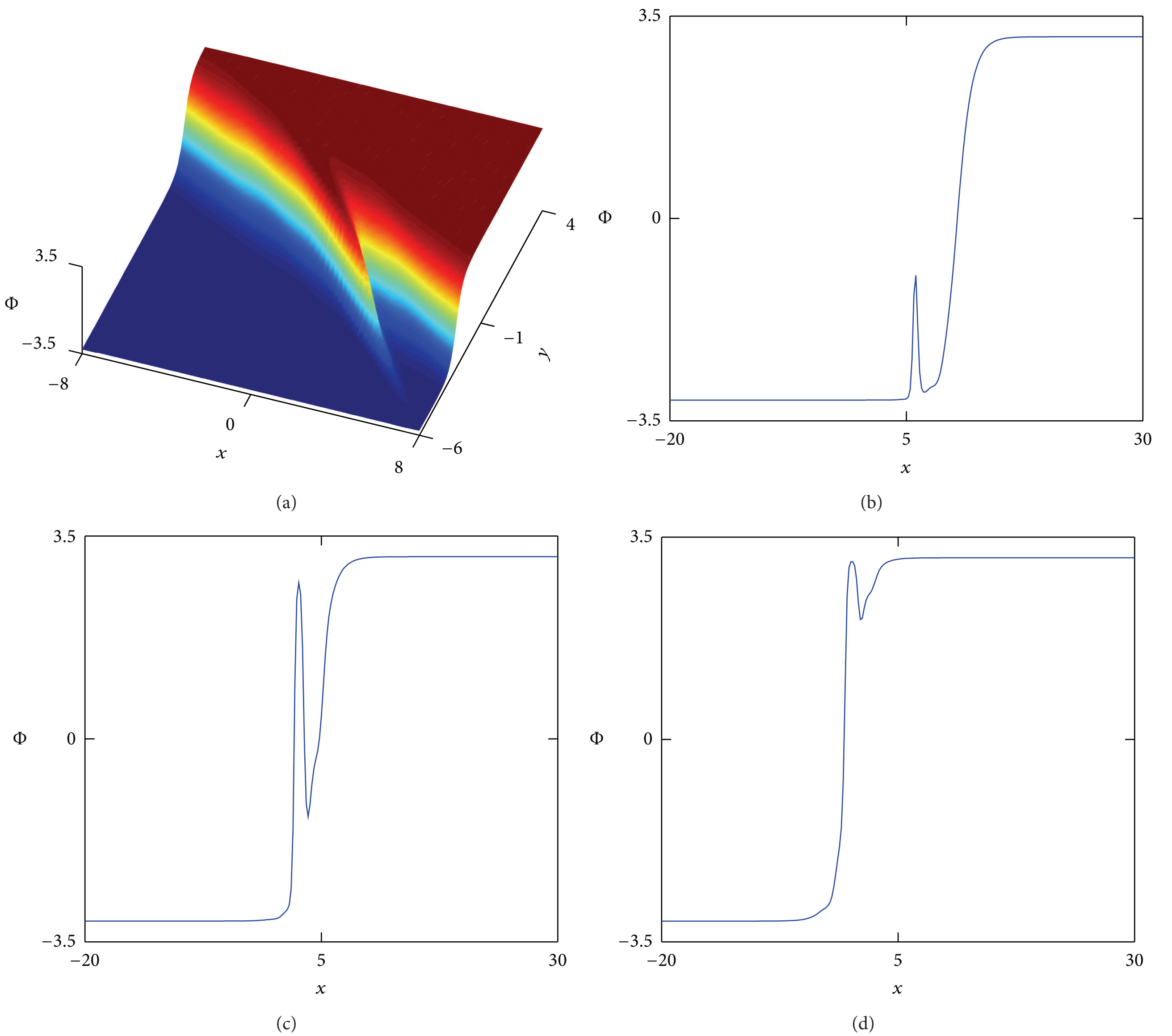


FIGURE 7: (a) A periodic solitoff-kink interaction solution (34) with (28) at time $t = 0$ except for the modulus $n = 1$. (b)–(d) show the propagation of the periodic solitoff-kink interaction solution along the x-axis when $y = -1.3$ at $t = -3$, $t = -0.5$, and $t = 1$.

the periodicity and the peak of the soliton keeps periodically changing. Figure 5(b) shows the density distribution of Φ on the x-y plane.

Furthermore, if we take

$$v(\xi) = \sqrt{\xi^2 + 1} + \frac{5}{4}\sin^3(\xi), \qquad v(\xi) = 3\cos^3(\xi)\,\mathrm{cn}(\xi) + 1, \tag{32}$$

then a two-sawtooth-solitoff solution and a periodic solitoff-kink interaction solution of 2DsG equation can be written as

$$\Phi = 4\arctan\left(\sqrt{n}\,\mathrm{sn}\left(\frac{\sqrt{|m|}}{1+n}\left(\frac{5}{4}\sin^3(k_{11}x + k_{12}y + \omega_1 t) + k_{01}x + k_{02}y + \omega_0 t + \sqrt{(k_{11}x + k_{12}y + \omega_1 t)^2 + 1}\right)\right)\right), \tag{33}$$

$$\Phi = 4\arctan\left(\sqrt{n}\,\mathrm{sn}\left(\frac{\sqrt{|m|}}{1+n}\left(k_{01}x + k_{02}y + \omega_0 t + 1 + 3\cos^3(k_{11}x + k_{12}y + \omega_1 t) \times \mathrm{cn}(k_{11}x + k_{12}y + \omega_1 t)\right)\right)\right), \tag{34}$$

respectively. Figure 6 shows a two-toothed-solitoff solution (33) with (31) in the limit case of the modulus $n = 1$. The two-toothed-solitoff structure is constructed by a kink soliton and an antikink soliton. Their travelling velocities are different, but group velocities are the same. And travelling directions of these two solitoff waves construct a constant acute angle during the propagation process.

Figure 7(a) displays a periodic solitoff-kink interaction solution constructed by a bright soliton and a kink soliton. Figures 7(b)–7(d) show that the bright soliton and the kink soliton have different travelling velocities, and they propagate along the negative x-axis. The peak of the bright soliton keeps increasing until it is arriving at the same amplitude of the kink soliton.

4. Summary and Discussion

First of all, we use the generalized tanh function expansion method to solve the 2DsG equation; a special new kink-periodic wave interaction solution is explicitly expressed both analytically and graphically. This interaction solution between tanh-type soliton and periodic wave of 2DsG equation is firstly obtained. Then, we use the direct method and obtain more new interaction solutions of the 2DsG equation, including the two-periodic solitoff solution (27), periodic soliton-periodic travelling wave interaction solution (29), two-toothed-solitoff solution (33), and periodic solitoff-kink interaction solution (34). The solution (34) is a generalization of a single straight-line kink soliton solution, while the solution (33) is an alternative generalization of periodic straight-line solitoff type of kink soliton solution. These types of interaction solutions are also firstly found for the 2DsG equation. All of these solutions indicate the interaction solution among solitary waves and periodic waves; their travelling velocities are different, but group velocities are same, and they propagate in different trajectories which contain linear shape, curve shape, and saw-tooth shape. In fact, the forms of (12) and (20) can be not only taken the sine Jacobi elliptic function (sn), more functions can be selected such as exp, cn, and cn/sn, and more explicit solutions can be gained. The abundant solutions solved by these two methods suggest that the rich structures of nonlinear systems do not only exist in the integrable systems but also in the nonintegrable systems. Furthermore, there are some types of localized solutions decaying in all directions, for instance, the dromions and ring solitons have not been found by these two methods; those will be left for us to do more research.

Conflict of Interests

The authors declare that they have no financial relationships with other people or organizations that can inappropriately influence this work or possible conflict of interests.

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