

Research Article

Dynamical Behavior of a Rumor Transmission Model with Psychological Effect in Emergency Event

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A rumor transmission model with nonmonotonic incidence rate was proposed, which provides excellent explanations of the “psychological” effect with rumor spreading in emergency event. By carrying out a global analysis of the model and studying the stability of the rumor-free equilibrium and the rumor-endemic equilibrium, we showed that either the number of infective individuals tends to zero as time evolves or the rumor persists. Finally, recommendations for policy makers and consulting advice for related commissions are explored in the case study of crazy rumors propagated for the iodized salt shortage panic in China.

1. Introduction

Rumors are part of our everyday life, and its spread has a significant impact on human lives. Hayakawa [1] defines rumor as a kind of social phenomenon that a similar remark spreads on a large scale in a short time through chains of communication. Rumors may contain confidential information about public figures or news which concerns important social issues; they can shape the public opinion of a society or a market by affecting the individual beliefs of its members, and its spread plays a significant role in a variety of human affairs [2]. Research of rumor has become an urgent and serious theory topic with practical significance.

Rumor spread is the social phenomenon that a remark spreads on a large scale in a short time through chain of communication. To analyze the spread and cessation of them, rumor transmissions are often modeled as social contagion processes. The classical models for the spread of rumor were introduced by Daley and Kendall [3] and Maki and Thompson [4], and then many researchers have used the model extensively in the past for their quantitative studies [5]. In classical models, people are divided into three classes: ignorant (those not aware of the rumor; let $x(t)$ be the number of ignorant individuals at time t), spreaders (those who are spreading it; $y(t)$, the number of infective individuals

at time t), and stiflers (those who know the rumor but have ceased communicating it after meeting somebody already informed, $z(t)$, the number of stifier individuals at time t), and they interact by pairwise contacts. In the Daley-Kendall (D-K) model, spreader-ignorant contact will convert the ignorant to spreader; spreader-spreader contact will convert both spreaders to stiflers and spreader-stifler contact will stifle the spreader. In the Maki-Thompson (M-T) model, the rumor is spread by direct contact of the spreaders with other individuals. Hence, when a spreader contacts another spreader, only the initiating one becomes a stifier. Bettencourt et al. [6] have worked on the spreading process of multiple varying ideas. Huang [7] studied the rumor spread process with denial and skepticism; two models are established to accommodate skeptics. Kawachi [8] proposed and mathematically analyzed deterministic models for rumor transmission, which are extensions of the deterministic D-K model. In Kawachi's other extension model [9], he and his cooperators studied a flexible spreader-ignorant-stifler model where spreader to ignorant and stifier to spreader transitions are possible, while Lebensztayn et al. [10] investigated the case where a new uninterested class of people exists. Huo et al. [11] considered a rumor transmission model with incubation that incorporates constant recruitment and has infectious

force in the latent period and infected period. Pearce [12] and Gani [13] analyzed the probability generating functions in the stochastic rumor models by means of block-matrix methodology. Nakamaru and Kawata [14] analyzed the effect of gossip on the evolution of cooperation by the agent-based simulation. They assume not only the rumor-spreader but also the rumor-starter, and other previous studies did not assume the rumor-starter.

In many real situations the parameters of the model are not constant, Dickinson and Pearce [15] studied stochastic models for more general transient processes including epidemics. Chichigina et al. [16] presented a noise source model which involves a pulse sequence at random times with memory. Chichigina et al. [17] investigated the stability of dynamical system subject to multiplicative one-side pulse noise with hidden periodicity. Exogenous variables will affect biological systems, and the statistical properties of the noise and the physical picture of the stochastic resonance phenomenon were systematically studied [18–21]. Independently of this series of studies, deterministic models for rumor transmission have been studied sporadically. For example, Castillo-Chávez and Song [22] proposed the transmission models for a fanatic behavior based on the models for sexually transmitted diseases and analyzed them qualitatively and numerically. In addition, a number of studies proposed more complex models of rumor spread based on several classical models of social networks, including homogeneous networks, Erdos-Renyi (ER) random graphs, uncorrelated scale-free networks and scale-free networks with assortative degree correlations [23–31], in particular those which were mediated by the internet, such as “virtual” communities and email networks.

Among the existed literature on rumor transmission, the incidence rate (the rate of new infections) is bilinear in the infective number y and ignorant number x . There are a variety of reasons that the standard bilinear form may require modification in rumor propagation model. This is important because the number of effective contacts between infective individuals and ignorant individuals may reduce at high infective levels due to weary infective individuals or due to the protection measures by the ignorant individuals. If the incidence rate function $g(y)$ is increasing when y is small and decreasing when y is large, it can also be used to interpret the “psychological” effects: for a very large number of the infective, the infection force may decrease as the number of infective individuals increases, because in the presence of large number of infective, the population may tend to reduce the number of contacts per unit time. Actually, incidence rates that increase more gradually than linear in y and x can also arise from saturation effects; if the number of the infective is very large, exposure to the rumor agent is virtually certain and then the incidence rate will respond more slowly than linearly to increases in y . For example, if multiple exposures to the rumor vector were necessary before infection occurred. The nonmonotone incidence rate seems to be better able to explain the practical processes of rumor propagation with psychological effect.

In this paper, we apply general model, inspired by epidemiology and informed by our knowledge of the sociology

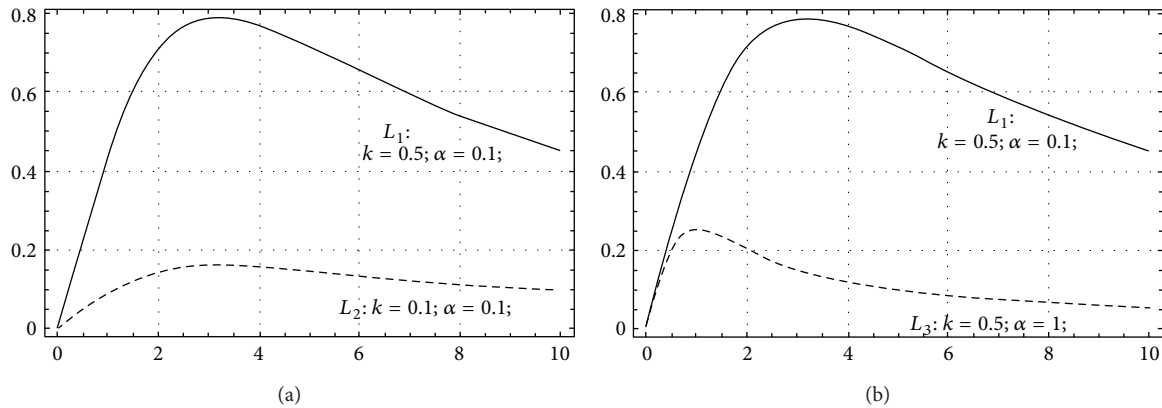
of the spread dynamics, to the diffusion of the rumor. It deals with the rumor transmission model with nonmonotone incidence rate. We provide a more detailed and realistic description of rumor spreading process with psychological effect for crazy rumors propagated in China. In the next section, we describe the psychological effect with nonmonotone incidence function. In Section 3, we present the model with nonmonotone incidence rate and derive the rumor-free equilibrium (RFE) and the rumor-endemic equilibrium (REE). In Section 4, we carry out a qualitative analysis of the model. Stability conditions for the rumor-free equilibrium and the rumor-endemic equilibrium are derived, respectively. A brief discussion and some numerical simulations are given in Section 5. Section 6 addresses a case study for demonstrating applications of the proposed model. Closing conclusions and remarks are given in the last section.

2. Psychological Effects

Rumors may be about natural disasters or threatening situations that evoke fear and insecurity; rumor transmission may be affected by various other social and behavioral factors. The symbolic achievement of psychological research on rumor was done by Allport and Postman [32] as there was some formative research in the following. Homans [33] points to a variety of theories concerning interpersonal relationships that are relevant to the sociability of people exposed to rumor. Rosnow [34] posits the idea that there are three stages to the phenomenon of rumor transmission: parturition, diffusion, and control. Pendleton [35] revisited previous research on rumor and make suggestions as to how the focus of rumor research needs to be expanded.

According to Homans, when two people communicate with each other, collective result is rewarding to both. By sharing a rumor with another person, the rumor disseminator may be attempting one-upmanship, looking for social approval for exchanging such important information and/or expecting the recipient to reciprocate at a later time. At the initial stage, most people do not have much information about emergency event, and spreaders could gain much satisfaction from telling the rumors; it was a major motive power for the spreading of news. The more people knew the rumor, the lower spreaders’ grades and levels of personal contentment are. Rumors expired because people grow weary of an issue and stop talking or thinking about it. Problem-solving rumors should dissipate when interests are drawn toward other news events or if attention turns from a problem out of boredom or frustration. Rumors should also terminate if the underlying tensions are played out.

The recent outbreak of the crazy rumors propagated for the iodized salt shortage panic in China had such psychological effects on the general public; aggressive measures and policies, such as government officials and specialists refuting, science propagandizing and popularizing, and information sharing, and have been proved to be very effective in reducing the propagative rate at the late stage of the rumor outbreak, even when the number of infective individuals were getting relatively larger. During the earlier days of the emergency event, rumor is viewed as an unverified account or

FIGURE 1: Nonmonotone incidence function $g(y)$.

explanation of events circulating in informal channels and pertaining to an emergency event in public concern. When the public are in great need for information to fill the gap between their anxiety and the lack of information from normal media channels, they are likely to be susceptible to any information; hence, the rumor is accepted to serve this purpose, and then most people are very interested in the rumor about crisis as they would not distinguish uncertain information from right and wrong as their interest is an important factor for deciding whether to spread rumor or not. Sometimes people prefer to believe the rumor is true. So the infection force rises rapidly at the early stage. As the spread of the rumor continues, more and more people in the world know the rumor, and the public may gradually be losing interest in spreading the rumor; even this minimal interest in the spreaders' message could fall further as the implications of the emergencies news start to sink in. In the presence of large number of infectives, the public may tend to reduce the number of contacts per unit time. In consequence, for a very large number of infectives, the infection force may decrease as the number of infective individuals increases. This is important because the alertness of the public is comprehensive in time and the general public will be more alert and aware of the rumor, and the transmission rate will be decreasing as the infective increases.

The nonmonotone function $g(y)$ has been used to describe the psychological effects in the process of rumor transmission (see Figure 1).

The function $g(y)$ is increasing when y is small and decreasing when y is large. The purpose of this paper is to present global qualitative for the model. We will perform a qualitative analysis of the model and study the stability of the rumor-free equilibrium and the rumor-endemic equilibrium. We will focus on

$$g(y) = \frac{ky}{(1 + \alpha y^2)}. \quad (1)$$

Consider the existence and nonexistence of limit cycles in system, which is crucial to determine the existence of

a persistence region of the rumor. In the equation (1), ky measures the probability of news rumor infection and $1/(1 + \alpha y^2)$ measures the inhibition effect from the behavioral change of the ignorant individuals when their number increases or from the weary effect of the others. It describes the psychological effect of the behavioral change of the susceptible individuals when the numbers of infective individuals change in an emergency event. Notice that when $\alpha = 0$, the nonmonotone incidence rate becomes the bilinear incidence rate. Parameter k is the probability infection of the rumor, and parameter α describes the psychological quality of the general public toward the infective. Figure 1(a) shows how the densities of incidence rate $g(y)$ change over time for different k ; specifically, the solid line L_1 represents the scenario that $k = 0.5$ and dashed line L_2 represents the scenario with $k = 0.1$. The peak value of incidence rate $g(y)$, the strongest infection force of people to spread rumors, can be used to measure the maximum rumor influence. From Figure 1(a), we can see that the higher the probability of news infection k , the larger the maximum rumor influence, and the smaller the probability of new infection k , the rumor terminates relatively quickly. Figure 1(b) illustrates the incidence rate $g(y)$ of the behavioral change of the susceptible individuals with different parameters α . Figure 1(b) shows that the stronger the effect of psychological α , the smaller the maximum rumor influence, and the influence force decline to zero in a short period of time. We can see that by increasing the psychological effect, we speed up the time at which the outbreak reaches its lower peak. With a smaller α value, which means that the people have poor psychological quality and showed great interest in rumors of unfathomable things, the influence force can have reached their maximum values subsequently, if the influence force decline to zero take a remarkably long time, rumor for crisis will swirling.

3. General Rumor Transmission Model with Psychological Effect

In the following, we will concentrate on the model based on "homogeneous mixing" with time dependent state variables.

With respect to the classical models, the proposed model is more general because investigate the role of psychological effect on rumor transmission. We divide the population into three classes: the ignorant class, the spreader class, and the stifler class. Each population at time t is denoted by $x(t)$, $y(t)$, and $z(t)$, respectively, each of which we call rumor-class. Those who belong to the ignorant class, whom we call ignorant, do not know about the rumor. Those who belong to the spreader class, whom we call spreaders, know the rumor and spread it actively. Those who belong to the stifler class, whom we call stiflers, know the rumor and do not spread it. The total population size at time t is denoted by $N(t)$, with $N(t) = x(t) + y(t) + z(t)$.

The model is described by the following system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= \Lambda - \frac{k y x}{(1 + \alpha y^2)} - \mu x, \\ \frac{dy}{dt} &= \frac{k y x}{(1 + \alpha y^2)} - \lambda y (y + z) - \mu y, \\ \frac{dz}{dt} &= \lambda y (y + z) - \mu z. \end{aligned} \quad (2)$$

We assume that no transition of rumor-class happens unless a spreader contacts someone, since the two people who are not spreaders do not talk about the rumor. That is, spreaders who are involved in the transition of rumor-class. When a spreader contacts an ignorant, the spreader transmits the rumor with psychological effect and the ignorant gets to know it. And so we assume that $g(y)x$ ignorant change their rumor-class and become spreaders during the small interval $(t, t + \Delta t)$, where $g(y)$ is the incidence rate, and $g(y) = ky/(1 + \alpha y^2)$. When two spreaders contact each other, both of them transmit the rumor at a constant frequency. After hearing it repeatedly, the spreader gets bored, gradually loses interest in it, and consequently becomes a stifler. And so we assume that $\lambda y^2 \Delta t$ spreaders become stiflers during the small interval $(t, t + \Delta t)$, where λ is a positive constant number. When a spreader contacts a stifler, the spreader transmits the rumor at a constant frequency, and after hearing it, the stifler tries to remove it, because the stifler shows no interest in it or denies it. As a result, the spreader becomes a stifler. And so we assume that $\lambda y z \Delta t$ spreaders become stiflers during the small interval $(t, t + \Delta t)$.

In the meantime; we assume that the rumor is “constant” that is, the same remark is transmitted at all times. To be more practical, we assume that the transmission of a constant rumor is with a variable population size; that is, the rumor is spreading in population with constant immigration and emigration. Let Λ be the immigration, such as the number of new internet accounts created (“births”) and μ the emigration rate such as the numbers of Internet accounts that are canceled or become void (“deaths”). Thus, the maximum value that $1/\mu$ can take is the average lifespan of the rumor within a generation of researchers in the relevant community. We assume that Λ, μ are positive constants; that is, the newcomers

are all ignorant and the emigration is independent of rumor-class. k is the proportionality constant and α is the parameter that measures the psychological or inhibitory effect.

Summing the equations in (2), we obtain that the total population N satisfy the differential equation

$$\frac{dN}{dt} = \Lambda - \mu N, \quad (3)$$

whose solution is given by

$$N(t) = N_0 e^{-\mu t} + \frac{(1 - e^{-\mu t}) \Lambda}{\mu}. \quad (4)$$

Thus, we assume that the initial value is $N_0 = x_0 + y_0 + z_0 = \Lambda/\mu$ in order to have population of constant size (i.e., $x(t) + y(t) + z(t) = N = \Lambda/\mu$). Obviously, the state variables $(x(t), y(t), z(t))$ remain in the sociologically meaningful set $\Omega = \{(x, y, z) \in R_+^3 \mid 0 \leq x + y + z \leq \Lambda/\mu\}$ for $(x(0), y(0), z(0)) \in R_+^3$, which is a positively invariant region.

We first consider the existence of equilibria of system (2). For any values of parameters, model (2) always has a rumor-free equilibrium $E_0 = (\Lambda/\mu, 0, 0)$. According to reference of Herbert and Hethcote [36], the rumor-free equilibrium (RFE) is calculated as if diseases were never introduced into the system. From this, we can see that there should be no people in either the infected classes or the recovered class.

To find the positive equilibria, set

$$\begin{aligned} \Lambda - \frac{k y x}{(1 + \alpha y^2)} - \mu x &= 0, \\ \frac{k y x}{(1 + \alpha y^2)} - \lambda y (y + z) - \mu y &= 0, \\ \lambda y (y + z) - \mu z &= 0. \end{aligned} \quad (5)$$

This yields

$$\alpha \mu^3 y^2 + k (\lambda \Lambda + \mu^2) y - k \Lambda \mu + \mu^3 = 0. \quad (6)$$

Define the basic reproduction number as follows:

$$R_0 = \frac{k \Lambda}{\mu^2}. \quad (7)$$

The basic reproductive number R_0 is defined as the average number of secondary infections produced when one infected individual is introduced into host population where everyone is susceptible [37]. Thus the basic reproduction number R_0 is often considered as the threshold quantity that determines when an infection can invade and persist in a new host population.

From (6) we can see the following:

- (1) if $R_0 \leq 1$, then there is no positive equilibrium;
- (2) if $R_0 > 1$, then there is a unique positive equilibrium (REE) $E^* = (x^*, y^*, z^*)$, which is called the rumor-endemic equilibrium and given by

$$x^* = \frac{[\Lambda(-1 + \lambda y^*/\mu) + \mu y^*]}{(\lambda y^* - \mu)}, \quad (8)$$

$$y^* = \frac{[-k(\lambda\Lambda + \mu^2) + \sqrt{\Delta}]}{(2\alpha\mu^3)}, \quad (9)$$

$$z^* = \frac{\lambda y^{*2}}{(\mu - \lambda y^*)}, \quad (10)$$

where $\Delta \equiv k^2(\lambda\Lambda + \mu^2)^2 - 4\alpha\mu^3(\mu^3 - k\Lambda\mu) = 4\alpha\mu^6(R_0 - 1) + k^2(\lambda\Lambda + \mu^2)^2$.

In the next section, we will study the property of these equilibria and perform a global qualitative analysis of model (2).

4. Mathematical Analysis

It is clear that the limit set of system (2) is on the plane $x + y + z = \Lambda/\mu$. Thus, we focus on the reduced system

$$\begin{aligned} \frac{dx}{dt} &= \Lambda - \frac{k y x}{(1 + \alpha y^2)} - \mu x \equiv P(x, y), \\ \frac{dy}{dt} &= \frac{k y x}{(1 + \alpha y^2)} - \lambda y \left(\frac{\Lambda}{\mu} - x \right) - \mu y \equiv Q(x, y). \end{aligned} \quad (11)$$

We have the following result regarding the nonexistence of periodic orbits in system (11), which implies the nonexistence of periodic orbits of system (2).

Theorem 1. System (11) does not have nontrivial periodic orbits.

Proof. Consider system (11) for $x > 0$ and $y > 0$. Take a Dulac function:

$$D(x, y) = \frac{(1 + \alpha y^2)}{ky}. \quad (12)$$

□

We have

$$\begin{aligned} &\frac{\partial(DP)}{\partial x} + \frac{\partial(DQ)}{\partial y} \\ &= -\frac{[\mu^2 + \mu k y + 3\alpha\mu^2 y^2 + 2\alpha\beta(\Lambda - \mu x)y^2]}{(\mu k y)}. \end{aligned} \quad (13)$$

Because $x < \Lambda/\mu$ and $\partial(DP)/\partial x + \partial(DQ)/\partial y < 0$, the conclusion follows.

In order to study the properties of the rumor-free equilibrium E_0 and the rumor-endemic equilibrium E^* , we rescale (11) by $u = \Lambda/\mu - x$, $v = y$, $\tau = \mu t$.

Then we obtain

$$\begin{aligned} \frac{du}{d\tau} &= \left[\frac{(kv/\mu)}{(1 + \alpha v^2)} \right] \left(\frac{\Lambda}{\mu} - u \right) - u, \\ \frac{dv}{d\tau} &= \left[\frac{(kv/\mu)}{(1 + \alpha v^2)} \right] \left(\frac{\Lambda}{\mu} - u \right) - \frac{\lambda uv}{\mu} - v. \end{aligned} \quad (14)$$

Note that the trivial equilibrium $(0, 0)$ of system (14) is the rumor-free equilibrium E_0 of model (2) and the unique positive equilibrium (u^*, v^*) of system (14) is the endemic equilibrium E^* of model (2) if and only if $R_0 > 1$, where $u^* = v^*/(1 - \lambda v^*/\mu)$, $v^* = [-k(\lambda\Lambda + \mu^2) + \sqrt{\Delta}]/(2\alpha\mu^3)$, $\Delta' \equiv 4\alpha\mu^6(R_0 - 1) + k^2(\lambda\Lambda + \mu^2)^2$.

We first determine the stability and topological type of $(\Lambda/\mu, 0)$. The Jacobian matrix of system (14) at $(0, 0)$ is

$$J_0 = \begin{pmatrix} -1 & -\frac{k\Lambda}{\mu^2} \\ 0 & -1 + \frac{k\Lambda}{\mu^2} \end{pmatrix}. \quad (15)$$

If $R_0 = k\Lambda/\mu^2 = 1$, then there exists a small neighborhood N_0 of $(0, 0)$ such that the dynamics of system (14) are equivalent to those

$$\begin{aligned} \frac{du}{d\tau} &= v - \frac{kuv}{\mu} - u + o((x, y)^3), \\ \frac{dv}{d\tau} &= \frac{-(k + \lambda)uv}{\mu} + o((x, y)^3). \end{aligned} \quad (16)$$

By Theorem 2.11.1 of Perko [38, page 150], we know that $(0, 0)$ is a saddle node. Hence, we obtain the following result.

Theorem 2. The rumor-free equilibrium $(0, 0)$ of system (14) is as following:

- (i) a stable hyperbolic node if $R_0 < 1$;
- (ii) a saddle node if $R_0 = 1$;
- (iii) a hyperbolic saddle if $R_0 > 1$.

When $R_0 > 1$, we discuss the stability and topological type of the rumor-endemic equilibrium (u^*, v^*) . The Jacobian matrix of (4.4) at (u^*, v^*) is

$$J^* = \begin{pmatrix} -1 - \frac{(kv^*/\mu)}{(1 + \alpha v^{*2})} & H \\ -\frac{(kv^*/\mu)}{(1 + \alpha v^{*2})} - \frac{\lambda v^*}{\mu} & -1 - \frac{\lambda v^*}{\mu} + H \end{pmatrix}, \quad (17)$$

where $H = -[2k\alpha v^{*2}(\Lambda/\mu - u)/\mu]/(1 + \alpha v^{*2})^2 + [(\Lambda/\mu - u)k/\mu]/(1 + \alpha v^{*2})$.

Note that $-1 - \lambda u^*/\mu + [(\Lambda/\mu - u)k/\mu]/(1 + \alpha v^{*2}) = 0$, and we have that

$$\begin{aligned} \text{Det}(J^*) &= \frac{[1 + kv^*(1 + \alpha v^{*2})/\mu][(\Lambda/\mu - u)k/\mu]}{(1 + \alpha v^{*2})} \\ &\quad - H\left(-1 - \frac{\lambda v^*}{\mu}\right) \\ &= \frac{kv^*(\Lambda - \mu u^*)M}{[u^*(1 + \alpha v^{*2})^2 \mu^4]}, \end{aligned} \quad (18)$$

where $M = k\Lambda + (\alpha v^{*2} - 1)\mu^2$. Because $x < \Lambda/\mu$ and $(\Lambda - \mu u^*) > 0$, the sign of $\text{Det}(J^*)$ is determined by M .

Substituting

$$v^* = \frac{[-k(\lambda\Lambda + \mu^2) + \sqrt{\Delta'}]}{(2\alpha\mu^3)}, \quad (19)$$

$$\Delta' = 4\alpha\mu^6(R_0 - 1) + k^2(\lambda\Lambda + \mu^2)^2$$

into M and using a straightforward calculation, we have

$$\begin{aligned} M &= k\Lambda + \mu^2 \left[\frac{(\sqrt{\Delta'} - k(\lambda\Lambda + \mu^2))^2}{(4\alpha\mu^6)} - 1 \right] \\ &= \frac{2\sqrt{\Delta'}[\sqrt{\Delta'} - k(\lambda\Lambda + \mu^2)]}{(4\alpha\mu^6)}. \end{aligned} \quad (20)$$

Since

$$\Delta' \equiv 4\alpha\mu^6(R_0 - 1) + k^2(\lambda\Lambda + \mu^2)^2 > k^2(\lambda\Lambda + \mu^2)^2, \quad (21)$$

it follows that $M > 0$. Hence, $\text{Det}(J^*) > 0$, and (u^*, v^*) is a node or a focus or a center. Furthermore, we have the following result on the stability of (u^*, v^*) .

Theorem 3. Suppose that $R_0 > 1$; then, there is a unique endemic equilibrium (u^*, v^*) of model (14), which is a stable node.

Proof. We know that the stability of (u^*, v^*) is determined by $\text{Tr}(J^*)$. We have

$$\begin{aligned} \text{Tr}(J^*) &= -1 - \frac{kv^*}{[\mu(1 + \alpha v^{*2})]} \\ &\quad - \frac{[2k\alpha v^{*2}(\Lambda/\mu - u)]}{[\mu(1 + \alpha v^{*2})^2]} < 0. \end{aligned} \quad (22)$$

□

Summarizing Theorems 1–3, we have the following results on the dynamics of the original model (2).

Theorem 4. Let R_0 be defined by (7).

- (i) If $R_0 < 1$, then model (2) has a unique rumor-free equilibrium $E_0 = (\Lambda/\mu, 0, 0)$, which is a global attractor in the first octant.

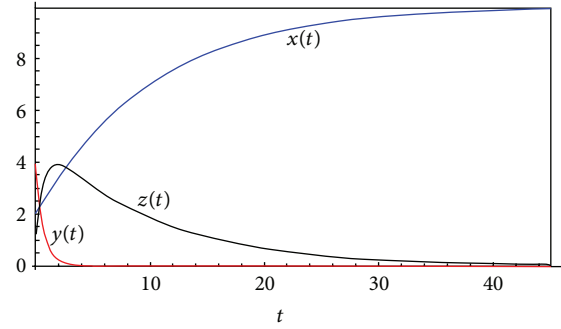


FIGURE 2: $R_0 < 1$; rumor “dies out.”

- (ii) If $R_0 = 1$, then model (2) has a unique rumor-free equilibrium $E_0 = (\Lambda/\mu, 0, 0)$, which attracts all orbits in the interior of the first octant.

- (iii) If $R_0 > 1$, then model (2) has two equilibria, a rumor-free equilibrium $E_0 = (\Lambda/\mu, 0, 0)$ and a rumor-endemic equilibrium $E^* = (x^*, y^*, z^*)$. The endemic equilibrium E^* is a global attractor in the interior of the first octant.

Thus, our model used R_0 to determine the average number of secondary transmissions of the rumor. If a person, on average, will tell more than one other person before they stop transmitting the rumor, then $R_0 > 1$ and the rumor-free equilibrium will be unstable. If this should occur, then there will be continuous presence of the rumor in the population.

5. Discussions and Simulations

5.1. The Relationship between Rumor Transmission and the Basic Reproduction Number R_0 . We have carried out a global qualitative analysis of the model with the nonmonotone and nonlinear incidence rate and studied the existence and stability of the rumor-free and rumor-endemic equilibria. The running results show that rumor transmission states of equilibrium are closely related to the basic reproduction number R_0 , but nothing on the arrived time for the states of equilibrium. Thus the basic reproduction number R_0 is often considered as the threshold quantity that determines when an infection can invade and persist in new host population; R_0 determine the average number of secondary transmissions of the rumor. In terms of the basic reproduction number $R_0 = k\Lambda/\mu^2$, $R_0 < 1$, the rumor-free equilibrium is globally attractive. When $R_0 > 1$, the rumor-equilibrium exists and is globally stable. Numerical simulations carried out for system (2) show that the rumor “dies out” when the basic reproduction number $R_0 < 1$ (the threshold) and the rumor persists at an “endemic” level when $R_0 > 1$. Figures show the general trends of the three kinds of agents in the rumor spreading model. From the simulation with $R_0 < 1$ (Figure 2), we can find that there is a sharp increase in the number of stiflers with further spreading of the rumor, and the number of stiflers reaches a peak and thereafter declines. In this whole process, the number of spreaders always reduces while the number of ignorant always increases until they reach

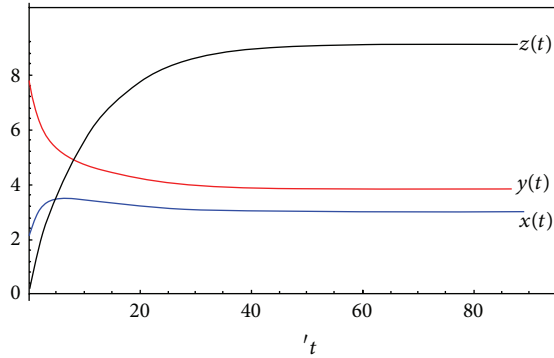


FIGURE 3: $R_0 > 1$; the rumor persists at an “endemic” level.

the balance, respectively. Finally, the number of spreaders is zero and this leads to the termination of rumor spreading. From the simulation with $R_0 > 1$ (Figure 3), we can find that the numbers of ignorant reaches a peak at first and thereafter declines until they reach the balance. The number of spreaders always reduce while the number of stifiers always increases until they reach the balance, respectively. Finally, the rumor persists at an “endemic” level.

Consider that $R_0 < 1$ is the condition necessary to insure the that the rumor-free equilibrium will be stable this is alarming. The only parameters we seem to be able to change so that the R_0 will be less than one are k , Λ , and μ . These indicate that when the probability infection constant k and/or the recruitment rate Λ is small enough and removal rate μ is large enough such that $R_0 < 1$, which means that the population is cycling very rapidly, then the rumor dies out. We can conclude here that to prevent the outbreak, we need either k , Λ and μ to be unrealistic, or a few people leave from the x to the y class. Note that the parameters for the unbelievable rumor case seem to be somewhat effective in controlling the initial outbreak. The aggressive control measures and policies, such as government officials and specialists refuting, science propagandizing and popularizing, and information sharing, can help in reducing the infection rate and increasing the removal rate and in the eventual eradication of rumor.

5.2. The Relationship between Rumor Transmission and the Psychological Effect. Recall that the parameter α ($\alpha > 0$) describes the psychological quality of the general public toward the infectives. Though the basic reproduction number R_0 does not depend on α explicitly, numerical simulations indicate that when the rumor is endemic, the steady state value y^* of the infectives decreases as α ($\alpha > 0$) increases (see Figure 4). Compare the results for $\alpha > 0$ with those with $\alpha = 0$, and in the first case, if $\alpha > 0$, the steady state value y^* will decline and it will remain steady. This shows that the stronger the psychological effect. On the other hand, if $\alpha = 0$, by analyzing the steady state value y^* that increases rapidly as the time t increased, after reaching the peak, and then decrease gradually, it will ultimately turn to a stable state. Based on the above, one can draw a conclusion that the parameter α describes the psychological quality of the general public with respect to the infectives; it would be a significant risk factor

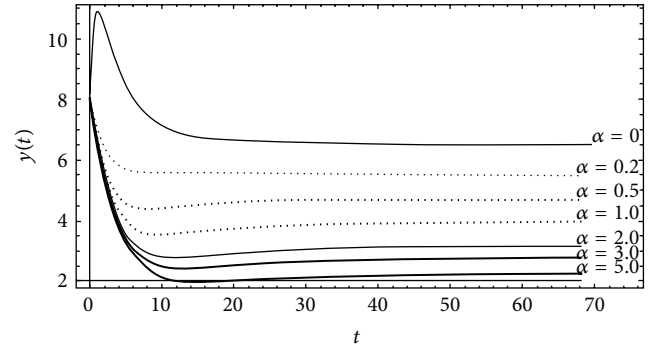


FIGURE 4: $\beta_0 = 0.9$, $\lambda_0 = 0.1$, $\alpha_0 = 0.5$, $\gamma_0 = 0.03$, $\theta = 0.6$, $\mu = 0.1$, and $R_0 = 3.46$.

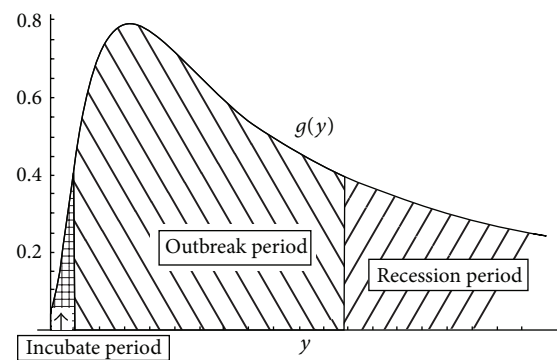


FIGURE 5: The whole process of the infection force of rumor spreading with psychological effect.

in the process in the rumor transmission. We believe that the psychological quality to be absolutely essential to minimize the bad influence of rumors.

With the increased psychological quality, we can also see that the population of spreaders seem to approach zero to a closer degree than population with the poor mental quality levels. From the steady state expression (9), we can see that y^* approaches zero as α tends to infinity. Larger α , the better psychological quality one in emergency event; the people with high psychological quality show good resistance to rumor spreading, while the ones with low psychological quality show fragility to rumor spreading.

6. A Case Study

The 9-magnitude earthquake in Fukushima incurring nuclear leakage accidents in 2011, which made tens of thousands people lose their homes, seriously impacted people's psychology and triggered a chain of disorders. The whole process of the infection force of rumor spreading is divided into three stages, that is, the incubate, outbreak, and recession periods (see Figure 5). The following figure shows the evolution process of the infection force of the rumor spreading with iodized salt shortage panic.

At first incubate period, this stage was soon after the crisis event occurred. Simultaneously the majority of public

is in shocks which are brought by the crisis event; as the public are in great need for information to fill the gap between their anxiety and the lack of information, they are likely to be susceptible to any information, and then most people are very interested in the rumor. In this period, the infection force of rumor spreading developed fiercely. As event evolved over time, more people knew the rumor, and their curiosity about the rumor around them is aroused over time. Two days after the earthquake in Fukushima, some rumors said that taking materials containing iodine could help ward off nuclear radiation, which led to the public rushing for everything containing iodine, such as Chinese snapping up iodized salt, Americans rushing for iodine pills, Russians hoarding iodine, and Korean residents rushing for seaweed. The most troublesome thing is harmful rumors and the psychological effect on consumers as a result of radiation concerns.

At outbreak period, the public eager to know the latest developments of the crisis, everyone talks about the crisis, and the psychological effect grows to a maximum and then began to subside rapidly. From the afternoon of March 16th, crazy rumors propagated in the coastline cities of China. In a sign of increasing public worries about the risks, people across much of China have been buying large amounts of iodized salt, emptying markets of the usually cheap and plentiful products. On March 17th, this irrational behavior of storing up iodized salt swept the whole country. Although some supermarkets took measures to restrict salt supply to 350 g for each person, the salt shelves in supermarkets were still emptied by worried residents, and salt prices even reached five times more than their normal price [39]. When iodized salt ran out, they bought iodine-free sea salt, then, even more bizarrely, soy sauce.

At recession period, the spread scope of the true crisis information gets more and more widely, and government agencies as well as authority media begin to join the true information spreading ranks, which strengthens the crisis information identify capacity of the public; then, after receiving the true information, the public will not be confused by the rumors. The public show steady decline needs for the crisis information and gradually lose interest for rumor. After March 18th, the Chinese government have increased awareness of rumor prevention efforts and spread scientific knowledge that eating iodized salt can not prevent people from radiation. The scare-buying emergency is under control when the rumor is debunked.

Actually, the collective behavior caused by Japan's nuclear crisis, it was described as psychological effect with nonmonotone and nonlinear incidence in rumor spreading model, and that is the main innovation in this paper.

7. Conclusions

In this paper, we proposed a nonmonotone and nonlinear incidence rate of the form $g(y) = ky/(1 + \alpha y^2)$, and it can be used to interpret the psychological effect with rumor transmission in emergency: the number of effective contacts between infective individuals and ignorant individuals decreases at high infective levels due to the quarantine of

infective individuals or due to the protection measures by the ignorant individuals. The people pay more and more attention to crisis information form authenticity and decrease the interest for rumor. Two valuable conclusion drawn from our analysis.

- (1) To decrease the probability of new infection value of the rumor spreading, authorities will intensify propaganda and direct public views by controlling rumor spreading and take some actions to relax public vigilance, reducing the focus of attention of rumors, and it is the important task for the authorities to avoid the negative effects for rumor spreading in emergency management. Generally, it can also enhance the transparency of the information in an emergency event, and then the rumor transmission can be controlled by implementing those strategies. Sometimes information disclosure can achieve management aims for rumor transmission more effectively and at far lower cost than traditional regulation.
- (2) Parameters α for psychological quality have no effect on the threshold R_0 , but it effect on the final size of a rumor infective. To reduce rumors impact on society, developing excellent psychological quality is essential in emergency management. The recommendations are to provide effective immunity against rumor through the timely publicity and popularize scientific knowledge. Excellent psychological quality provides the public with strong mental power and the ability to control rumor in emergency event.

Since some rumors spread in a certain group of people, we then can assume that the transmission coefficient is a function of parameters for special populations and time, which may take the nonautonomous system instead of the autonomous system; we leave this for future work.

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