

Research Article

Location-Price Competition in Airline Networks

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This paper addresses location-then-price competition in airline market as a two-stage game of n players on the graph. Passenger's demand distribution is described by multinomial logit model. Equilibrium in price game is computed through best response dynamics. We solve location game using backward induction, knowing that airlines will choose prices from equilibrium for the second-stage game. Some numerical results for airline market under consideration are presented.

1. Introduction

The aim of this paper is to characterize airline behavior in the market in both strategic and operational levels. In a competitive environment an airline makes a strategic decision on how to allocate planes among available routes. Starting operations on chosen routes airlines compete for passengers using ticket prices.

Location-then-price competition was first introduced by Hotelling [1]. This classical model examines behavior for two firms producing homogenous goods on a line segment. Passenger demand depends on the firm's price and transportation costs. Hotelling found price equilibrium and raised a problem of firms' competitive location. The nonexistence of location equilibrium in Hotelling model was shown in [2]. Extensions to Hotelling's duopoly were studied in several directions. Different forms of transportation costs were used in [2, 3]. In [4–6], Hotelling's model is examined on more complex set than line segment. Location-then-price competition on the plane was studied in [6–8], where different forms of transportation costs are used. These models can be applied in transportation networks (with Euclidian distance) and in mobile and telecommunication networks (with Manhattan distance). Price competition among more than two firms is addressed in [9], where sufficient conditions on the existence

of Nash equilibrium in price game for any number of firms are introduced.

In this paper we examine location-then-price Hotelling model on the graph for the case of $n \geq 2$ players. Results of theoretical analysis for proposed model are applied to study competition in airline market, where airlines first decide plane allocation and then choose ticket prices.

This paper is organized as follows. In Section 2 we describe model notations and assumptions. In Section 3 the main results about the equilibrium existence in price and location games with logit analysis application are given. Airline market data is described in Section 4. In this section analysis of price and location equilibrium is performed and results are applied to find location-then-price equilibrium in airline markets under consideration. Some remarks are given in the final section.

2. Location Game-Theoretic Model on Graph

Consider a market where the customers are distributed in the vertexes of the transportation graph $G(V, E)$. The edges of the graph are transportation links (railways, car, air lines, etc.). The vertexes are the hubs (bus stops, airports, railway stations, etc.). The customers are the passengers, who use this

kind of transportation. The demand is determined by the flow of passengers. Notice that graph G can be unconnected.

There are n companies (players), who make a service in this market. A service is possible only if there is a link $e_j \in E$ between two vertexes in graph $G(V, E)$. The demand is determined by the number of customers in vertexes $v_1, v_2 \in V$ connected by the link e_j :

$$d(e_j) = d(v_1, v_2), \quad e_j = (v_1, v_2). \quad (1)$$

Assume that player i has m_i units of a resource. He distributes the resource among links in graph $G(V, E)$. Suppose that each player i distributes all m_i units of resource and forms transportation network E^i , which is a subset of the links in graph $G(V, E)$.

A competition exists on the link e_j only if it is included into several transportation networks:

$$\exists i, j : E^i \cap E^j \neq \emptyset, \quad i, j \in N, \quad i \neq j. \quad (2)$$

The demand on the link e_j is distributed between players. Each of them makes a service of the part M_{ij} of the customers on this link. Players announce prices for the service on the link e_j . The part of customers, which prefer the service of player i , depends on the price p_{ij} and the prices of other players on this link:

$$M_{ij} = M_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right), \quad |M_{ij}| \leq 1, \quad (3)$$

where N_j is the number of the rival players on the link e_j .

The number of customers who prefer the service i on the link e_j is

$$S_{ij} \left(\{p_{rj}\}_{r \in N_j} \right) = M_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) d(e_j). \quad (4)$$

Let x_{ij} be a distribution of player i on the link e_j ; that is,

$$x_{ij} = \begin{cases} 1, & e_j \in E^i, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Player i with m_i units of the resource on graph $G(V, E)$ can attract customers, whose number equals

$$S_i = \sum_{j=1}^{|E|} M_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) d(e_j) x_{ij}. \quad (6)$$

The gain of player i on the link e_j depends on the price for the service and the share in the customer demand:

$$h_{ij} \left(\{p_{rj}\}_{r \in N_j} \right) = p_{ij} M_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) d(e_j). \quad (7)$$

Denote by c_{ij} the costs of player i on the link e_j . The costs are proportional to the number of customers, who use the resource. Then the payoff of player i on graph $G(V, E)$ is

$$\begin{aligned} H_i \left(\{p_r\}_{r \in N}, \{x_r\}_{r \in N} \right) \\ = \sum_{j=1}^{|E|} \left(h_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) \right. \\ \left. - c_{ij} S_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) \right) x_{ij}, \end{aligned} \quad (8)$$

where p_r is a vector of prices of player r in his network E^r

and x_r is a vector, which defines allocation of m_r units of the resource on graph $G(V, E)$ ($r \in N$).

First, players form their transportation networks and then they announce the prices for the service. The objective of a player is to maximize the payoff.

We determine the noncooperative game Γ_G for n players. Strategy of player i is a pair of vectors (x_i, p_i) . Player determines the allocation x_i of m_i units of the resource:

$$\forall j \in \{1, \dots, |E|\} : x_{ij} \in \{0, 1\}, \quad \sum_{r=1}^{|E|} x_{ir} = m_i. \quad (9)$$

Then player i announces the prices in his network E^i :

$$\forall j \ p_{ij} \in [0, \infty), \quad e^j \in E^i. \quad (10)$$

The game has three stages.

- (1) The players simultaneously distribute the resources given by $\{x_i\}_{i \in N}$.
- (2) The players simultaneously announce the prices $\{p_i\}_{i \in N}$.
- (3) Customers choose a service and the players receive the payoffs $\{H_i\}_{i \in N}$ depending on their transportation networks and prices.

We seek the Nash equilibrium $\{x_i^*\}_{i \in N}$, that is, x_i^* , such that $\forall x_i, i \in N$ it satisfies the condition

$$\begin{aligned} H_i \left(\{\tilde{p}_r(x_i, \{x_r^*\}_{r \in N \setminus \{i\}})\}_{r \in N}, x_i, \{x_r^*\}_{r \in N \setminus \{i\}} \right) \\ \leq H_i \left(\{\tilde{p}_r(x_i^*, \{x_r^*\}_{r \in N \setminus \{i\}})\}_{r \in N}, x_i^*, \{x_r^*\}_{r \in N \setminus \{i\}} \right), \end{aligned} \quad (11)$$

where $\{\tilde{p}_r(\{x_i\}_{i \in N})\}_{r \in N}$ is an equilibrium in price game for fixed resource distribution on graph $G(V, E)$.

For fixed resource allocation $\{\tilde{x}_r\}_{r \in N}$ we find the Nash equilibrium $\{p_i^*\}_{i \in N}$; that is, p_i^* , such that $\forall p_i, i \in N$ holds. Consider

$$H_i(p_i, \{p_r^*\}_{r \in N \setminus \{i\}}, \{\tilde{x}_r\}_{r \in N}) \leq H_i(p_i^*, \{p_r^*\}_{r \in N \setminus \{i\}}, \{\tilde{x}_r\}_{r \in N}). \quad (12)$$

3. Logit Analysis in Price Game on Graph

Consider the number of players N_j , who choose the link e_j in graph $G(V, E)$. Player $i \in N_j$ announces the price p_{ij} for the service on the link e_j . Suppose that demand $d(e_j)$ is distributed among services in the logit manner. Then

$$M_{ij} = \frac{e^{\alpha p_{ij} + (a, v_i)}}{\sum_{s=1}^{|N_j|} e^{\alpha p_{sj} + (a, v_s)} + e^\rho}, \quad e_j \in E^i, \quad i \in N_j, \quad (13)$$

where v_i is a vector of characteristics of the service i , $\alpha < 0$, a is a constant vector of weights, and ρ corresponds to the customers, who prefer not to use any service at all.

On the link e_j we obtain price game of N_j players with the payoffs:

$$h_{ij} \left(\{p_{rj}\}_{r \in N_j} \right) = (p_{ij} - c_{ij}) M_{ij} d(e_j), \quad i \in N_j. \quad (14)$$

Let us denote by $\{p_{ij}^*\}_{i \in N_j}$ an equilibrium in price game. It can be found as a solution of the system of equations

$$\frac{\partial h_{ij}}{\partial p_{ij}} = M_{ij} d(e_j) (1 + \alpha (p_{ij} - c_{ij}) (1 - M_{ij})) = 0. \quad (15)$$

Imagine that a new player appears on the link e_j . What happens with equilibrium prices and payoffs? Denote by γ a new player and consider the new set of players on the link e_j $\tilde{N}_j = N_j \cup \{\gamma\}$. Let $\{\tilde{p}_{ij}^*\}_{i \in \tilde{N}_j}$ be an equilibrium in the price game with additional player.

Theorem 1. *In the price game with additional player the equilibrium prices for all players except the new one are decreasing; that is, $\forall i \in N_j : p_{ij}^* > \tilde{p}_{ij}^*$.*

Proof. Equilibrium prices in the game on the link e_j with N_j players satisfy the following equations:

$$1 + \alpha (p_{ij} - c_{ij}) (1 - M_{ij}) = 0, \quad i \in N_j. \quad (16)$$

Rewrite it in the following form:

$$\sum_{r \in N_j} e^{\alpha p_{rj} + (a, v_r)} + e^\rho + \alpha (p_{ij} - c_{ij}) \times \left(\sum_{r \in N_j \setminus \{i\}} e^{\alpha p_{rj} + (a, v_r)} + e^\rho \right) = 0, \quad i \in N_j. \quad (17)$$

It yields

$$e^\rho + \sum_{r \in N_j \setminus \{i\}} e^{\alpha p_{rj} + (a, v_r)} = \frac{-e^{\alpha p_{ij} + (a, v_i)}}{\alpha (p_{ij} - c_{ij}) + 1}, \quad i \in N_j. \quad (18)$$

The right-hand side of this equation is the function $g(x) = -e^{\alpha x + b} / (\alpha(x - c_{ij}) + 1)$, where b is a constant. The derivative of this function is

$$g'(x) = \frac{-\alpha^2 e^{\alpha x + b} (x - c_{ij})}{(\alpha(x - c_{ij}) + 1)^2}. \quad (19)$$

Consequently, $g(x)$ is a decreasing function for $x > c_{ij}$.

So the right side of the optimality equation (18) is a decreasing function in the equilibrium price p_{ij} of player i . At the left side of the equation we have an expression, which depends on the prices of other players. If we introduce a new player to the link e_j , then the left side of the equation increases. Therefore, the root of the right side of the equation decreases. It can be proven by induction. We demonstrate it for the case $|N_j| = 1$.

Let $|N_j| = 1$. The equilibrium p_{1j}^* satisfies the equation

$$e^\rho = \frac{-e^{\alpha p_{1j}^* + (a, v_1)}}{\alpha (p_{1j}^* - c_{1j}) + 1}. \quad (20)$$

In the game with additional player the equilibrium price \tilde{p}_{1j}^* of the first player satisfies the equation

$$e^\rho + e^{\alpha \tilde{p}_{2j}^* + (a, v_2)} = \frac{-e^{\alpha \tilde{p}_{1j}^* + (a, v_1)}}{\alpha (\tilde{p}_{1j}^* - c_{1j}) + 1}. \quad (21)$$

Monotonicity of the function $g(x)$ gives $p_{1j}^* > \tilde{p}_{1j}^*$. \square

Corollary 2. *In the price game with additional player the optimal payoffs of all players except the new one are decreasing; that is, $\forall i \in N_j$ holds. Consider*

$$h_{ij} \left(\{p_{ij}^*\}_{i \in N_j} \right) > h_{ij} \left(\{\tilde{p}_{ij}^*\}_{i \in \tilde{N}_j} \right). \quad (22)$$

Proof. In the equilibrium (18) holds for player i . Substituting it in to the payoff of player i gives

$$h_{ij} \left(\{p_{ij}^*\}_{i \in N_j} \right) = (p_{ij}^* - c_{ij}) \frac{e^{\alpha p_{ij}^* + (a, v_i)}}{e^\rho + \sum_{r \in N_j \setminus \{i\}} e^{\alpha p_{rj}^* + (a, v_r)} + e^{\alpha p_{ij}^* + (a, v_i)}} d(e_j). \quad (23)$$

Hence, we obtain

$$h_{ij} \left(\{p_{ij}^*\}_{i \in N_j} \right) = \left(p_{ij}^* - c_{ij} + \frac{1}{\alpha} \right) d(e_j). \quad (24)$$

According to the theorem the equilibrium prices in the game with additional player are decreasing. Consequently, the optimal payoffs in (24) also decrease if a new service is introduced on the link e_j . \square

Let us return to location game. On the first stage the players distribute their resources among links. After resource allocation price game takes place on each link and the players receive some payoffs. We obtain in the theorem that these payoffs are decreasing functions in the number of players choosing the same link.

The game of this type is a so-called congestion game. In paper [10] it was proven that equilibrium in pure strategies exists in these games. Equilibrium point can be found as a result of best response process.

4. Location-Price Competition in Airline Networks

Let V be a finite set of airports and let E be a finite set of routes between airports. Under route we mean that any airline can perform operations between these two airports. An undirected graph $G(V, E)$ represents possible routes between airports in airline market.

An airline is considered as a player in the market. Each airline i allocates m_i planes among routes in $G(V, E)$. We make a restriction that airline cannot allocate more than one plane to a single route. Note that one plane can serve many airline routes, if it performs a flight with several connections.

Let us define x_i as an airline allocation vector and E^i as an airline route network.

Each route e_j in $G(V, E)$ is characterized by potential passenger demand $d(e_j)$. Airline share in route passenger demand M_{ij} depends on airline own price p_{ij} and prices of competitive airlines. For simplicity, we assume that airline operating costs on the route e_j are proportional to passenger demand.

TABLE 1: Summary statistics (Russian market).

Factor	Mean	Standard deviation	Median	Min	Max
Price (rub.)	9831	4111	9425	1500	21630
Flight time (h.)	3.34	2.33	2.4	0.4	15.3
Frequency (flights per week)	2.8	2.04	1	1	14
Distance (km)	1774	1263	1486	215	7314
Income (rub./year)	27 053	10 390	22 224	14 167	50 991
Population	499 430	394 948	327 423	44 334	1 498 921

TABLE 2: Summary statistics (Chinese market).

Factor	Mean	Standard deviation	Median	Min	Max
Price (rmb.)	1366	537	1300	540	2910
Flight time (h.)	2.23	1	2.08	0.75	5.72
Frequency (flights per week)	6.4	1.44	7	1	7
Distance (km)	1298	657	1233	351	3388
Income (rmb./year)	29501	6903	28731	18400	40742
Population (ths.)	10968.7	7347.4	9325.05	2141.3	29190

Hence, the gain, which an airline gets from operating on the route $e_j \in E^i$, can be written down as (7) and total airline payoff in the market equals (8).

To describe airline competitive behavior, we will consider the following game, where prices and allocation vectors are set as airlines strategies.

- (1) Airlines simultaneously select planes allocation $\{x_i\}_{i \in N}$ among routes.
- (2) Airlines simultaneously select prices in their networks $\{p_i\}_{i \in N}$.
- (3) As a result of these decisions, passengers select flights and airlines receive payoffs $\{H_i\}_{i \in N}$.

We study Nash equilibrium for allocation vectors $\{x_i\}_{i \in N}$, knowing that in the second stage of the game players choose prices $\{p_i\}_{i \in N}$ from Nash equilibrium for fixed airline networks on graph $G(V, E)$.

4.1. Passenger Demand. In order to proceed with analyzing equilibrium prices and locations we need to specify how passenger demand is distributed between competitive airlines. We consider here that potential demand for the route e_j depends on the population size in airport regions and equals

$$d(e_j) = \frac{\sqrt{P(v_j^1)P(v_j^2)}}{2}, \quad e_j = (v_j^1, v_j^2), \quad (25)$$

where P defines population size in airport region.

Multinomial logit model is used to compute airline shares in passenger demand. We assume that a passenger considers price and airline route characteristics to choose between competitive airlines. Airline route characteristics are

combined in vector k_{ij} . Hence, airline share in passenger demand on the route e_j equals

$$M_{ij} \left(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}} \right) = \frac{e^{a_1 p_{ij} + (a, k_{ij})}}{\sum_{s=1}^{|N_j|} e^{a_1 p_{sj} + (a, k_{sj})} + e^\rho}, \quad e_j \in E^i, \quad (26)$$

where $a_1 < 0$ and a is a vector of constants. We include the additional term in denominator to capture passenger alternative not to travel by plane.

4.2. Examples of Airline Markets. We illustrate presented model with an application to Russian and Chinese airline markets (Figures 1 and 2). Market statistics are shown in Tables 1 and 2.

The Russian market contains 27 airports with 95 routes in $G(V, E)$. There are 239 single-trip flights and 74 flights with connections. The number of airlines is 11 and the maximum number of competitive airlines on a single route equals 5. The Chinese market has 14 airports with 61 routes in $G(V, E)$. There are 351 single-trip flights and 14 flights with connections. The number of airlines is 5 and the maximum number of competitive airlines on a single route equals 3.

Airline route characteristics in Russian case include flight time, dummy variable γ_{ij} to indicate direct flight, geometric mean of income rates in airport city pair, and distance between airports:

$$\tilde{k}_{ij} = a_2 t_{ij} + a_3 \gamma_{ij} + a_4 \text{income}_{ij} + a_5 \ln(\text{dist}_{ij}), \quad e_j \in E^i. \quad (27)$$

Airline share is expected to drop on price and flight time increase. On the distance growth planes become more competitive between other means of transport and airline share should increase.

We include the same route characteristics for the Chinese market and also add airline loading indicator, which equals

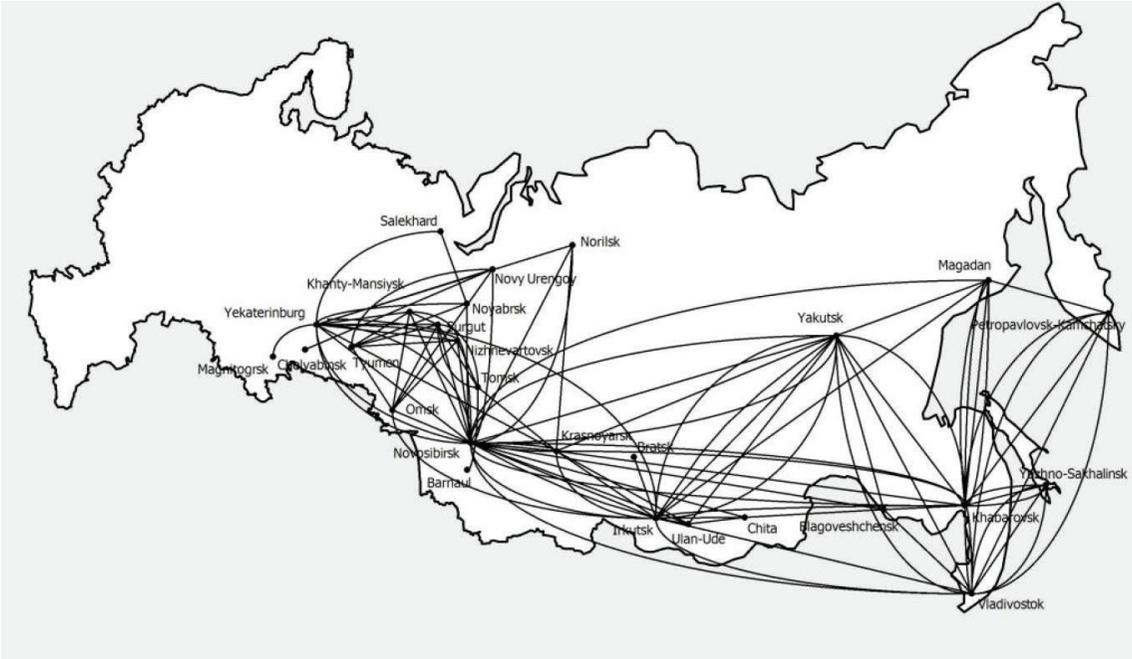


FIGURE 1: Russian airline market.



FIGURE 2: Chinese airline market.

TABLE 3: Parameter estimation results.

	a_1	a_2	a_3	a_4	a_5	a_6	Constant
Russia	-0.000656	-0.288	0.628	0.000141	3.83		-28.305
China	-0.00196	-1.138	0.135	3.845	-6.571	1.142	3.845

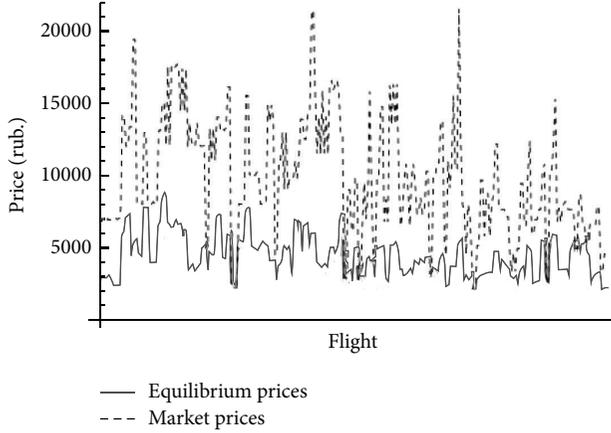


FIGURE 3: Equilibrium prices (Russian market).

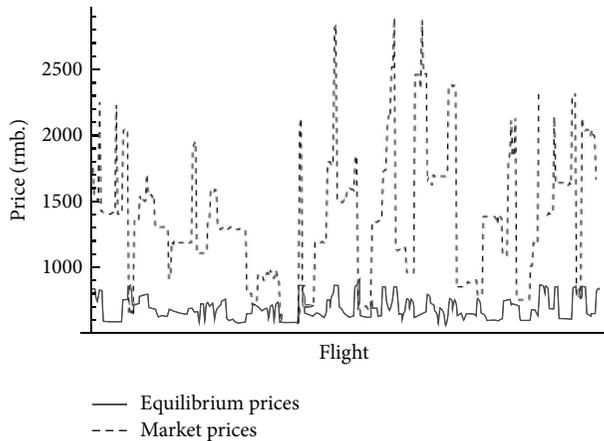


FIGURE 4: Equilibrium prices (Chinese market).

1 if average seat occupancy is more than 80%. Time, income, and distance values represent these factor values divided by the minimum value of the same factor in the considered route. Hence,

$$\begin{aligned} \tilde{k}_{ij} = & a_2 t_{ij}^{\text{ratio}} + a_3 \gamma_{ij} + a_4 \text{income}_{ij}^{\text{ratio}} \\ & + a_5 \text{dist}_{ij}^{\text{ratio}} + a_6 \text{loading}_{ij}, \quad e_j \in E^i. \end{aligned} \quad (28)$$

We performed parameter estimation in (26) using BLP method [11] with flight fuel consumption taken as instrumental variable. Estimation results are presented in Table 3.

4.3. Price Competition. Location-price competition in airline market is examined using backward induction. First, we analyze price game, when airlines have already specified

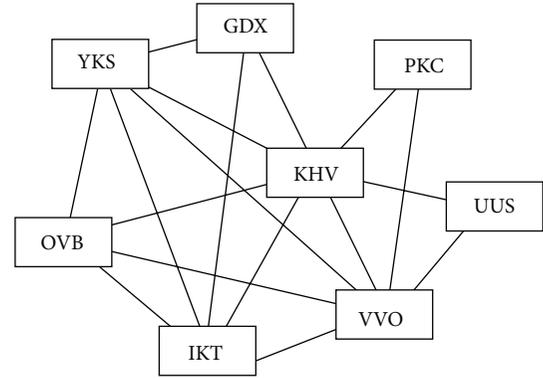


FIGURE 5: Market graph (Russia).

planes allocation among routes in $G(V, E)$. After that, Nash equilibrium in location game is considered.

Airline share in route passenger demand depends on its own price and the prices of other airlines, which compete on this route. The existence and uniqueness of equilibrium for prices on the route e_j follows from [9].

In Nash equilibrium airline price is a best response to the equilibrium prices of other airlines on the considered route e_j . The first-order condition for airline best response is given in the following equation:

$$(1 - M_{ij})(c_{ij} - p_{ij}) = \frac{1}{a_1}. \quad (29)$$

In Figures 3 and 4 prices in equilibrium, which were computed from (29) for each route, are compared with real market prices used by airlines. Flights are ordered by airport city pairs. Note that only fuel consumption is included in airline costs, which leads to smaller price values in equilibrium.

4.4. Location Game. Let us return to the first stage of the game, when airlines select operating routes in $G(V, E)$. We picked out a subgraph in both considered markets for two competitive airlines (Figures 5 and 6). Airline A has 11 (7) planes with 158 (164) seats and airline B has 9 (6) planes with 150 (164) seats in these two examples. Operating networks are presented in Figure 7 for the Russian market and in Figure 8 for the Chinese market.

We derive allocation vectors in equilibrium with the following procedure. First, airline A optimally allocates all planes among routes. Airline B planes enter a market consequentially. Each time we find location equilibrium in the market for all planes of airline A and present planes of airline B using best response dynamics. One can show that the best response sequence converges to an equilibrium using the fact that airline payoff in equilibrium decreases, when a new airline enters the market on the route e_j .

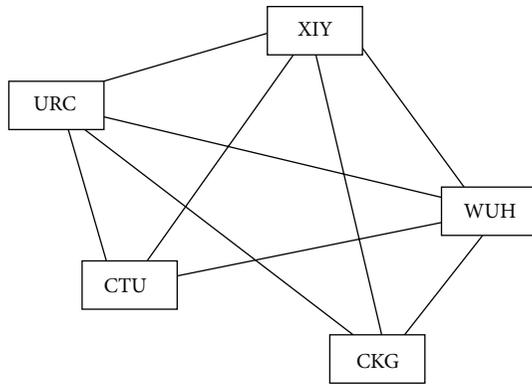
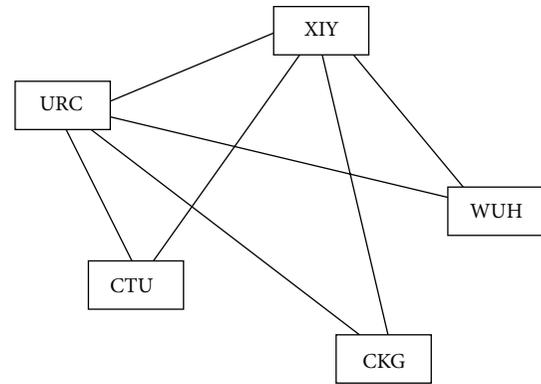
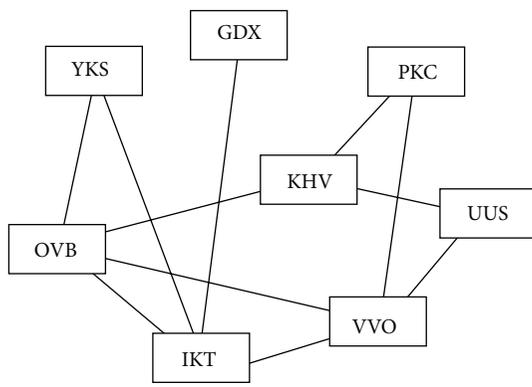


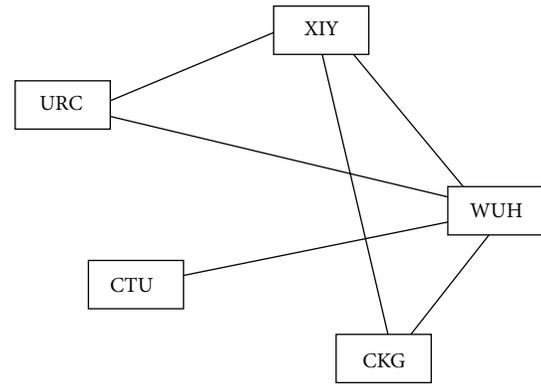
FIGURE 6: Market graph (China).



(a) Network A

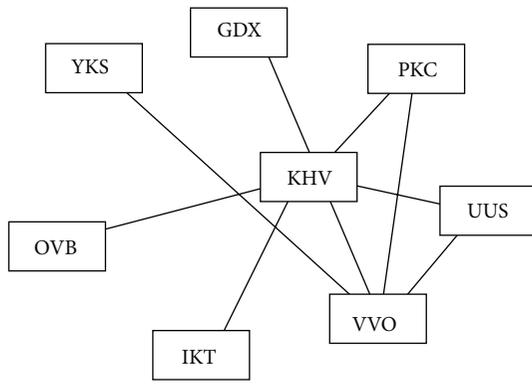


(a) Network A



(b) Network B

FIGURE 8: Operating networks (China).



(b) Network B

FIGURE 7: Operating networks (Russia).

Equilibrium airline networks are shown in Figures 9 and 10. Note that in the real market airline B behaves the same as in the model location equilibrium in the Russian case and switches 2 planes in the Chinese case. Airline A switches 2 planes to new routes in the equilibrium in both examples.

5. Conclusion

This paper studies location-then-price game as a two-stage game of n players on the graph. Logit analysis is used to model demand distribution between competitors. The use of

multinomial logit model allows us to compute shares in case of n players and proceed with studying price equilibrium. Price and location equilibria are constructed using best response dynamics. We apply proposed model to Russian and Chinese airline markets and find location and price equilibria for competitive airlines.

Our research can be extended in several directions. Airline competitiveness is also affected by the choice of base airports and hubs in operating network. Our model does not address these issues. Multinomial logit model belongs to a class of discrete choice models and development of location-then-price game for other models in discrete choice analysis can extend model applicability. Routes scheduling, airline mergers, and other airline market specific features are not considered. Complex analysis of competitive behavior in airline market is an open problem for further research.

Appendix

The list of IATA codes that are used in the text are presented in Tables 4 and 5.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

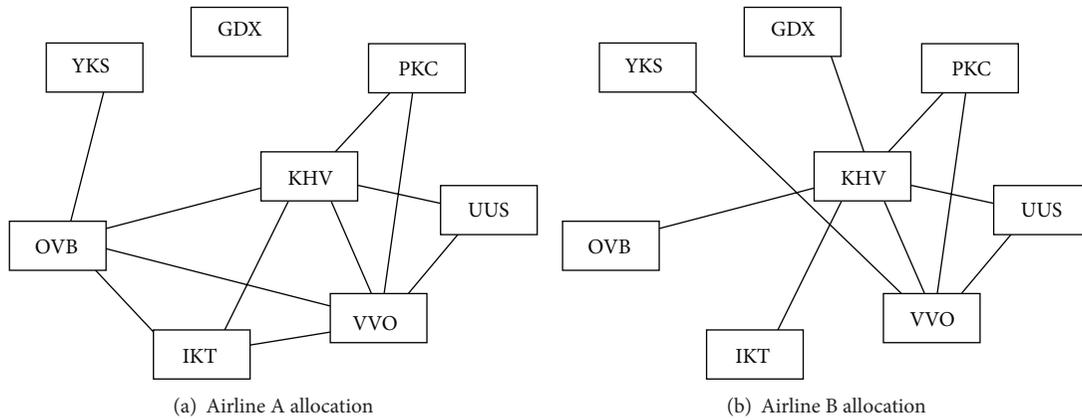


FIGURE 9: Equilibrium in location game (Russia).

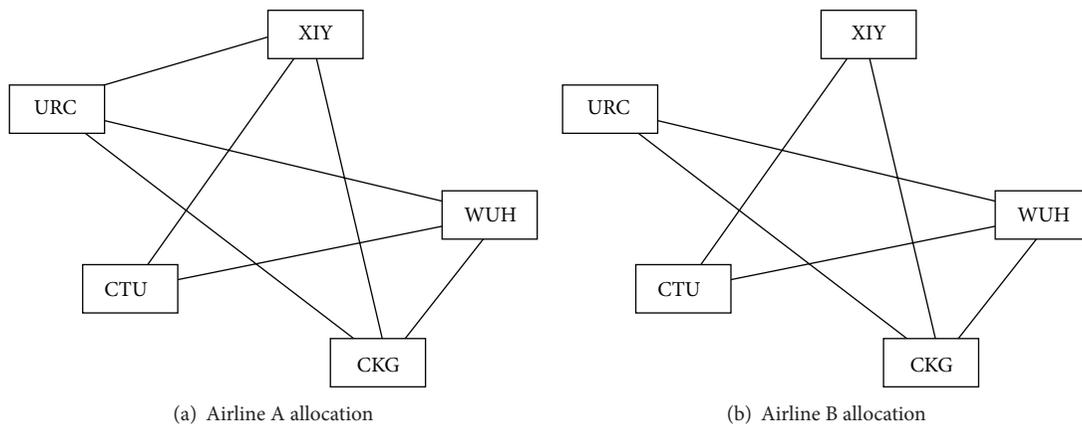


FIGURE 10: Equilibrium in location game (China).

TABLE 4: Russian airports.

IATA	City
GDX	Magadan
IKT	Irkutsk
KHV	Khabarovsk
OVB	Novosibirsk
PKC	Petropavlovsk-Kamchatsky
UUS	Yuzhno-Sakhalinsk
VVO	Vladivostok
YKS	Yakutsk

TABLE 5: Chinese airports.

IATA	City
CKG	Chongqing
CTU	Chengdu
URC	Urumqi
WUH	Wuhan
XIY	Xi'an

Acknowledgments

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