Research Article

An Adaptive Estimation Scheme for Open-Circuit Voltage of Power Lithium-Ion Battery

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Open-circuit voltage (OCV) is one of the most important parameters in determining state of charge (SoC) of power battery. The direct measurement of it is costly and time consuming. This paper describes an adaptive scheme that can be used to derive OCV of the power battery. The scheme only uses the measurable input (terminal current) and the measurable output (terminal voltage) signals of the battery system and is simple enough to enable online implement. Firstly an equivalent circuit model is employed to describe the polarization characteristic and the dynamic behavior of the lithium-ion battery; the state-space representation of the electrical performance for the battery is obtained based on the equivalent circuit model. Then the implementation procedure of the adaptive scheme is given; also the asymptotic convergence of the observer error and the boundedness of all the parameter estimates are proven. Finally, experiments are carried out, and the effectiveness of the adaptive estimation scheme is validated by the experimental results.

1. Introduction

Electric vehicles (EVs) and hybrid electric vehicles (HEVs) have been getting more and more attention in recent years because they have the potential of improving the fuel efficiency and reducing the pollutant emissions [1]. The merits of the high energy density, high power density, no memory effect, and experience of low self-discharge when not in use compared to other batteries make the lithium-ion batteries the main contender for energy storage in EV/HEVs [2, 3].

Open-circuit voltage (OCV) is one of the most important parameters of a battery due to the intrinsic relationship between state of charge (SoC) and OCV [4, 5], which is defined as the measured terminal voltage when battery reaches steady-state. To the authors' best knowledge there only exist two methods to get the value of OCV: experiment test and online estimation. The former method is very time consuming because the battery needs rest 10 h or more to reach the steady-state when charging or discharging at each measurement. Reference [6] has proposed a rapid test method, and the pause time at different SoC needs only one minute before the new experiment test is carried out. The voltage that reaches steady-state during the pauses at different SoC during discharging and charging is connected with line, and then the value of OCV takes the mean of the lines. This way can significantly save test time, but however the conventional experiment method or the new experiment method is suitable for measurement in laboratory, but not suitable for online measurement. The latter method is based on the equivalent circuit model to estimate OCV. The voltage of the electromotive force [7] or the voltage across the bulk capacitor [8, 9] is used to denote OCV. In this method, the parameter (capacitors and resistances) values of the equivalent circuit are generally considered to be constant, which contradict the fact that all parameter values usually change along with the SoC, temperature, and usage history change, and then the estimate precision will be degraded due to the inaccuracy of parameter values. Thus new theory and new method are demanded to enrich the estimate method and enhance the estimate accuracy. Adaptive technology is undoubtedly the best choice to solve such problem.

Battery is a complex electrochemical system; the accurate modeling for it is more difficult. As the adaptive scheme is implemented easily and the modeling error is within an acceptable range, the Thevenin battery model that includes one RC network is adopted here. The state-space formulation is obtained based on the equivalent circuit model, and an adaptive observer is designed according to the feature of the state-space. The designed adaptive observer together with the update laws gives accurate estimate of the true states but does not give the accurate estimate of the true parameters, which can only guarantee the boundedness of all the parameter estimates. Fortunately, the estimate of some parameters change in the same proportion and the estimate of OCV is the quotient of them, thus the accurate estimate of OCV can be obtained.

The remainder of this paper is organized as follows. Section 2 presents the main results of this paper. In Section 2.1, a dynamic equivalent circuit model is chosen, and the state-space formulation is obtained based on the equivalent circuit model; in Section 2.2, the adaptive observer and adaptive laws are designed. Section 3 provides the experimental results to demonstrate the correctness of the proposed adaptive estimate scheme. This paper ends with some conclusions.

2. Modeling, Adaptive Observer, and Adaptive Laws Design

In this section, an appropriate model of the battery is chosen firstly, and then the dynamic characteristic of the battery system is described by differential equations. The design procedure of the adaptive observer and adaptive laws will be shown step by step at last.

2.1. Modeling. The principle of selecting a battery model is as simple as possible but precise enough for the investigated problem. The model chosen here is the first-order Thevenin model (see Figure 1), which is simple yet accurate enough for the control-oriented purpose in hybrid electric vehicles [1]. The parallel *RC*-branch, comprising R_p and C_p , is used to model battery polarization effect; *R* denotes the ohmic resistance, u_{oc} denotes the open circuit voltage, and u_t denotes the terminal voltage. *I* denotes the terminal current and assumes it is positive when discharge otherwise is negative. Based on the Kirchhoff's law, the electrical behavior of the circuit can be characterized as follows:

$$\dot{u}_p = -\frac{1}{C_p R_p} u_p + \frac{1}{C_p} I,$$

$$u_t = u_{oc} - RI - u_p.$$
(1)

From (1), the derivative of u_t can be obtained as

$$\begin{split} \dot{u}_{t} &= \dot{u}_{oc} - \dot{R}I - R\dot{I} - \dot{u}_{p} \\ &\approx -R\dot{I} + \frac{1}{C_{p}R_{p}}u_{p} - \frac{1}{C_{p}}I \\ &= -R\dot{I} + \frac{1}{C_{p}R_{p}}\left(u_{oc} - RI - u_{t}\right) - \frac{1}{C_{p}}I \\ &= -\frac{1}{C_{p}R_{p}}u_{t} - \frac{R + R_{p}}{C_{p}R_{p}}I - R\dot{I} + \frac{1}{C_{p}R_{p}}u_{oc}. \end{split}$$
(2)

Remark 1. The parameters *R*, R_p , and C_p are functions of time, SoC, and temperature, but the partial differential of them is treated as zero due to the reason that the change rate is very slow with respect to the signals sampling period (the sampling period is 1 s). For the same reason, $\dot{u}_{oc} \approx 0$.

2.2. Adaptive Observer and Adaptive Laws Design. This subsection is devoted to the constructive design of the adaptive observer and the update laws. To this end, choose $\theta_1 = 1/C_p R_p$, $\theta_2 = (R + R_p)/C_p R_p$, $\theta_3 = R$, $\theta_4 = (1/C_p R_p)u_{oc}$; then (2) can be rewritten as

$$\dot{u}_t = -\theta_1 u_t - \theta_2 I - \theta_3 \dot{I} + \theta_4, \tag{3}$$

obviously $u_{\rm oc} = \theta_4/\theta_1$.

Enlightened by [10], we design adaptive observer (4) for system (3) as well as for system (2) as follows:

$$\dot{\hat{u}}_t = -\widehat{\theta}_1 u_t - \widehat{\theta}_2 I - \widehat{\theta}_3 \dot{I} + \widehat{\theta}_4 + L \left(u_t - \widehat{u}_t \right), \qquad (4)$$

where \hat{u}_t is the estimate of u_t and L is the observer gain. The parameters $\hat{\theta}_i$ (i = 1, ..., 4) are estimates of the parameters θ_i (i = 1, ..., 4), which will be adjusted adaptively such that $\hat{u}_t \rightarrow u_t$ as $t \rightarrow \infty$.

Subtracting (4) from (3), the dynamic of observer error $e = u_t - \hat{u}_t$ is as follows:

$$\dot{e} = -\tilde{\theta}_1 u_t - \tilde{\theta}_2 I - \tilde{\theta}_3 \dot{I} + \tilde{\theta}_4 - Le, \qquad (5)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ (i = 1, ..., 4).

The following Barbalat's lemma that will be used in the development of our main results is given first.

Lemma 2 (see [11]). If the differentiable function f(t) has a finite limit as $t \to \infty$ and if \dot{f} is uniformly continuous, then $\dot{f}(t) \to 0$ as $t \to \infty$.

The proof can be found in the reference and is omitted here.

To guarantee the asymptotical convergence of observer error system (5) and the boundedness of signals $\hat{\theta}_i$ (i = 1, ..., 4), we have the following theorem.

Theorem 3. Consider battery system (2) as well as its parametric form (3), design the observer (4), if one chooses the parameter update laws for $\hat{\theta}_i$ (i = 1, ..., 4) as

$$\begin{aligned} \dot{\hat{\theta}}_1 &= -c_1 e u_t, \qquad \hat{\hat{\theta}}_2 &= -c_2 e I, \\ \dot{\hat{\theta}}_3 &= -c_3 e \dot{I}, \qquad \dot{\hat{\theta}}_4 &= c_4 e, \\ c_i &> 0, \qquad i = 1, \dots, 4 \end{aligned}$$
(6)

and the observer gain L > 0, then the signal e is convergent to zero as $t \to \infty$ and all the signals $\hat{\theta}_i$ (i = 1, ..., 4) are uniformly bounded.

Proof. Define a candidate Lyapunov function

$$V = \frac{1}{2}e^{2} + \frac{1}{2}\sum_{i=1}^{4} \left(\frac{1}{c_{i}}\widetilde{\theta}_{i}^{2}\right), \quad c_{i} > 0, \ i = 1, \dots, 4;$$
(7)

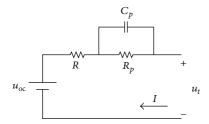


FIGURE 1: Thevenin equivalent model for lithium-ion battery.

from this together with (5) and (6), we have

$$\begin{split} \dot{V} &= e\dot{e} - \sum_{i=1}^{4} \frac{1}{c_i} \vec{\hat{\theta}}_i \\ &= -e\tilde{\theta}_1 u_t - e\tilde{\theta}_2 I - e\tilde{\theta}_3 \dot{I} + e\tilde{\theta}_4 \\ &- eL \left(u_t - \hat{u}_t \right) - \frac{1}{c_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 - \frac{1}{c_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2 - \frac{1}{c_3} \tilde{\theta}_3 \dot{\hat{\theta}}_3 - \frac{1}{c_4} \tilde{\theta}_4 \dot{\hat{\theta}}_4 \\ &= -\tilde{\theta}_1 \left(eu_t + \frac{1}{c_1} \dot{\hat{\theta}}_1 \right) - \tilde{\theta}_2 \left(eI + \frac{1}{c_2} \dot{\hat{\theta}}_2 \right) \\ &- \tilde{\theta}_3 \left(e\dot{I} + \frac{1}{c_3} \dot{\hat{\theta}}_3 \right) - \tilde{\theta}_4 \left(e - \frac{1}{c_4} \dot{\hat{\theta}}_4 \right) - eL \left(u_t - \hat{u}_t \right), \\ &= -Le^2 \le 0. \end{split}$$
(8)

Thus V(t) is a nonincreasing function of time; that is,

$$\sup_{t \ge 0} V(t) \le V(0).$$
(9)

This proves that *e* and $\overline{\theta}_i$ (*i* = 1, ..., 4) remain bounded for all $t \ge 0$, namely,

$$\lim_{t \to \infty} \|e(t)\| < \infty, \quad \lim_{t \to \infty} \left| \widetilde{\theta}_i(t) \right| < \infty, \quad i = 1, \dots, 4.$$
 (10)

The derivative of \dot{V} is $\ddot{V}(t) = 2Le(\tilde{\theta}_1u_t + \tilde{\theta}_2I + \tilde{\theta}_3\dot{I} - \tilde{\theta}_4 + L(u_t - \hat{u}_t))$; this shows that $\ddot{V}(t)$ is bounded since *e* and $\tilde{\theta}_i$ (i = 1, ..., 4) are bounded. Hence, $\dot{V}(t)$ is uniformly continuous. After the application of Barbalat's lemma we obtain the conclusion that $e \to 0$ as $t \to \infty$.

From (10) and the definition $\hat{\theta}_i = \theta_i - \hat{\theta}_i$, (i = 1, ..., 4), we know that $\hat{\theta}_i$ (i = 1, ..., 4) are bounded.

Thus, we obtain the conclusion of the asymptotical convergence to zero of the signal *e* and the uniform boundedness of the signals $\hat{\theta}_i$ (*i* = 1, ..., 4).

3. Experimental Results

There are three purposes for the experiment: the first one is to verify the adaptive observer which can give an accurate estimate of the system states; the second one is to verify the estimates of the parameters θ_1 and θ_4 which change with the same scaling proportion. The fact that OCV can be estimated by $\hat{\theta}_4/\hat{\theta}_1$ will be verified at last.

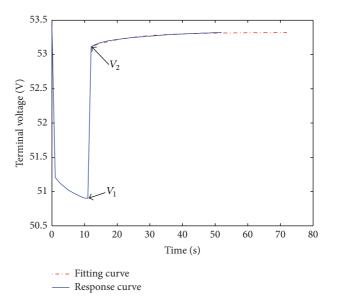


FIGURE 2: Terminal voltage response under step-current discharge.

3.1. Parameter Extraction. The extraction experiment that can obtain the parameters value is firstly carried out. The experimental sample is a large size lithium iron phosphate battery. The nominal capacity and voltage of the selected battery is 50 Ah and 51.2 V, respectively. The tests are carried out on an AVL e-Storage Tester. The tester is capable of charging and discharging the battery at a maximum rate of 285 A. It can operate in several modes such as alternating current and constant current. It comprises power, current, voltage, and temperature measurements. The battery tester and the data logger are connected to a PC through a bus system. The host computer with the installed software can program the experiment procedure. Several limits for the battery can be defined, which are necessary in order to avoid damage, for example, caused by overcharging.

The pulsed-discharge experiments are carried out at different SoC, and the terminal voltage is monitored simultaneously (see Figure 2). The battery parameters can be extracted through the transient response.

The terminal voltage response at different SoC can be fitted as in the following form:

$$u_t(t) = V_2 + c_1 \left(1 - e^{-c_2 t} \right), \tag{11}$$

where c_1 and c_2 are the least-squares curve fitting coefficients. V_2 is as shown in Figure 2. Then the parameters value of battery model (see Figure 1) can be derived from

$$R = \frac{V_2 - V_1}{I}, \qquad R_p = \frac{c_1}{I}, C_p = \frac{1}{c_2 R_p}.$$
(12)

The parameter values identified for the battery module at different SoC are given in Table 1. From Table 1, it can be seen that the parameters R, R_p , and C_p are all varying with the SoC changes, which further confirms that the system state estimation needs adaptive scheme.

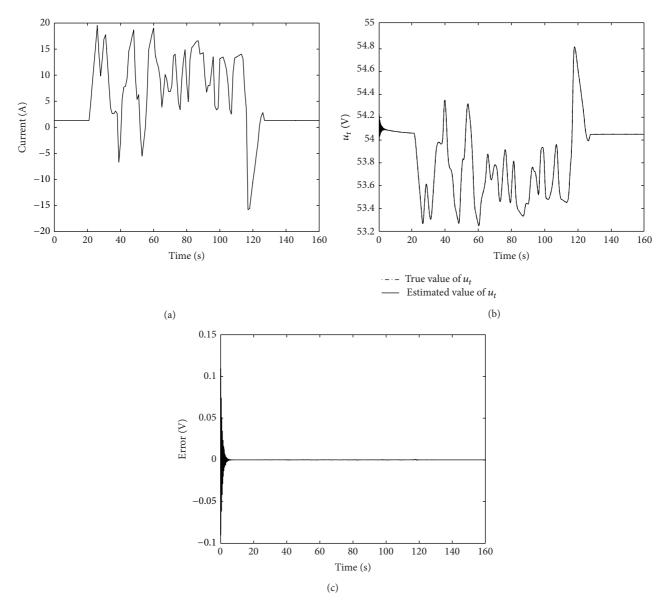


FIGURE 3: Comparison of the true and estimated battery terminal voltage.

TABLE 1: Model parameter values list identified for the battery module.

SoC	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$R(\Omega)$	0.05028	0.04882	0.04796	0.0471	0.04626	0.04564	0.04542	0.0448	0.04418	0.0438
$R_{p}\left(\Omega\right)$	0.008786	0.007328	0.00647	0.005808	0.005254	0.005232	0.004908	0.00481	0.004142	0.003978
C_p (F)	960	1297	1569	1906.7	2150.6	2222.5	3059.3	2315	2847	2913
$U_{\rm oc}$ (V)	51.459	52.18	52.658	52.758	52.808	53.078	53.387	53.394	53.406	54.108

3.2. Experiment Validation. This subsection is to validate the correctness of the theory of Section 2.2. The urban dynamometer driving schedule (UDDS) driving cycle of Advisor is adopted to generate terminal current (see Figure 3(a)). According to the parameter values at SoC = 1.0 (see Table 1), set the initial conditions of system (3) to be $u_t(0) =$ $54.108, \theta_1 = 0.0863, \theta_2 = 0.00412, \theta_3 = 0.0438, and \theta_4 = 4.67,$ and set the initial conditions of observer system (4) to be $\hat{u}_t = 51.459, \hat{\theta}_1(0) = 0.0833, \hat{\theta}_2(0) = 0.0035, \hat{\theta}_3(0) = 0.043,$ and $\hat{\theta}_4(0) = 4$. Set coefficients of update laws (6) to be $c_1 = 0.1$, $c_2 = 0.1, c_3 = 0.1$, and $c_4 = 0.1$, and set the observer gain to be L = 2. The experiment results are shown in Figures 3–5.

Figure 3(a) is the system input, namely, battery terminal current. Figure 3(b) is the original and observer system outputs. The observer error is shown in Figure 3(c), from which we can see that the observer error converges to zero

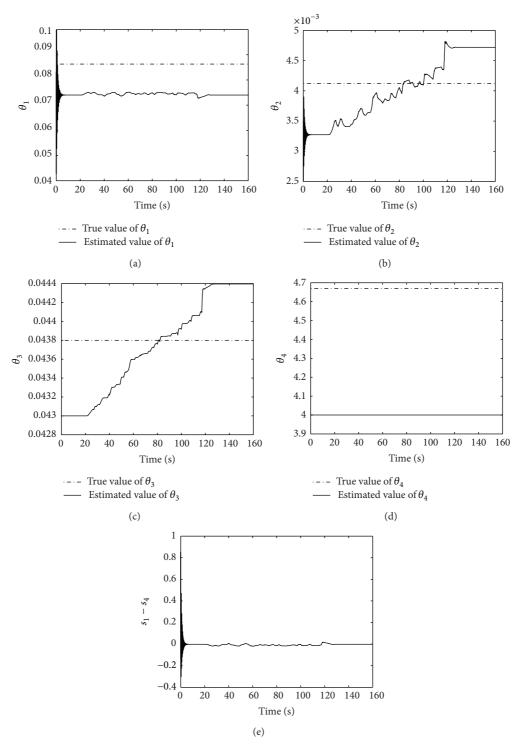


FIGURE 4: Comparison of the true and estimated parameters value for the equivalent circuit model.

within twelve seconds although the parameter values of original system and observer system are different.

Figures 4(a)-4(d) are the true and estimated values of the parameters θ_i (i = 1, ..., 4), from which we can see that the estimated values of the parameters are bounded. Assume that the scaling of the estimated values of θ_1 and θ_4 is $s_1 = \hat{\theta}_1/\theta_1$ and $s_4 = \hat{\theta}_4/\theta_4$, accordingly, Figure 4(e) shows that the difference of s_1 and s_4 is approximately zero. In other words $s_1 \approx s_4$; so the estimated value of $u_{\rm oc}$ can be obtained from $\hat{\theta}_4/\hat{\theta}_1$.

Figures 5(a) and 5(b) are the true and estimated battery OCV and the estimated error. It can be seen that the adaptive observer designed here can estimate the OCV accurately.

Remark 4. Battery discharging at a constant current value 15 A from SoC = 1.0 to SoC = 0.1 takes 10800 s (which does

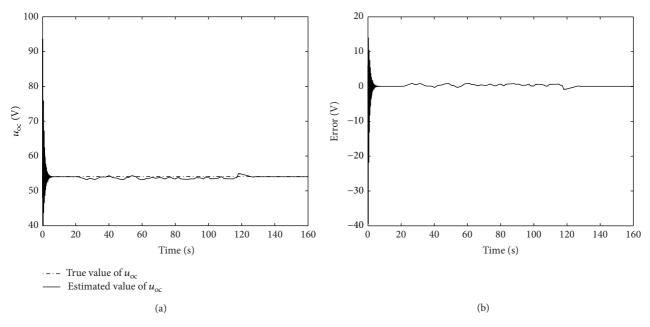


FIGURE 5: Comparison of the true and estimated battery open-circuit voltage.

not include the standing time of the discharge experiment). u_{oc} changes from the maximum 54.108 V to the minimum 51.459 V; so u_{oc} is considered to be a constant when discharging and charging for tens of seconds.

4. Conclusions

In this paper, an adaptive observer is designed to estimate OCV of a lithium-ion battery pack. The parameter values of the battery equivalent circuit model are extracted through nonlinear fitting. The performance of the proposed estimator is confirmed by a series of experiment results. The validity results show that the estimator has a good performance under the condition that it has no requirement for knowing the exact parameter values.

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