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Research Article

Sign Stability for Switched Linear Systems and Its Application in Flight Control

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The sign stability concept in ecological systems is introduced into the analysis and synthesis of switched linear system to explore new control design technique. The necessary and sufficient condition for sign stability of a switched linear system under arbitrary switching is achieved via the notion of complete isogenous sign stable set (CISSS). A new approach for the stabilization of switched system is presented. Although the controllers are devised for each subsystem, respectively, the switched system is sign stabilized by the constitution of CISSS. The provided method has natural robustness and more design freedoms than the familiar Lyapunov function method, which bears relative conservativeness as the requirement of solving LMIs. The presented technique is validated by an example of flight control within a large-scale flight envelop. Simulation results indicate that the proposed method can stabilize the flight attitude under large variations of system parameters and external perturbations.

1. Introduction

The sign stability (or qualitative stability) concept is first proposed in bionomics and utilized to analyze the interactions of different species in a large-scale ecosystem which lacks exact model but presents high robustness under various perturbations [1, 2]. The sign stability approach is also applied to population biology and economics in respect that these systems are also short of quantitative mathematical models. This qualitative analysis technique provides an avenue to research the linear system stability by the Jacobian matrix with only signs, and still attracts growing attention in system science community [3, 4].

For a given matrix, the signs ($+, -,$ or 0) of its elements are taken to make up a new matrix named the sign-pattern. A matrix is called sign stable if arbitrary matrix which has the same sign-pattern is Hurwitz stable, regardless of the elements' magnitudes. Hence, the sign stability of a matrix equates to that of the corresponding sign-pattern. Jeffries developed necessary and sufficient conditions for sign stability of matrix and proposed an approach named the color test to verify an arbitrary matrix in ecological terms [5, 6]. Yedavalli translated the color test conditions in matrix theory notation and devised a programmable set of conditions for the color test in an irreducible matrix [7]. It is worth noticing that the conclusions in this work are based on the conditions and criterions for the matrix sign stability presented in the aforementioned literatures.

Based on theoretical researches, the sign stability technique is applied in control engineering, especially the robust controllers design for aerospace flight control system. A three-axis attitude stabilization controller for an axisymmetric satellite is provided in [7]. The closed-loop system matrix is designed to possess the specific sign-pattern and sign stability property which will bring the system robust stability under arbitrarily large variations in the spin angular velocity. In [8], the effect of the elements' signs on the matrix properties such as eigenvalues and condition number is shown. Efforts are made to identify target closed-loop systems that incorporate the desirable features of ecological systems, and an algorithm for the design of controller is given. The control design procedure is illustrated with two applications in the aerospace field: satellite attitude control and aircraft lateral dynamics control.

However, the existing applications of sign stability only consider linear time-invariant systems which cannot approximate the flight dynamics within the full flight envelope. One of the alternative solutions is the switched linear system which is a good approximation of complicated system characteristics. Flight-control-oriented analysis and synthesis of switched systems are studied in the prior literatures and show large potential in engineering practice [9–12]. For example, the gain schedule control that is widely applied in flight control systems can be abstracted as a switching control law if variations of system parameters are regarded as a series of switching with the transition of the schedule variable. Hence, the aim of this paper is to generalize the sign stability concept to switched linear system and develop a new and practicable control scheme.

Compared to the prior references, the significant contribution of this work is the conclusion on sign stability analysis of switched linear system, which is presented by a necessary and sufficient condition. The system model treated with sign stability approach is extended from LTI system to a hybrid system. The mix of continuous linear systems and discrete signals make switched system suitable to describe a wide range of engineering systems such as power systems, automotive engine control, flight control, and networked control systems. As a result, the application areas of sign stability approach are enlarged remarkably.

Another important contribution is that a new stabilization technique of switched system is presented. As for the asymptotic stability of switched system under arbitrary switching, the common Lyapunov function is a necessary and sufficient condition and a usual method. In [13, 14], the existing condition of common quadratic Lyapunov function for switched linear system is discussed. Furthermore, multiple Lyapunov functions method [15], dwell-time method [16], and average dwell-time method [17] are proposed to adapt different design situations. In recent years, Zhao and his coworkers propose the mode-dependent average dwell-time method in order to decrease conservativeness [18, 19]. To utilize the above methods, the LMI technique is widely used, whereas the feasible solution of LMI is only a sufficient condition for the existence of common Lyapunov function, and an expectant controller may not exist or may not be found even if one does exist [20]. That leads to uncertainty of the controller solvability, even though modern mathematic tools are utilized. In contrast, the sign stability theory offers a fire-new approach with which designers can configure target sign-patterns of closed-loop switched subsystems via state feedback and guarantee the stability under arbitrary switching. The primary advantage of the sign stability technique is that there is no need to solve LMIs. That provides more design freedoms than Lyapunov function approach. Besides, the provided control synthesis technique possesses natural robustness because the sign stability is independent of the elements' magnitudes. This property allows the sign stabilization controllers to fit discretionarily large sign-preserving parameter perturbations, especially in the flight control system.

With this consideration, the paper is organized as follows. In Section 2, the basic concepts and conditions for the sign stability of matrix are reviewed. The definition of sign stability for switched linear system is presented to formulate the main issue of this paper. In Section 3, the main result of this paper is given in the form of necessary and sufficient condition for the sign stability of switched system along with a new approach for the stabilization problem. In Section 4, the proposed technique is illustrated by an application to flight control of the HiMAT vehicle. Finally, Section 5 provides a detailed conclusion of this work.

2. Problem Formulation

In this section, preliminary knowledge about the sign stability is reviewed, and then the definition of sign stability for switched linear system is proposed to formulate the main problem of this paper.

2.1. Preliminary Knowledge. A switched linear autonomous system is usually described as

$$\dot{x}(t) = A_{\sigma(t)} x(t), \tag{1}$$

in which $\sigma(t) : [0, t) \rightarrow \Omega = \{1, 2, \dots, n\}$ is the switching signal and $\{A_i, i \in \Omega\}$ is the set of switching subsystem matrices. Then the aim of this section is to establish the concept of sign stability for the switched system (1). Above all, basic knowledge and results are needed.

For the $n \times n$ real matrix $A = (a_{ij})$ and $B = (b_{ij})$, if $\operatorname{sgn} a_{ij} = \operatorname{sgn} b_{ij}$, for all i, j, it is defined that the two matrices have the same sign-pattern. The sign-pattern can be represented as a matrix, of which all the entries simply consist of signs $+$, $-$, or 0. The $\operatorname{sgn}(A)$ is defined as the sign-pattern matrix of A. Similarly, $\operatorname{sgn}\{A_i, i \in \Omega\}$ is defined as the sign-pattern set of the matrix set $\{A_i, i \in \Omega\}$; that is, $\operatorname{sgn}\{A_i\} = \{\operatorname{sgn}(A_i), i \in \Omega\}$. For two sets of matrices $\{A_i, i \in \Omega\}$ and $\{B_j, j \in \Delta\}$, if for all $i \in \Omega \operatorname{sgn}(A_i) \in \operatorname{sgn}\{B_j\}$ and for all $j \in \Delta \operatorname{sgn}(B_j) \in \operatorname{sgn}\{A_i\}$, it is defined that the two matrix sets have the same sign-pattern set.

Definition 1. The $n \times n$ real matrix $A = (a_{ij})$ is sign stable if each matrix $B = (b_{ij})$ of the same sign-pattern as A is Hurwitz stable.

The necessary and sufficient condition for the sign stability of matrix is provided in referenced researches [5, 21–23]. The main necessary conditions are listed in Lemma 2 as the foundation of main results in this paper.

Lemma 2. *The following are necessary conditions for sign stability of matrix $A = (a_{ij})$.*

(1) *For all i, $a_{ii} \leq 0$.*

(2) *$a_{ii} < 0$ for at least one i.*

(3) *For all $i \neq j$, $a_{ij} a_{ji} \leq 0$.*

(4) $a_{ij}a_{jk}\cdots a_{qr}a_{ri} = 0$ *for any sequences of three or more distinct indices* $i, j, k, \ldots, q, r$.

(5) $\det A \neq 0$.

It is worth announcing that the color test, given by Jeffries in [5], is confirmed as the criterion for the sign stability of any matrix or sign-pattern in this work.

2.2. Problem Statement. Compared with the definition of sign stability for matrix, sign stability of switched linear system is defined as below.

Definition 3. The switched linear system (1) is sign stable under arbitrary switching if each switched system $\dot{x}(t) = \overline{A}_{\sigma(t)}x(t)$ which has the same sign-pattern set as $\{A_{\sigma(t)}\}$ is asymptotically stable under arbitrary switching.

From Definition 3 it is seen that, for a given switched system, that is, sign stable, it is obviously asymptotically stable under arbitrary switching. Besides, the determinant of sign stability is the sign-pattern set, and the concept of sign stability for the sign-pattern set $\mathrm{sgn}\{A_\sigma\}$ is similar to that for the switched system. We can also know that a sign stable switched system will be achieved by assigning arbitrary values for elements of sign-patterns in a sign-pattern set which is sign stable. In addition, the number of subsystems will not affect the stability of the switched system.

Remark 4. In a sign stable switched system, there may be several subsystems of which the matrices have the same sign-pattern. In other words, the switched system derived from a given sign stable sign-pattern set is stable even if more than one matrices are built according to the same sign-pattern.

Remark 5. The subset of an arbitrary sign stable switched system (or sign-pattern set) is also sign stable. The conclusion is obtained under arbitrary switching laws.

With the concept given in Definition 3, we are now interested in the properties of a sign stable switched linear system, and it is also important to develop a technique to constitute a sign stable switched system. Towards the above targets, the main results are provided in the next section. To accomplish the proof of the main theorem, a primary conclusion given by [24] is used and presented here as a lemma.

Lemma 6. *The following statements are equivalent:*

(1) *the switched linear system*

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{2}$$

where $A_{\sigma(t)} \in \{A_1, A_2, \ldots, A_N\}$ *is asymptotically stable under arbitrary switching;*

(2) *the linear time-variant system*

$$\begin{aligned} &\dot{x}(t) = F(t)x(t), \\ &F(t) = \left\{\sum_{i=1}^{N}\alpha_i(t)A_i \mid \alpha_i(t) \geq 0, \ \sum_{i=1}^{N}\alpha_i(t) = 1\right\} \end{aligned} \tag{3}$$

is asymptotically stable.

3. Main Results

3.1. Necessary Condition. From Definition 3 and Lemma 6, the sign stability of the switched linear system (2) under arbitrary switching implies that each sign-pattern in $\mathrm{sgn}\{A_\sigma\}$ is sign stable. The necessary conditions of a sign stable switched linear system are given in the following theorem, where a_{ij}^k denotes the $i \times j$th element of the kth subsystem matrix of switched system (2).

Theorem 7. *The following are necessary conditions for sign stability of switched linear system* (2) *under arbitrary switching.*

(1) *For all* A^k, *which is the* k*th subsystem matrix of switched system* (2), *the necessary conditions in Lemma 2 are satisfied.*

(2) *For all* $p \neq q$, *for all* $i \neq j$, *one of the following three conditions is satisfied: (i)* $a_{ij}^p = a_{ij}^q = 0$; *(ii)* $a_{ji}^p = a_{ji}^q = 0$; *(iii)* $a_{ij}^p a_{ij}^q \geq 0$.

Proof. For (1), noting that each subsystem of switched system (2) is sign stable, the conclusion is obvious.

For (2), with no loss of generality, it is supposed that there are 2 subsystems in switched system (2). From Lemma 6, the linear time-variant system (3) is asymptotically stable, and the system matrix can be written as

$$\begin{aligned} F(t) = \big\{&\alpha_1(t)A^1 + \alpha_2(t)A^2 \mid \\ &\alpha_1(t), \alpha_2(t) \geq 0, \ \alpha_1(t) + \alpha_2(t) = 1\big\}. \end{aligned} \tag{4}$$

It is easily understood that $F(t)$ is sign stable because the elements of A^1 and A^2 may take arbitrary values. Define f_{ij} as the $i \times j$th element of $F(t)$, and it holds that

$$f_{ij}(t) = \alpha_1(t)a_{ij}^1 + \alpha_2(t)a_{ij}^2. \tag{5}$$

Due to condition (3) of Lemma 2, the following inequality holds for all $i \neq j$:

$$\begin{aligned} f_{ij}(t)f_{ji}(t) &= \left(\alpha_1(t)a_{ij}^1 + \alpha_2(t)a_{ij}^2\right)\left(\alpha_1(t)a_{ji}^1 + \alpha_2(t)a_{ji}^2\right) \\ &\leq 0. \end{aligned} \tag{6}$$

(i) For the situation that $f_{ij}(t)f_{ji}(t) = 0$, since $\alpha_1(t)$ and $\alpha_2(t)$ may take arbitrary values, it must hold that $a_{ij}^1 = a_{ij}^2 = 0$ or $a_{ji}^1 = a_{ji}^2 = 0$. That is, for all $p \neq q$, for all $i \neq j$, $a_{ij}^p = a_{ij}^q = 0$ or $a_{ji}^p = a_{ji}^q = 0$ is satisfied.

(ii) For the situation that $f_{ij}(t)f_{ji}(t) < 0$, with no loss of generality, supposing $\alpha_1(t) = \alpha_2(t) \equiv 0.5$ gives

$$\left(a_{ij}^1 + a_{ij}^2\right)\left(a_{ji}^1 + a_{ji}^2\right) < 0, \quad \forall i \neq j. \tag{7}$$

That is,

$$a_{ij}^1 a_{ji}^1 + a_{ij}^2 a_{ji}^1 + a_{ij}^1 a_{ji}^2 + a_{ij}^2 a_{ji}^2 < 0, \quad \forall i \neq j. \tag{8}$$

Considering that all the elements permit arbitrary sign-preserving variations, let

$$\left|a_{ij}^1\right| = \left|a_{ji}^2\right|, \qquad \left|a_{ij}^2\right| = \left|a_{ji}^1\right| \tag{9}$$

which leads to

$$\begin{aligned} a_{ij}^1 a_{ji}^1 &= \operatorname{sgn}\left(a_{ji}^1 a_{ij}^2\right) a_{ij}^1 a_{ij}^2, \\ a_{ij}^2 a_{ji}^1 &= \operatorname{sgn}\left(a_{ji}^1 a_{ij}^2\right) \left(a_{ij}^2\right)^2, \\ a_{ij}^1 a_{ji}^2 &= \operatorname{sgn}\left(a_{ij}^1 a_{ji}^2\right) \left(a_{ij}^1\right)^2, \\ a_{ij}^2 a_{ji}^2 &= \operatorname{sgn}\left(a_{ij}^1 a_{ji}^2\right) a_{ij}^1 a_{ij}^2, \end{aligned} \tag{10}$$

with $\operatorname{sgn}(a)$ denoting the sign of a.

From condition (3) of Lemma 2, it is known that a_{ij}^1 and a_{ji}^1 have the opposite signs (including 0), so do a_{ij}^2 and a_{ji}^2. It implies that $a_{ij}^1 a_{ji}^2$ and $a_{ji}^1 a_{ij}^2$ have the same sign. That is, $\operatorname{sgn}(a_{ij}^1 a_{ji}^2) = \operatorname{sgn}(a_{ji}^1 a_{ij}^2)$. Then (8) can be rewritten as

$$\operatorname{sgn}\left(a_{ij}^1 a_{ji}^2\right)\left(a_{ij}^1 a_{ij}^2 + \left(a_{ij}^2\right)^2 + \left(a_{ij}^1\right)^2 + a_{ij}^1 a_{ij}^2\right) < 0, \quad \forall i \neq j. \tag{11}$$

That is,

$$\operatorname{sgn}\left(a_{ij}^1 a_{ji}^2\right)\left(a_{ij}^1 + a_{ij}^2\right)^2 < 0, \quad \forall i \neq j. \tag{12}$$

To ensure (12), a_{ij}^1 and a_{ji}^2 must have the opposite signs. Therefore, a_{ij}^1 and a_{ij}^2 have the same sign (including 0). That is, for all $p \neq q$, for all $i \neq j$, $a_{ij}^p a_{ij}^q \geq 0$.

Synthesizing situations (i) and (ii), condition (2) is proved. □

It can be concluded by Theorem 7 that, for all the subsystem matrices of a sign stable switched system, the elements in the same position must satisfy specific conditions. According to this, the definition of an isogenous sign-pattern set and correlative corollary are given as below.

Definition 8. A sign-pattern set is called an isogenous set if the elements in the same position of all the sign-patterns satisfy one of the following terms: (i) they have the same sign (including 0); (ii) they are all 0; (iii) the elements in the opposite position are all 0.

Corollary 9. *The sign-pattern set of a sign stable switched system is an isogenous set.*

Remark 10. We also say that two sign-patterns are isogenous with each other if they conform to the condition of Definition 8. Apparently, if arbitrary two sign-patterns of a known set are isogenous, this set is an isogenous set. Corollary 9 can be acquired from Theorem 7 and Definition 8 easily. However, it should be noticed that the isogenous set is only a necessary condition for sign stable switched system. For example, an isogenous set by 3×3 is in (13). However, the set is not sign stable as the first sign-pattern is not sign stable obviously:

$$\left\{\begin{bmatrix} - & + & + \\ - & 0 & - \\ 0 & + & - \end{bmatrix}, \begin{bmatrix} - & 0 & + \\ - & - & - \\ 0 & 0 & - \end{bmatrix}, \begin{bmatrix} 0 & + & - \\ - & - & 0 \\ 0 & + & - \end{bmatrix}\right\}. \tag{13}$$

Considering an arbitrary isogenous sign-pattern set, we define the joining operation of all the sign-patterns to develop a special sign-pattern named the original sign-pattern of the set. The joining operation should keep the following rules.

(1) If elements in a certain position of all the sign-patterns have the same nonzero sign or 0, then the corresponding element of the original sign-pattern is the same nonzero sign.

(2) If elements in a certain position of all the sign-patterns are all 0, then the corresponding element of the original sign-pattern is 0.

(3) If elements in a certain position of all the sign-patterns have different signs, then the corresponding element of the original sign-pattern is written as $*$ to denote arbitrary signs.

The original sign-pattern obtained from a known isogenous set may be a new sign-pattern or an existent one which is already in the set. The original sign-pattern of set (13) is given in as follows as an illustration:

$$\begin{bmatrix} - & + & * \\ - & - & - \\ 0 & + & - \end{bmatrix}. \tag{14}$$

Furthermore, it is obvious that the new original sign-pattern and the known isogenous set can be joined to build a new set which is still isogenous. Actually, new sign-patterns can be derived to extend the isogenous set by replacing some nonzero elements of the original sign-pattern by 0. To demonstrate the aforementioned operation, the set which is developed from (13) is given below with the first matrix as the original sign-pattern and the last two as accessorial ones:

$$\begin{aligned} \Bigg\{&\begin{bmatrix} - & + & * \\ - & - & - \\ 0 & + & - \end{bmatrix}, \begin{bmatrix} - & + & + \\ - & 0 & - \\ 0 & + & - \end{bmatrix}, \begin{bmatrix} - & 0 & + \\ - & - & - \\ 0 & 0 & - \end{bmatrix}, \\ &\begin{bmatrix} 0 & + & - \\ - & - & 0 \\ 0 & + & - \end{bmatrix}, \begin{bmatrix} - & + & - \\ - & - & - \\ 0 & + & 0 \end{bmatrix}, \begin{bmatrix} - & + & 0 \\ 0 & - & - \\ 0 & + & - \end{bmatrix}\Bigg\}. \end{aligned} \tag{15}$$

Of course, we can continue to add new isogenous sign-patterns of (15) on the basis of the original sign-pattern and list all the other ones. However, among all these sign-patterns, the sign stable ones are what we focused on and can be selected out. That leads to another important definition in this paper.

Definition 11. A sign-pattern set is named a complete isogenous sign stable set (CISSS) if it contains a sign stable original sign-pattern and all of the derivative isogenous sign-patterns that are sign stable.

Remark 12. Definition 11 indicates that we can obtain a CISSS from an isogenous set, for example, (15), if the original sign-pattern and some other ones are sign stable. Unfortunately, as the original sign-pattern of (15) is not sign stable, the subset of (15) cannot be a CISSS. It can be seen that a sign stable original sign-pattern is necessary for a CISSS. This notion can be interpreted by another example. Set (16) is an isogenous set, but not a CISSS even though all the sign-patterns are sign stable. Actually, the original sign-pattern given by (17) is not included in (16), and it is more important that the original sign-pattern is not sign stable:

$$\left\{ \begin{bmatrix} - & 0 \\ - & - \end{bmatrix}, \begin{bmatrix} - & - \\ 0 & - \end{bmatrix} \right\}, \tag{16}$$

$$\begin{bmatrix} - & - \\ - & - \end{bmatrix}. \tag{17}$$

Remark 13. It also should be emphasized that the selection of the uncertain element $*$ does not affect the sign stability, which is related to the products of the off-diagonal pairs. Since the sign $*$ must appear in the form of an off-diagonal pair "0 and $*$," leading the product to 0 at all time, the above pair can still exist in CISSS.

3.2. Necessary and Sufficient Condition. In what follows, the main result of this work is proposed.

Theorem 14. *The necessary and sufficient condition for sign stability of switched linear system* (2) *under arbitrary switching is that the sign-pattern set of* (2) *is a subset of a CISSS.*

Proof. (1) The necessity can be easily proved by Corollary 9 and Definition 11.

(2) If the sign-pattern set $\mathrm{sgn}\{A_\sigma\}$ of (2) is a subset of a CISSS, then all the sign-patterns of $\mathrm{sgn}\{A_\sigma\}$ are isogenous and sign stable. For another arbitrary matrix set $\{\overline{A}_i\}$ that has the same sign-pattern set $\mathrm{sgn}\{A_\sigma\}$ and $\{\alpha_i \mid \alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1\}$, let

$$A^* = \sum_{i=1}^{N} \alpha_i \overline{A}_i. \tag{18}$$

According to Definition 11, it is certain that $\mathrm{sgn}(A^*)$ belongs to the foregoing CISSS. Hence, A^* is sign stable and thus Hurwitz stable. By Lemma 6, the switched system $\dot{x}(t) = \overline{A}_{\sigma(t)} x(t)$ is asymptotically stable. At last, by Definition 3, the sufficiency is proved. □

With Theorem 14, it is easy to judge whether a given switched system is sign stable. To construct a sign stable switched system, the original sign-pattern and the CISSS are required first. Furthermore, a sign stable original sign-pattern with the least element "0" is the basis to constitute a CISSS.

3.3. Simple Examples. Two simple examples are given to illuminate the constitution of CISSS.

Example 1. For 2×2 matrices, the sets Δ_1, Δ_2 given by (19) are CISSS with the first sign-pattern as the original sign-pattern of each set, respectively:

$$\begin{aligned} \Delta_1 &= \left\{ \begin{bmatrix} - & + \\ - & - \end{bmatrix}, \begin{bmatrix} 0 & + \\ - & - \end{bmatrix}, \begin{bmatrix} - & + \\ - & 0 \end{bmatrix}, \begin{bmatrix} - & + \\ 0 & - \end{bmatrix}, \begin{bmatrix} - & 0 \\ - & - \end{bmatrix}, \begin{bmatrix} - & 0 \\ 0 & - \end{bmatrix} \right\}, \\ \Delta_2 &= \left\{ \begin{bmatrix} - & - \\ + & - \end{bmatrix}, \begin{bmatrix} 0 & - \\ + & - \end{bmatrix}, \begin{bmatrix} - & - \\ + & 0 \end{bmatrix}, \begin{bmatrix} - & - \\ 0 & - \end{bmatrix}, \begin{bmatrix} - & 0 \\ + & - \end{bmatrix}, \begin{bmatrix} - & 0 \\ 0 & - \end{bmatrix} \right\}. \end{aligned} \tag{19}$$

It has been summarized that the original sign-pattern is needed to have as few "0" elements as possible. Hence, all the diagonal elements are chosen to be "−," and the off-diagonal elements are selected to possess as many pairs of opposite signs as possible. For 2 × 2 matrices, it is not necessary for us to configure any "0" element in the original sign-pattern as shown in Example 1, whereas, for 3×3 matrix $A_{3\times3} = (a_{ij})_{3\times3}$, according to condition (4) of Lemma 2, there must be at least two "0" elements in the off-diagonal positions to guarantee $a_{12}a_{23}a_{31} = 0$ and $a_{13}a_{32}a_{21} = 0$. It then can be concluded that, in the 3 × 3 matrix below, there must be at least one 0 in the positions represented by & and one 0 in the positions represented by #, respectively:

$$\begin{bmatrix} - & \& & \# \\ \# & - & \& \\ \& & \# & - \end{bmatrix}. \tag{20}$$

Example 2. Consider the 3×3 sign stable original sign-pattern given in (21), where each $*$ presents an uncertain sign that may be +, −, or 0. The aim is to discover the corresponding CISSS:

$$\begin{bmatrix} - & 0 & 0 \\ * & - & + \\ * & - & - \end{bmatrix}. \tag{21}$$

Utilizing the criterion in [5] to select sign stable sign-patterns in the isogenous set of (21), the CISSS is acquired and shown in (22), with each $*$ denoting an arbitrary sign. As each sign-pattern in (22) contains 9 (= 3 × 3) sign-patterns, there is a total of 54 (= 6 × 9) sign-patterns in the CISSS:

$$\begin{aligned} &\left\{ \begin{bmatrix} - & 0 & 0 \\ * & - & + \\ * & - & - \end{bmatrix}, \begin{bmatrix} - & 0 & 0 \\ * & 0 & + \\ * & - & - \end{bmatrix}, \begin{bmatrix} - & 0 & 0 \\ * & - & + \\ * & - & 0 \end{bmatrix}, \right. \\ &\left. \begin{bmatrix} - & 0 & 0 \\ * & - & 0 \\ * & - & - \end{bmatrix}, \begin{bmatrix} - & 0 & 0 \\ * & - & + \\ * & 0 & - \end{bmatrix}, \begin{bmatrix} - & 0 & 0 \\ * & - & 0 \\ * & 0 & - \end{bmatrix} \right\}. \end{aligned} \tag{22}$$

In fact, in the above process of finding CISSS, the two pairs of "0 and $*$" do not need to be changed. The first term

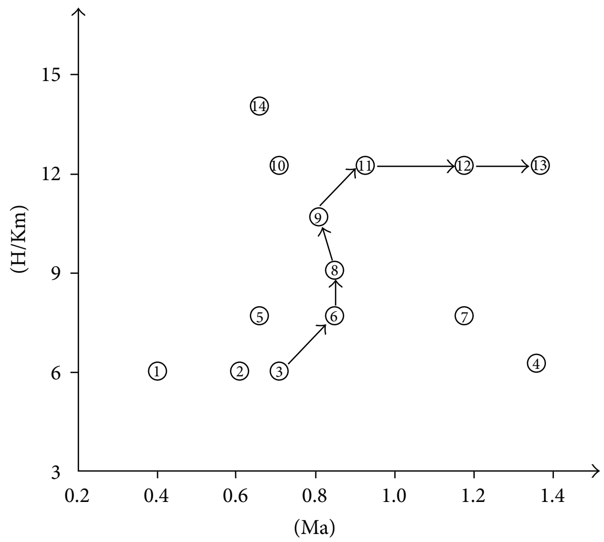

Figure 1: Partial flight envelope and operating points.

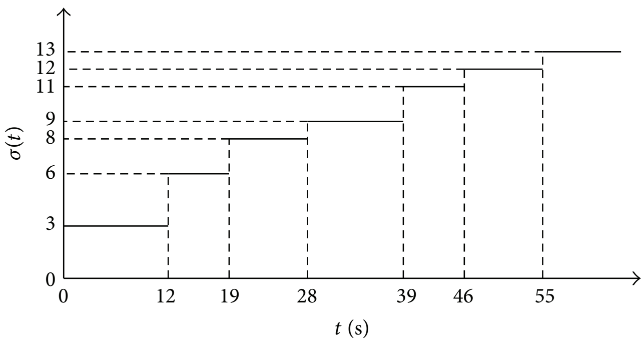

Figure 2: Switching law of the flight trajectory.

of the diagonal is also unchangeable. As a result, the only way is to transform the 2×2 block on the lower-right. Indeed, it can be seen that the lower-right blocks are the same as the set Δ_1 of Example 1.

4. Example

The controller design problem for the highly maneuverable technology (HiMAT) vehicle is presented to demonstrate the main results in this paper. As declared in [10], a switched system can be applied to describe flight conditions with fast varying parameters. Controllers for the subsystems can be synthesized via a switching law to adapt the parameter variations among different operating points. In this section, the sign stability approach is utilized to design a switching control scheme for the unstable longitudinal dynamics. The considered flight envelope and 14 operating points within it are depicted in Figure 1. We suppose that the dynamics in the contiguous region of each operating point can be approximated by the corresponding linear subsystem. These linear models are given in [25].

The linearization model for each operating point is regarded as a subsystem of the switched system. The switched system is then given by

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t), \quad \sigma \in \{1, 2, \ldots, 14\}, \tag{23}$$

where $x(t) \in R^3$ is the state vector consisting of three state variables α (angle of attack), q (pitch rate), and ϕ (pitch angle), and $u(t) \in R^4$ is the control vector consisting of δ_e (elevator), δ_v (elevon), δ_c (canard), and δ_s (symmetric aileron).

The system matrices and control matrices for several operating points are shown to illustrate the sign-patterns:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -0.8435 & 0.97505 & -0.0048 \\ 8.7072 & -1.1643 & 0.0026 \\ 0 & 1 & 0 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} -0.1299 & -0.092 & -0.0107 & -0.0827 \\ -7.6833 & -4.7974 & 4.8178 & -5.7416 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
A_6 &= \begin{bmatrix} -1.8997 & 0.98312 & -0.00073 \\ 11.720 & -2.6316 & 0.00088 \\ 0 & 1 & 0 \end{bmatrix}, \\
B_6 &= \begin{bmatrix} -0.2436 & -0.1708 & -0.00497 & -0.1997 \\ -46.206 & -31.604 & 22.396 & -31.179 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
A_{13} &= \begin{bmatrix} -1.2206 & 0.99411 & -0.00084 \\ -64.071 & -1.8876 & 0.00046 \\ 0 & 1 & 0 \end{bmatrix}, \\
B_{13} &= \begin{bmatrix} -0.0662 & -0.0315 & -0.0141 & -0.0749 \\ -27.333 & -13.163 & 11.058 & -26.878 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{24}$$

Furthermore, the sign-pattern set of the open-loop system matrices is given by the following:

$$\left\{ \begin{bmatrix} - & + & - \\ + & - & + \\ 0 & + & 0 \end{bmatrix}, \begin{bmatrix} - & + & - \\ - & - & + \\ 0 & + & 0 \end{bmatrix} \right\}. \tag{25}$$

As explained in Section 3, a subset of a CISSS is required for the closed-loop switched system matrices. According to the structure of $\{B_\sigma\}$, the original sign-pattern is chosen as follows:

$$\begin{bmatrix} - & * & * \\ 0 & - & - \\ 0 & + & 0 \end{bmatrix}. \tag{26}$$

Although the complete form of the CISSS derived from (26) has more sign-patterns, we can regard (26) as the target sign-pattern of closed-loop subsystems directly. Compared to the sign-patterns in (25), it is only needed to change two signs in each subsystem matrix. A state feedback control gain

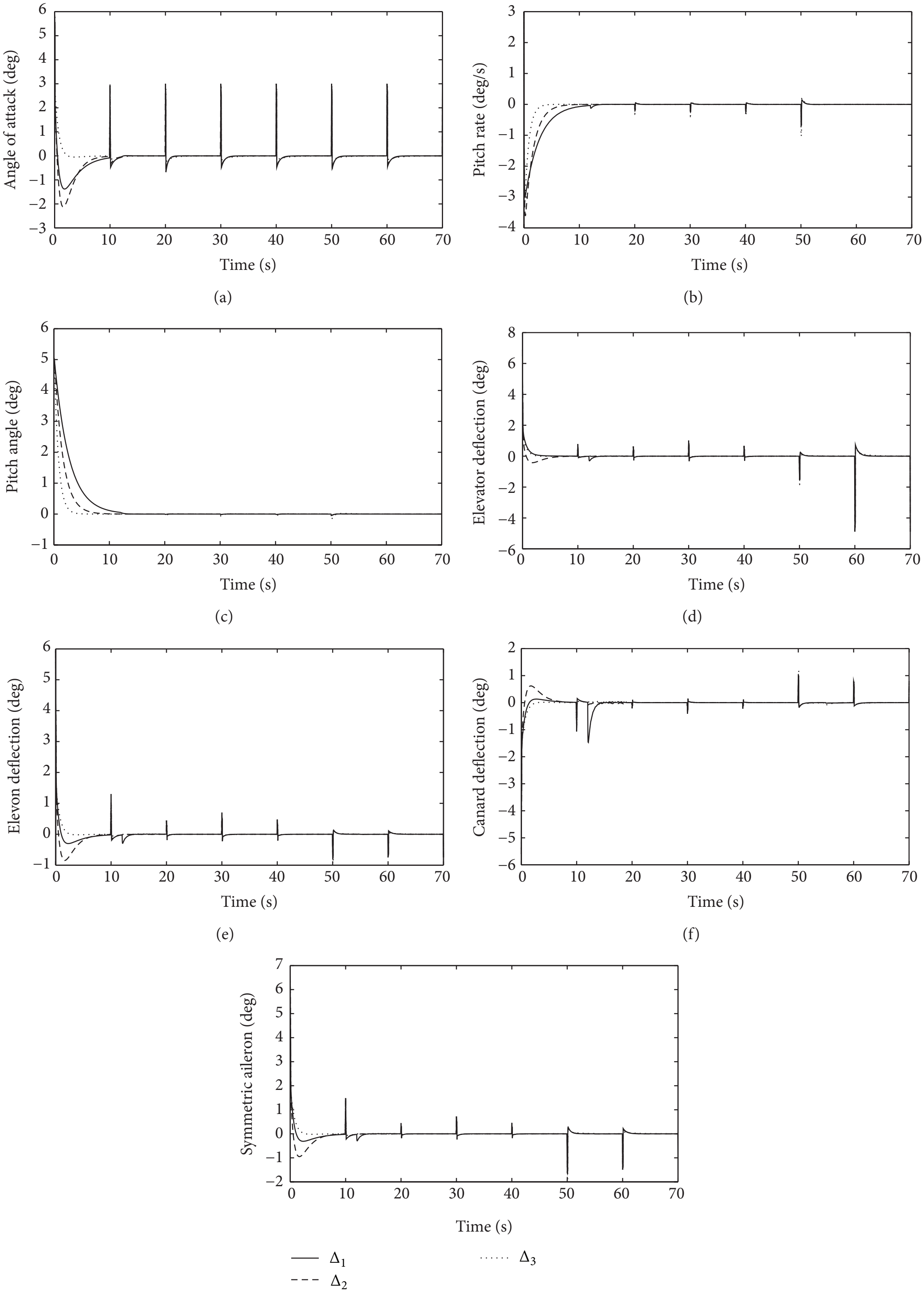

Figure 3: Time histories of system states and control inputs under perturbations: (a) angle of attack, (b) pitch rate, (c) pitch angle, (d) elevator, (e) elevon, (f) canard, and (g) symmetric aileron.

is designed for each operating point in the envelope, and the results for several points are given as follows:

$$A_{1c} = \begin{bmatrix} -0.9555 & 0.8066 & -0.1616 \\ 0 & -13.6041 & -11.7700 \\ 0 & 1 & 0 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0.2618 & 0.6935 & 0.6468 \\ 0.4362 & 0.4339 & 0.4005 \\ -0.3589 & -0.4278 & -0.4323 \\ 0.5007 & 0.5171 & 0.4875 \end{bmatrix},$$

$$A_{6c} = \begin{bmatrix} -1.9582 & -0.6336 & -0.3349 \\ 0 & -18.4138 & -67.8792 \\ 0 & 1 & 0 \end{bmatrix},$$

$$K_6 = \begin{bmatrix} -0.0031 & 2.6675 & 0.6795 \\ 0.1577 & 2.4566 & 0.4644 \\ -0.0794 & 11.6769 & -0.3393 \\ 0.1636 & 2.4505 & 0.4557 \end{bmatrix}, \quad (27)$$

$$A_{13c} = \begin{bmatrix} -1.0714 & 0.9009 & 0.4040 \\ 0 & -42.1910 & -28.9097 \\ 0 & 1 & 0 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -1.6277 & 0.6241 & -1.3513 \\ -0.2504 & 0.3005 & -1.6873 \\ 0.2634 & -0.2534 & -11.2900 \\ -0.4974 & 0.6134 & -1.3687 \end{bmatrix}.$$

It can be seen that the closed-loop system matrices A_{1c}, A_{6c}, and A_{13c} have different sign-patterns. However, these sign-patterns (also including those of other operating points) belong to the CISSS derived from the original sign-pattern (26). Based on Theorem 14, the closed-loop switched system is sign stable under arbitrary switching.

The simulation across different regions of the flight envelope is taken to validate the proposed technique. The flight trajectory travels through vicinities of 7 operating points ($3 \rightarrow 6 \rightarrow 8 \rightarrow 9 \rightarrow 11 \rightarrow 12 \rightarrow 13$; see Figure 1). The switching law is depicted in Figure 2.

To validate the stability within the full flight envelope, gust disturbance is considered in the form of perturbations acted on the angle of attack, which is a group of pulse signals with a period of 10 seconds, a width of 1 second, and a magnitude of 3 degrees. The initial values of the angle of attack, pitch rate, and pitch angle are set to 3 degrees, 3 degrees per second, and 5 degrees, respectively. To illustrate the robustness of the proposed approach, multiplicative perturbations on the closed-loop subsystem matrices are introduced. Each element in the matrices is multiplied by an arbitrary positive value, which will preserve all the signs of the products. The perturbation matrices are given by the following:

$$\Delta_1 = \begin{bmatrix} 1.5 & 1.2 & 0.9 \\ 0.8 & 1.8 & 1.3 \\ 1.4 & 0.6 & 0.7 \end{bmatrix},$$
$$\Delta_2 = \begin{bmatrix} 1.2 & 1.6 & 1.3 \\ 0.5 & 1.2 & 1.1 \\ 1.5 & 0.8 & 1.4 \end{bmatrix}, \qquad \Delta_3 = \begin{bmatrix} 0.5 & 0.6 & 1.1 \\ 1.4 & 2.2 & 1.8 \\ 0.5 & 1.6 & 2.4 \end{bmatrix}. \quad (28)$$

The resulted closed-loop subsystem matrices are obtained by multiplying each element of the original matrix with the corresponding element in the perturbation matrix. Thus, it leads to 3 closed-loop switched systems with different multiplicative perturbations. The time histories of the system states and control surface deflections are depicted in Figure 3.

As depicted in Figure 3, the system states converge quickly under gust disturbances and keep stable while the switching occurs along the flight trajectory. There are vibrations in the deflections of the control surfaces at the time of the first switching, nevertheless the amplitudes are acceptable. It is concluded that the devised approach is robust for the sign-preserving parameter perturbations on the closed-loop systems.

5. Conclusion

A new concept of sign stable switched linear system is established to develop novel techniques for flight control. The main result is provided in the form of necessary and sufficient condition for the sign stability of switched system under arbitrary switching. Hence, the application areas of sign stability approach are enlarged remarkably. A new robust stabilization approach for switched linear system is proposed via the notion of CISSS. The sign stabilization controllers are devised for each subsystem, respectively; therefore the design process possesses more freedoms. Compared with the traditional Lyapunov function method, the proposed technique has natural robustness and decreased conservativeness. The aforementioned points are verified by a stabilization control problem of aircraft with a large-scale flight envelope.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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