Research Article

New Rational Homoclinic and Rogue Waves for Davey-Stewartson Equation

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A new method, homoclinic breather limit method (HBLM), for seeking rogue wave solution of nonlinear evolution equation is proposed. A new family of homoclinic breather wave solution, and rational homoclinic solution (homoclinic rogue wave) for DSI and DSII equations are obtained using the extended homoclinic test method and homoclinic breather limit method (HBLM), respectively. Moreover, rogue wave solution is exhibited as period of periodic wave in homoclinic breather wave approaches to infinite. This result shows that rogue wave can be generated by extreme behavior of homoclinic breather wave for higher dimensional nonlinear wave fields.

1. Introduction

In recent years, rogue waves, as a special type of solitary waves, has been triggered much interest in various physical branches, although there is no exact definition up to now. Rogue waves is a kind of waves that seems abnormal which is first observed in the deep ocean, it has been the subject of intensive research in oceanography [1, 2], optical fibres [3–5], superfluids [6], Bose-Einstein condensates, financial markets, and related fields [7–10]. A possible mechanism for the formation of rogue waves is associated with modulation instability [11–14].The mysteriousness of rogue wave events mainly lies in the phenomenon which appears out of nowhere and disappears without trace. As is known, there are some methods to seek rogue wave such as Darboux dressing technique, Hirota bilinear method. Based on Hirota bilinear equation of nonlinear evolution equation, for Schrödinger type complex systems, there are some effective techniques such as the Peregrine breather method (PB) [11], whose representation is mathematically a ratio of two polynomials, Ma solitons [4] (MS) and Akhmediev breather methods (ABs) [3]. The main difference between these methods is the test function to Hirota

bilinear equation. The test functions of PB, MS, and ABs are $E_1(x,t) = e^{i\phi(x,t)}(1 - ((G(x,t) + iH(x,t))/D(x,t)))$; $E_2(x,t) = e^{i\phi(x,t)}((a_1\cosh(px) + a_2\cos(kt) + ia_3\sin(kt))/(b_1\cosh(px) + b_2\cos(kt)))$; and $E_3(x,t) = e^{i\phi(x,t)}((a_1\cosh(\alpha t) + a_2\cos(kx) + ia_3\sinh(\alpha t))/(b_1\cosh(\alpha t) + b_2\cos(kx)))$, respectively. Here $\phi(x,t)$ is real function and $G(x,t), H(x,t)$, and $D(x,t)$ are polynomials of (x,t), and $E_i(x,t)$, $i = 2,3$, may generate the rogue wave similar to E_1 as $k \to 0$. The above three methods have been successfully applied to complex system such as Hirota equation, Sasa-Satsuma equation, Davey-Stewartson equation, coupled Gross-Pitaevskii equation, coupled NLS Maxwell-Bloch equation, and coupled Schrödinger-Boussinesq equation [11–18].

In this work, we propose a homoclinic (heteroclinic) breather limit method for seeking rogue wave solution. We take $E(x,t) = e^{i\phi(x,t)}((e^{-p(x-\alpha t)} + a_1\cos(p_1(x - \beta t)) + a_2 e^{p(x-\alpha t)})/(e^{-p(x-\alpha t)} + a_3\cos(p_1(x - \beta t)) + a_4 e^{p(x-\alpha t)}))$ as a test function to Hirots bilinear equation. $E(x,t)$ can generate one or two rogue waves as $p_1 \to 0$. It is obvious that the E is different from E_2 and E_3, comparing with E_2, E_3, and E has more complicated structure. Now we consider the application of HBLM to Davey-Stewartson equation.

Davey-Stewartson (DS) equation is written as [19]

$$iu_t = -u_{xx} - \frac{1}{\alpha_0^2} u_{yy} - \frac{2\epsilon}{\alpha_0^2} |u|^2 u - \frac{2}{\alpha_0^2} uv,$$

$$v_{yy} - \alpha_0^2 v_{xx} - 2\alpha_0^2 \epsilon \left(|u|^2 \right)_{xx} = 0, \tag{1}$$

where $u : R_x \times R_y \times R_t^+ \rightarrow C$, $v : R_x \times R_y \times R_t^+ \rightarrow R$, and ϵ and α_0 are constants. DS equation was introduced in a paper by Davey and Stewartson (1974) to describe the evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow. The function $u(x, y, t)$ stands for the complex amplitude, and $v(x, y, t)$ describes the perturbation of the real velocity. DS equation is called the DSI as $\epsilon = 1$, $\alpha_0 = \pm 1$ and DSII as $\epsilon = 1$, $\alpha_0 = \pm i$. There are known results due to local well-posed, global existence and blow-up of some solutions, exact periodic soliton solutions, solitoff and dromion solutions [20–29]. Recently, homoclinic and heteroclinic tube solutions were obtained [29–33].

We consider DSI equation:

$$iu_t + u_{xx} + u_{yy} = -2|u|^2 u - 2uv,$$

$$v_{xx} - v_{yy} = -2\left(|u|^2 \right)_{xx}, \tag{2}$$

and DSII equation:

$$iU_t + U_{xx} - U_{yy} = 2|U|^2 U + 2UV,$$

$$V_{xx} + V_{yy} = -2\left(|U|^2 \right)_{xx}. \tag{3}$$

2. Homoclinic Breather and Rogue Wave Solution of DSI

Making transformation $u = (a/\sqrt{2}) \exp(ia^2 t)Q, v = -\varphi/2$ and substituting it into (2), we can get

$$iQ_t + Q_{xx} + Q_{yy} = -a^2 \left(|Q|^2 - 1 \right) Q + Q\varphi,$$

$$\varphi_{xx} - \varphi_{yy} = 2a^2 \left(|Q|^2 \right)_{xx}, \tag{4}$$

where $Q = Q(x, y, t)$ is a complex function and φ is a real. By the dependent variable transformation

$$Q = \frac{G}{F}, \qquad \varphi = -4(\ln F)_{xx}, \tag{5}$$

with G being a complex and F being a real, then (4) can be converted into the form

$$iG_t F - iF_t G + G_{xx} F - 2G_x F_x + GF_{xx} + G_{yy} F$$

$$- 2G_y F_y + GF_{yy} - \left(a^2 + B \right) GF = 0, \tag{6}$$

$$2\left(F_{yy} F - F_y^2 - F_{xx} F + F_x^2 \right) - BF^2 - a^2 GG^* = 0,$$

where B is an integration constant and an asterisk denotes the complex conjugation.

By means of the extended homoclinic test approach [33], we take the test function as follows:

$$G = e^{-p(x+y/2+\alpha t)} + a_1 \cos\left(p_1 \left(x + 2y - \alpha t \right) \right)$$

$$+ a_2 e^{p(x+y/2+\alpha t)},$$

$$F = e^{-p(x+y/2+\alpha t)} + a_3 \cos\left(p_1 \left(x + 2y - \alpha t \right) \right)$$

$$+ a_4 e^{p(x+y/2+\alpha t)}, \tag{7}$$

where all of a_3, a_4, p, p_1, β, β_1, and α are real and a_1, a_2 are complex. Substituting (7) into (6) and equating the coefficients of all powers of $e^{jp(x+y/2+\alpha t)} \cos(p_1(x + 2y - \alpha t))$, $e^{jp(x+y/2+\alpha t)} \sin(p_1(x+2y-\alpha t))$ and $e^{\pm 2(p(x+y/2+\alpha t))}$ ($j = 0, \pm 1$) to zero, we can obtain a set of algebraic equations for p, p_1, β, β_1, α, and a_j, $j = 1, 2, 3, 4$, with

$$B = -a^2, \qquad (4pp_1 - ip_1\alpha) a_1 + (4pp_1 + ip_1\alpha) a_3 = 0,$$

$$(-ip_1\alpha - 4pp_1) a_4 a_1 + (ip_1\alpha - 4pp_1) a_3 a_2 = 0,$$

$$\left(ip\alpha + \frac{5p^2}{4} - 5p_1^2 \right) a_1 + \left(\frac{5p^2}{4} - 5p_1^2 - ip\alpha \right) a_3 = 0,$$

$$\left(\frac{5p^2}{4} - ip\alpha - 5p_1^2 \right) a_4 a_1 + \left(ip\alpha + \frac{5p^2}{4} - 5p_1^2 \right) a_3 a_2 = 0,$$

$$-10a_1 a_3 p_1^2 + \left(5p^2 + 2ip\alpha \right) a_2 + \left(5p^2 - 2ip\alpha \right) a_4 = 0,$$

$$a^2 \left(a_3^2 - a_1 a_1^* \right) = 0, \qquad a^2 \left(a_4^2 - a_2 a_2^* \right) = 0,$$

$$\left(2a^2 - \frac{3p^2}{2} - 6p_1^2 \right) a_3 - a^2 \left(a_1 + a_1^* \right) = 0,$$

$$\left(2a^2 - \frac{3p^2}{2} - 6p_1^2 \right) a_3 a_4 - a^2 \left(a_1 a_2^* + a_1^* a_2 \right) = 0,$$

$$\left(2a^2 - 6p^2 \right) a_4 - 6a_3^2 p_1^2 - a^2 \left(a_2 + a_2^* \right) = 0. \tag{8}$$

Solving these equations, we obtain the relations between the parameters as

$$B = -a^2, \qquad p_1^2 = \frac{21p^2}{20},$$

$$p^2 = \frac{320a^2 - 39\alpha^2}{624}, \qquad a_1 = \frac{(i\alpha + 4p) a_3}{i\alpha - 4p},$$

$$a_2 = \frac{(i\alpha + 4p)^2 a_4}{(i\alpha - 4p)^2}, \qquad a_3^2 = \frac{4\left(21\alpha^2 - 80p^2 \right) a_4}{21\left(\alpha^2 + 16p^2 \right)}. \tag{9}$$

From $p^2 \geq 0$ and $a_3^2 \geq 0$ in (9), we have $(800a^2/507) \leq \alpha^2 \leq (320a^2/39)$. Substituting (9) into (7) and then (5) and taking $a_4 > 0$, we obtain the solution for DSI equation as

$$u = \frac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \frac{2 \cosh\left[\xi + \gamma + i\theta\right] + (a_3/\sqrt{a_4}) \cos(\eta)}{2 \cosh(\xi + \gamma) + (a_3/\sqrt{a_4}) \cos(\eta)},$$

$$v = 2\left(2a_3 \sqrt{a_4} \left(p^2 - p_1^2\right) \cos(\eta) \cosh(\xi + \gamma)\right.$$

$$+ 4a_3 p p_1 \sqrt{a_4} \sin(\eta) \sinh(\xi + \gamma) \qquad (10)$$

$$+ 4a_4 p^2 - a_3^2 p_1^2\bigg)$$

$$\times \left(\left[a_3 \cos(\eta) + 2\sqrt{a_4} \cosh(\xi + \gamma)\right]^2\right)^{-1},$$

where $\xi = p(x + y/2 + \alpha t)$, $\eta = p_1(x + 2y - \alpha t)$, $\gamma = \ln \sqrt{a_4}$, $e^{i\theta} = (i\alpha + 4p)/(i\alpha - 4p)$, and p, p_1, α, a_3, and a_4, are given by (9). Note that if $(u(x, y, t), v(x, y, t))$ is the solution of DSI equation, then $(u(x, -y, t), v(x, -y, t))$ is the solution as well. So, we also obtain solution of DSI equation:

$$u_1 = \frac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \frac{2 \cosh\left[\xi_1 + \gamma + i\theta\right] + (a_3/\sqrt{a_4}) \cos(\eta_1)}{2 \cosh(\xi_1 + \gamma) + (a_3/\sqrt{a_4}) \cos(\eta_1)},$$

$$v_1 = 2\left(2a_3 \sqrt{a_4} \left(p^2 - p_1^2\right) \cos(\eta_1) \cosh(\xi_1 + \gamma)\right.$$

$$+ 4a_3 p p_1 \sqrt{a_4} \sin(\eta_1) \sinh(\xi_1 + \gamma)$$

$$+ 4a_4 p^2 - a_3^2 p_1^2\bigg)$$

$$\times \left(\left[a_3 \cos(\eta_1) + 2\sqrt{a_4} \cosh(\xi_1 + \gamma)\right]^2\right)^{-1},$$

$$(11)$$

where $\xi_1 = p(x - y/2 + \alpha t)$, $\eta_1 = p_1(x - 2y - \alpha t)$, and $\gamma = \ln \sqrt{a_4}$. Solution (11) is the homoclinic solution of DSI equation. Indeed, we have

$$(u_1, v_1) \longrightarrow \left(\frac{a}{\sqrt{2}} \exp\left(i\left(a^2 t + 2\theta\right)\right), 0\right),$$

$$\text{as } t \longrightarrow +\infty; \qquad (12)$$

$$(u_1, v_1) \longrightarrow \left(\frac{a}{\sqrt{2}} \exp\left(ia^2 t\right), 0\right), \quad \text{as } t \longrightarrow -\infty,$$

where 2θ is a phase shift and $(a \exp(ia^2 t), 0)$ is a fixed circle of DSI [30]. Note that solution (11) contains not only a periodic wave $\cos(p_1(x - 2y - \alpha t))$, so its amplitude periodically oscillates with the evolution of time (the breather effect), but also a solitary wave $1/\cosh(p(x - y/2 + \alpha t) + \gamma)$, which shows that interaction between a solitary wave and a periodic wave with the same velocity α and opposite propagation direction can form a new family of homoclinic solution. This is a new phenomenon of evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow (Figure 1).

In the above two cases, set $a_3 = -(2/21)\sqrt{(441r^2 - 1680p^2)/(16p^2 + r^2)}$ and $a_4 = 1$, when $\alpha = (8/39)\sqrt{195}a$. Let $p \to 0$, and we can obtain two rogue wave solutions for DSI as follows (Figures 2 and 3):

$$u_1 = \frac{a}{\sqrt{2}} \exp\left(ia^2 t\right)$$

$$\times \left(\left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3\right.\right.$$

$$+ \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2$$

$$- 18560\bigg)$$

$$\times \left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3\right.$$

$$+ \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2$$

$$+ 8320\bigg)^{-1}$$

$$+ i\left(-6720\alpha^2 t + (-3360y - 6720x)\alpha\right)$$

$$\times \left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3\right.$$

$$+ \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2 \qquad (13)$$

$$+ 8320\bigg)^{-1}\bigg),$$

$$v_1 = -84\alpha^2 \left(-35259\alpha^4 t^2\right.$$

$$+ \left(50736yt - 1722xt\right)\alpha^3$$

$$+ \left(35301x^2 + 89544xy + 36939y^2\right)\alpha^2$$

$$- 341120\bigg)$$

$$\times \left(\left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3\right.\right.$$

$$+ \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2$$

$$+ 8320\bigg)^2\bigg)^{-1},$$

$$u_2 = \frac{a}{\sqrt{2}} \exp\left(ia^2 t\right)$$

$$\times \left(\left(861\alpha^4 t^2 + (1344yt - 42xt)\alpha^3\right.\right.$$

$$+ \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2$$

$$- 18560\bigg)$$

$$\times \left(861\alpha^4 t^2 + (1344yt - 42xt)\alpha^3\right.$$

$$+ \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2$$

$$+ 8320\bigg)^{-1}$$

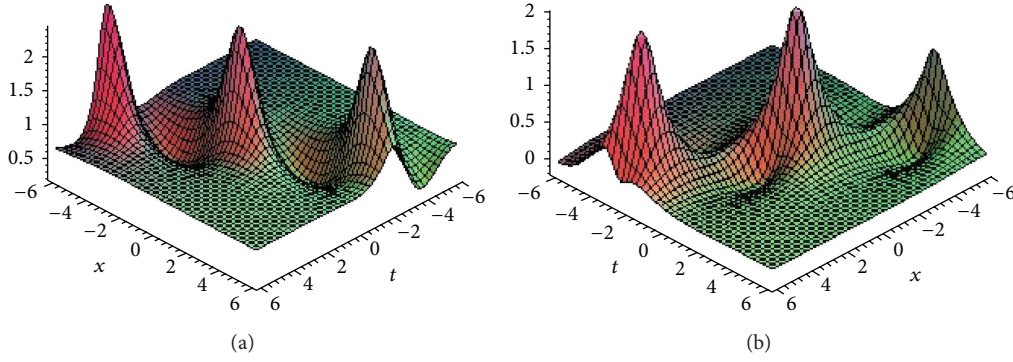$$+ i\left(-6720\alpha^2 t + (3360y - 6720x)\alpha\right)$$

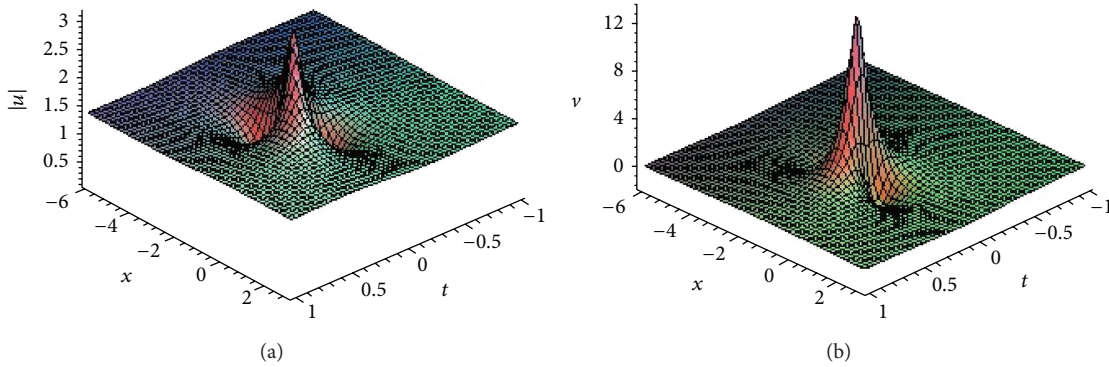FIGURE 1: (a) Homoclinic breather wave of $|u_1|$ in DSI. (b) Homoclinic breather wave of v_1 in DSI.

FIGURE 2: (a) Homoclinic rogue wave $|u_1|$ in solution (13). (b) Homoclinic rogue wave v_1 in solution (13).

$$\times \left(861\alpha^4 t^2 + \left(1344 yt - 42xt\right)\alpha^3 \right.$$
$$+ \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2$$
$$\left. + 8320 \right)^{-1} \Bigg),$$

$$v_2 = -84\alpha^2 \left(-35259\alpha^4 t^2 \right.$$
$$+ \left(-50736 yt - 1722xt\right)\alpha^3$$
$$+ \left(35301x^2 - 89544xy + 36939y^2\right)\alpha^2$$
$$- 341120 \Big)$$
$$\times \left(\left(861\alpha^4 t^2 + \left(-1344 yt - 42xt\right)\alpha^3 \right.\right.$$
$$+ \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2$$
$$\left.\left. + 8320 \right)^2 \right)^{-1} \Bigg),$$

(14)

where $\alpha = (8/39)\sqrt{195}a$.

Both (u_1, v_1) and (u_2, v_2) are rational homoclinic (rogue) wave solutions of DSI equation. In fact, we have $(u_j, v_j) \rightarrow ((a/\sqrt{2})\exp(ia^2 t), 0)$ as $t \rightarrow \infty$, $j = 1, 2$.

3. Homoclinic Breather and Rogue Wave Solution of DSII

As we know, the $(ae^{-2a^2 it}, 0)$ is hyperbolic fixed cycle of DSII equation when the period of y is larger than the period of x [30]. Similar to the argument in [30], we can analyze the linear stability of fixed cycle $(ae^{-2|a|^2 it}, 0)$. Similar to the dealing with process of (2), by means of transformation of functions
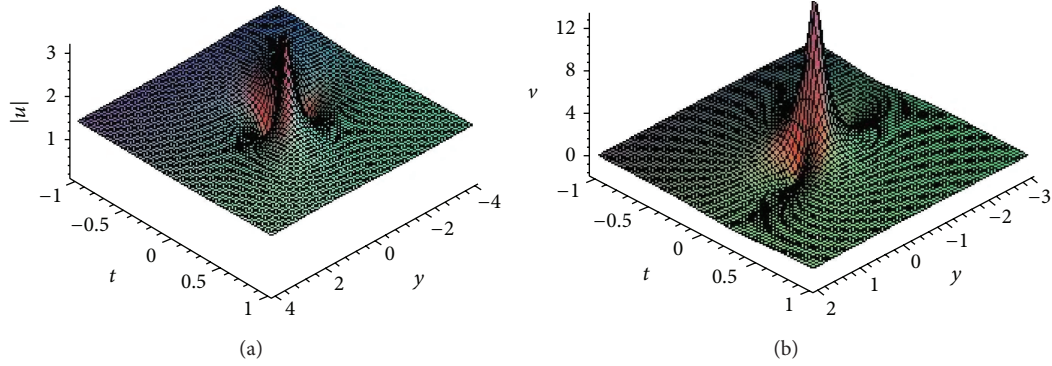
$$U = \frac{G}{F}, \qquad V = -2(\ln F)_{xx}, \qquad (15)$$

Equation (3) can be converted into the bilinear form

$$\left(iD_t + D_x^2 - D_y^2\right)G \cdot F = \lambda G \cdot F,$$
$$\left(D_x^2 + D_y^2 + \lambda\right)F \cdot F = 2GG^*, \qquad (16)$$

where G is a complex function and F is a real. Now, we take the following ansatz:

$$G = ae^{-2a^2 it}\left[e^{-p_2(mx+ny+\alpha t)} + b_1 \cos p_1\left(kx + ly - \alpha t\right)\right.$$
$$\left. + b_2 e^{p_2(mx+ny+\alpha t)}\right],$$
$$F = e^{-p_2(mx+ny+\alpha t)} + b_3 \cos p_1\left(kx + ly - \alpha t\right)$$
$$+ b_4 e^{p_2(mx+ny+\alpha t)}, \qquad (17)$$

Figure 3: (a) Behaviour of $|u_2|$ in solution (14). (b) Behaviour of v_2 in solution (14).

where p_1, p_2, m, n, k, l, α, b_3, and b_4 are real and b_1, b_2 are complex. Substituting (17) into (16), then we obtain

$$\lambda = 2a^2,$$

$$p_2^2 = \left(8\left(k^2 - l^2\right)(km - nl)^2 a^2 \right.$$

$$+ \left(l^3 n - \left(km + m^2\right)l^2 \right.$$

$$\left. + k^2 nl + n^2 k^2 - k^3 m\right)\alpha^2 \right)$$

$$\times \left(4\left(k^3 m - k^2 nl - n^2 k^2 \right. \right.$$

$$+ l^2 km + m^2 l^2 - l^3 n\right)$$

$$\times (km - nl)^2 \bigg)^{-1},$$

$$p_1^2 = \frac{\left(m^2 - n^2 - 2nl + 2km\right)p_2^2}{k^2 - l^2},$$

$$b_1 = \frac{b_3\left(i\alpha + 2p_2 mk - 2\,p_2 nl\right)}{i\alpha - 2\,p_2 mk + 2\,p_2 nl}, \tag{18}$$

$$b_2 = \frac{b_4\left(i\alpha + 2p_2 mk - 2p_2 nl\right)^2}{\left(i\alpha - 2p_2 mk + 2p_2 nl\right)^2},$$

$$b_3^2 = -4b_4$$

$$\times \left(\left(m^2 - n^2\right)(mk - nl)^2 p_2^2 \right.$$

$$\left. - \alpha^2\left(2mk - n^2 - 2nl + m^2\right)\right)$$

$$\times \left(\left(2mk - n^2 - 2nl + m^2\right) \right.$$

$$\times \left(\left(4m^2 k^2 - 8nlmk + 4n^2 l^2\right)p_2^2 + \alpha^2\right)\bigg)^{-1},$$

$$nl + km = 0.$$

So we can obtain breather solution of DSII equation as follows:

$$U = ae^{-i(2a^2 t - \theta)}$$

$$\times \left(2\sqrt{b_4}\cosh\left(p_2\left(mx + ny + \alpha t\right) + i\theta + \phi\right) \right.$$

$$\left. + b_3\cos\left(p_1\left(kx + ly - \alpha t\right)\right)\right)$$

$$\times \left(2\sqrt{b_4}\cosh\left(p_2\left(mx + ny + \alpha\,t\right) + \phi\right) \right.$$

$$\left. + b_3\cos\left(p_1\left(kx + ly - \alpha t\right)\right)\right)^{-1},$$

$$V = -2\left(2\sqrt{b_4} \right.$$

$$\times \cosh\left(p_2\left(mx + ny + \alpha t\right) + \phi\right)p_2^2 m^2$$

$$\left. - b_3\cos\left(p_1\left(kx + ly - \alpha t\right)\right)p_1^2 k^2\right)$$

$$\times \left(2\sqrt{b_4}\cosh\left(p_2\left(mx + ny + \alpha t\right) + \phi\right) \right.$$

$$\left. + b_3\cos\left(p_1\left(kx + ly - \alpha t\right)\right)\right)^{-1}$$

$$+ 2\left(\left(2\sqrt{b_4}\sinh\left(p_2\left(mx + ny + \alpha t\right) + \phi\right)p_2 m \right. \right.$$

$$\left. - b_3\sin\left(p_1\left(kx + ly - \alpha t\right)\right)p_1 k\right)^2\right)$$

$$\times \left(\left(2\sqrt{b_4}\cosh\left(p_2\left(mx + ny + \alpha t\right) + \phi\right) \right. \right.$$

$$\left. + b_3\cos\left(p_1\left(kx + ly - \alpha t\right)\right)\right)^2\bigg)^{-1}\bigg),$$

$$\tag{19}$$

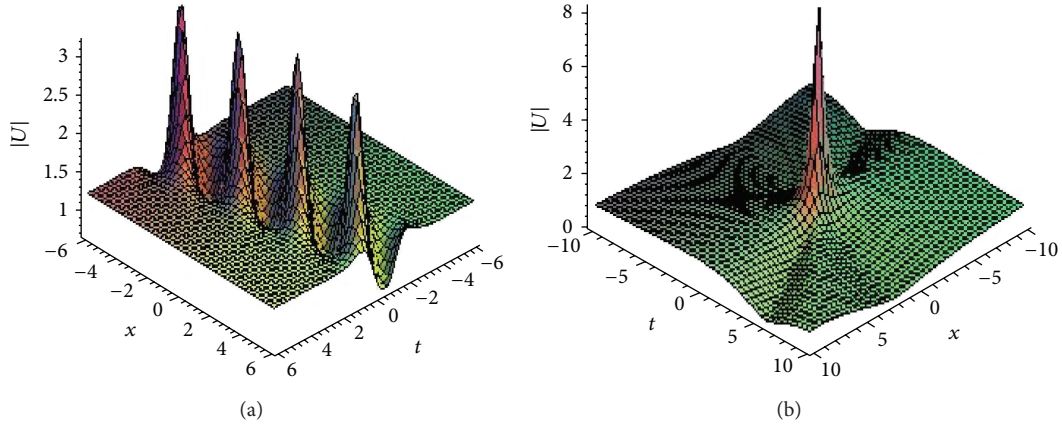where $\theta = \arctan b_1$ and $\phi = \ln(\sqrt{b_4})$.

FIGURE 4: (a) Bright breather structure $|U|$ in DSII. (b) Bright rogue wave $|U|$ in DSII.
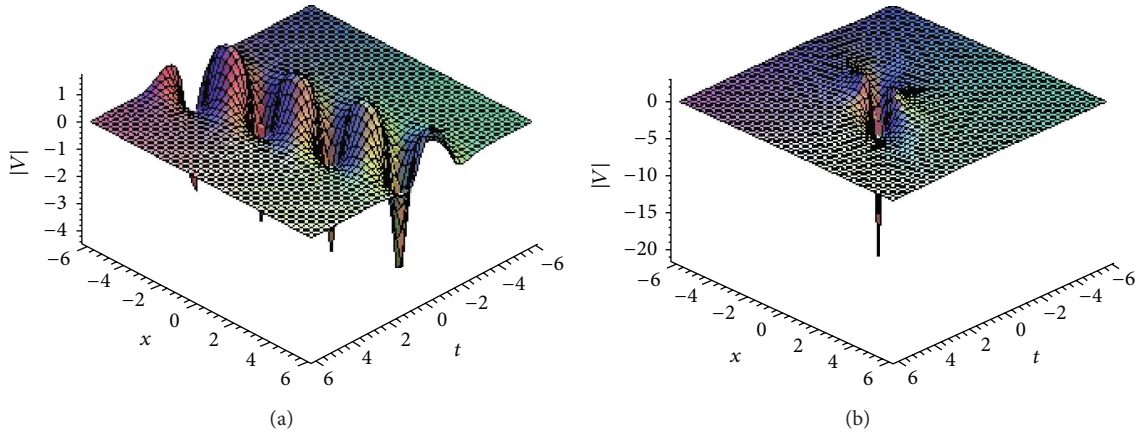

FIGURE 5: (a) Dark breather structure V in DSII. (b) Dark rogue wave V in DSII.

The rogue wave of the DSII system is derived when the period of periodic wave goes to infinite. Indeed, by letting $p_1 \to 0$ in solution (19), solution (19) becomes rogue waves:

$$U = ae^{2\,ia^2t}$$
$$\times \left(-(mx + ny + \alpha t)^2 + 2(kx + ly - \alpha t)^2 \right.$$
$$\left. + 16i\,(mx + ny + \alpha t)\,\delta_1\delta_2 + \delta_4 \right)$$
$$\times \left(2(mx + ny + \alpha t)^2\delta_1 \right.$$
$$\left. + 2(kx + ly - \alpha t)^2 + \delta_3 \right)^{-1},$$

$$V = -2\left(4m^2\delta_1 + 4k^2 \right)$$
$$\times \left(2(mx + ny + \alpha t)^2\delta_1 \right.$$
$$\left. + 2\,(kx + ly - \alpha t)^2 + \delta_3 \right)^{-1}$$

$$+ 2\left(4\,(mx + ny + \alpha t)\,\delta_1 m \right.$$
$$\left. + 4\,\left(kx + ly - \alpha t \right)k \right)^2$$
$$\times \left(\left(2(mx + ny + \alpha t)^2\delta_1 \right. \right.$$
$$\left. \left. + 2(kx + ly - \alpha t)^2 + \delta_3 \right)^2 \right)^{-1},$$

(20)

where

$$\delta_1 = \frac{k^2 - l^2}{m^2 - n^2 - 2ln + 2mk},$$

$$\delta_2 = \frac{mk - ln}{\alpha},$$

$$\delta_3 = 16\,\frac{\left(k^2 - l^2\right)\left(mk + m^2 - n^2 - ln\right)(mk - ln)^2}{\left(m^2 - n^2 - 2ln + 2mk\right)^2\alpha^2},$$

$$\delta_4 = 32 \frac{\left(k^2 - l^2\right)\left(5mk + 3\ m^2 - 5ln - 3\ n^2\right)(mk - ln)^2}{\left(m^2 - n^2 - 2ln + 2mk\right)^2 \alpha^2},$$

$$m = \frac{1312\alpha^2}{9\left(41\alpha^2 + 512a^2\right)},$$

$$n = -\frac{1640\alpha^2}{9\left(41\alpha^2 + 512a^2\right)}.$$

(21)

Figure 4(a): The dynamical evolution of bright breather $|U(x,t)|$ in solution (19) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 4(b): The dynamical evolution of bright rogue wave $|U(x,t)|$ in solution (20) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 5(a): The dynamical evolution of dark breather $V(x,t)$ in solution (19) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 5(b): The dynamical evolution of dark rogue wave $V(x,t)$ in solution (20) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

It is easy to verify that both solution (19) and (20) are solutions of DSII. Similar to DSI, we can show that solution (20) is a rational homoclinic rogue waves.

4. Conclusion

In summary, based on Hirota bilinear form, applying homoclinic breather limit method to DSI and DSII equations, we obtain a new kind of homoclinic solutions with locally oscillatory structure and rational homoclinic rogue wave solutions. We also investigate and exhibit the different homoclinic rogue wave structures of solutions. These results show the complexity and variety of dynamical behavior of the DS system. Following these ideas in this work, the problem needed to further study is other types of nonlinear evolution equations whether have this kind of rational homoclinic solutions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

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