Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 276972, 6 pages
http://dx.doi.org/10.1155/2013/276972

Research Article

Periodic Oscillation Analysis for a Coupled FHN Network Model with Delays

Yuanhua Lin

Department of Mathematics, Hechi University, Yizhou, Guangxi 546300, China

Correspondence should be addressed to Yuanhua Lin; lyh4773@163.com

Received 16 January 2013; Accepted 23 September 2013

Academic Editor: Chunhua Feng

The existence of periodic oscillation for a coupled FHN neural system with delays is investigated. Some criteria to determine the oscillations are given. Simple and practical criteria for selecting the parameters in this network are provided. Some examples are also presented to illustrate the result.

1. Introduction

Recently, several researchers have studied the dynamics of coupled FHN neural systems [1–5]. Wang et al. have investigated the following model [6]:

$$\begin{aligned} v_1'(t) &= -v_1^3(t) + av_1(t) - w_1(t) + c_1 \tanh(v_2(t-\tau)), \\ w_1'(t) &= v_1(t) - b_1 w_1(t), \\ v_2'(t) &= -v_2^3(t) + av_2(t) - w_2(t) + c_2 \tanh(v_1(t-\tau)), \\ w_2'(t) &= v_2(t) - b_2 w_2(t). \end{aligned} \tag{1}$$

The effects of time delay on bifurcation and synchronization in the two synaptically coupled FHN neurons have been investigated. The authors found that time delay can control the occurrence of bifurcation in the coupled FHN neural model and synchronization is sometimes related to bifurcation transition. Fan and Hong introduced second time delay in model (1) as follows [7]:

$$\begin{aligned} x_1'(t) &= -x_1^3(t) + ax_1(t) - x_2(t) + c_1 \tanh(x_3(t-\tau_1)), \\ x_2'(t) &= x_1(t) - b_1 x_2(t), \\ x_3'(t) &= -x_3^3(t) + ax_3(t) - x_4(t) + c_2 \tanh(x_1(t-\tau_2)), \\ x_4'(t) &= x_3(t) - b_2 x_4(t). \end{aligned} \tag{2}$$

Let $\tau = \tau_1 + \tau_2$ be a parameter. The authors have shown that there is a critical value of the parameter; the steady state of model (2) is stable when the parameter is less than the critical value and unstable when the parameter is greater than the critical value. Thus, the zero equilibrium loses its stability when the parameter passes through the critical value, and a Hopf bifurcation occurs and oscillations induced by the Hopf bifurcation appeared. Zhen and Xu generated models (1) and (2) to a three coupled FHN neurons network with time delay as follows [8]:

$$\begin{aligned} u_1' &= -\frac{1}{3}u_1^3 + cu_1^2 + du_1 - u_2 + \alpha u_1^2 \\ &\quad + \beta [f(u_3(t-\tau)) + f(u_5(t-\tau))], \\ u_2' &= \varepsilon(u_1 - bu_2), \\ u_3' &= -\frac{1}{3}u_3^3 + cu_3^2 + du_3 - u_4 + \alpha u_3^2 \\ &\quad + \beta [f(u_1(t-\tau)) + f(u_5(t-\tau))], \\ u_4' &= \varepsilon(u_3 - bu_4), \end{aligned}$$

$$u_5' = -\frac{1}{3}u_5^3 + cu_5^2 + du_5 - u_6 + \alpha u_5^2 + \beta\left[f\left(u_1\left(t-\tau\right)\right) + f\left(u_3\left(t-\tau\right)\right)\right],$$
$$u_6' = \varepsilon\left(u_5 - bu_6\right), \tag{3}$$

where α, β represent the synaptic strength of self-connection and neighborhood interaction, respectively, and $f(x)$ is a sufficiently smooth sigmoid amplification function such as $\tanh(x)$ and $\arctan(x)$. The method of Lyapunov functional is used to obtain the synchronization conditions of the neural system. Noting that, for each neuron of model (3), the synaptic strength of self-connection and neighborhoodinteraction are the same under the same restrictive condition, the dynamics of (3) are completely characterized by the following system:

$$\begin{aligned} u_1' &= -\frac{1}{3}u_1^3 + (c+\alpha)u_1^2 + du_1 - u_2 + 2\beta f\left(u_1\left(t-\tau\right)\right), \\ u_2' &= \varepsilon\left(u_1 - bu_2\right), \end{aligned} \tag{4}$$

where $[u_1, u_2]^T$ is a completely synchronous solution of system (4). The Bautin bifurcation of synchronous solution for this neural system (4) in which α, β are regarded as the bifurcating parameters is investigated. However, generally speaking, the synaptic strength of self-connection, neighborhoodinteraction for each neuron, and the time delays are different. Therefore, in this paper, we will discuss the following model:

$$\begin{aligned} u_1' &= -\frac{1}{3}u_1^3 + c_1u_1^2 + d_1u_1 - u_2 + \alpha_1u_1^2 + \beta_1\left[f\left(u_3\left(t-\tau_3\right)\right) + f\left(u_5\left(t-\tau_5\right)\right)\right], \\ u_2' &= \varepsilon_1\left(u_1 - b_1u_2\right), \\ u_3' &= -\frac{1}{3}u_3^3 + c_2u_3^2 + d_2u_3 - u_4 + \alpha_2u_3^2 + \beta_2\left[f\left(u_1\left(t-\tau_1\right)\right) + f\left(u_5\left(t-\tau_5\right)\right)\right], \\ u_4' &= \varepsilon_2\left(u_3 - b_2u_4\right), \\ u_5' &= -\frac{1}{3}u_5^3 + c_3u_5^2 + d_3u_5 - u_6 + \alpha_3u_5^2 + \beta_3\left[f\left(u_1\left(t-\tau_1\right)\right) + f\left(u_3\left(t-\tau_3\right)\right)\right], \\ u_6' &= \varepsilon_3\left(u_5 - b_3u_6\right), \end{aligned} \tag{5}$$

where $b_i, c_i, d_i, \alpha_i, \beta_i, \varepsilon_i$ $(i = 1, 2, 3)$ are constants. $\tau_j > 0$ $(j = 1, 3, 5)$ represent the time delays in signal transmission. System (5) can be rewritten as follows:

$$\begin{aligned} u_1' &= \left[d_1 + (\alpha_1 + c_1)u_1 - \frac{1}{3}u_1^2\right]u_1 - u_2 + \beta_1\left[f\left(u_3\left(t-\tau_3\right)\right) + f\left(u_5\left(t-\tau_5\right)\right)\right], \\ u_2' &= \varepsilon_1u_1 - \varepsilon_1b_1u_2, \\ u_3' &= \left[d_2 + (\alpha_2 + c_2)u_3 - \frac{1}{3}u_3^2\right]u_3 - u_4 + \beta_2\left[f\left(u_1\left(t-\tau_1\right)\right) + f\left(u_5\left(t-\tau_5\right)\right)\right], \\ u_4' &= \varepsilon_2u_3 - \varepsilon_2b_2u_4, \\ u_5' &= \left[d_3 + (\alpha_3 + c_3)u_5 - \frac{1}{3}u_5^2\right]u_5 - u_6 + \beta_3\left[f\left(u_1\left(t-\tau_1\right)\right) + f\left(u_3\left(t-\tau_3\right)\right)\right], \\ u_6' &= \varepsilon_3u_5 - \varepsilon_3b_3u_6. \end{aligned} \tag{6}$$

It is known that if all solutions of system (6) are bounded and there exists a unique unstable equilibrium point of system (6), then this particular instability will force system (6) to generate a limit cycle, namely, a periodic oscillation [9]. We will provide some restrictive conditions which are easy to check to ensure the existence of periodic oscillation. It was pointed out that bifurcating method to determine the periodic solution of system (6) is very difficult.

In the following, we first assume that $f(u_i(t - \tau_i))$ $(i = 1, 3, 5)$ are continuous bounded monotone increasing functions, satisfying

$$\lim_{u_i \to 0}\frac{f\left(u_i\left(t\right)\right)}{u_i\left(t\right)} = \gamma_i\,(>0), \quad i = 1, 3, 5; \;\; f(0) = 0. \tag{7}$$

For example, activation functions $\tanh(u_i(t))$, $\arctan(u_i(t))$, and $(1/2)(|u_i(t) + 1| - |u_i(t) - 1|)$ satisfy condition (7). From assumption (7), the linearization of system (6) about the zero point leads to the following:

$$\begin{aligned} u_1' &= d_1u_1 - u_2 + \beta_1\left[\gamma_3u_3\left(t-\tau_3\right) + \gamma_5u_5\left(t-\tau_5\right)\right], \\ u_2' &= \varepsilon_1u_1 - \varepsilon_1b_1u_2, \\ u_3' &= d_2u_3 - u_4 + \beta_2\left[\gamma_1u_1\left(t-\tau_1\right) + \gamma_5u_5\left(t-\tau_5\right)\right], \\ u_4' &= \varepsilon_2u_3 - \varepsilon_2b_2u_4, \\ u_5' &= d_3u_5 - u_6 + \beta_3\left[\gamma_1u_1\left(t-\tau_1\right) + \gamma_3u_3\left(t-\tau_3\right)\right], \\ u_6' &= \varepsilon_3u_5 - \varepsilon_3b_3u_6. \end{aligned} \tag{8}$$

The matrix form of system (8) is as follows:

$$U'(t) = AU(t) + BU(t-\tau), \tag{9}$$

where $U(t) = (u_1(t), u_2(t), \ldots, u_6(t))^T$, $U(t-\tau) = (u_1(t-\tau_1), 0, u_3(t-\tau_3), 0, u_5(t-\tau_5), 0)^T$,

$$A = \begin{pmatrix} d_1 & -1 & 0 & 0 & 0 & 0 \\ \varepsilon_1 & -\varepsilon_1 b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_2 & -1 & 0 & 0 \\ 0 & 0 & \varepsilon_2 & -\varepsilon_2 b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_3 & -1 \\ 0 & 0 & 0 & 0 & \varepsilon_3 & -\varepsilon_3 b_3 \end{pmatrix},$$
$$B = \begin{pmatrix} 0 & 0 & \beta_1\gamma_3 & 0 & \beta_1\gamma_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2\gamma_1 & 0 & 0 & 0 & \beta_2\gamma_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_3\gamma_1 & 0 & \beta_3\gamma_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (10)$$

2. Preliminaries

Lemma 1. *Suppose that* $b_i > 0$, $0 < \varepsilon_i \ll 1$, $d_i < 0$, $(\alpha_i + c_i)^2 + (4/3)d_i < 0$ $(i = 1, 2, 3)$; *then each solution of system* (6) *is bounded.*

Proof. Note that the activation functions are bounded continuous nonlinear functions. Therefore, there exist $N_j > 0$ such that $|f(u_j(t - \tau_j))| \le N_j$ $(j = 1, 3, 5)$. Since $d_i < 0$, $(\alpha_i + c_i)^2 + (4/3)d_i < 0$ $(i = 1, 2, 3)$, this implies that there exist constants $k_i > 0$ such that for any values u_i we have $d_i + (\alpha_i + c_i)u_i - (1/3)u_i^2 \le -k_i < 0$ $(i = 1, 2, 3)$. From (6) we get

$$\begin{aligned} \frac{d|u_1(t)|}{dt} &\le -k_1|u_1| + |u_2| + |\beta_1|(N_3 + N_5), \\ \frac{d|u_2(t)|}{dt} &\le \varepsilon_1|u_1| - \varepsilon_1 b_1|u_2|, \\ \frac{d|u_3(t)|}{dt} &\le -k_2|u_3| + |u_4| + |\beta_2|(N_1 + N_5), \\ \frac{d|u_4(t)|}{dt} &\le \varepsilon_2|u_3| - \varepsilon_2 b_2|u_4|, \\ \frac{d|u_5(t)|}{dt} &\le -k_3|u_5| + |u_6| + |\beta_3|(N_1 + N_3), \\ \frac{d|u_6(t)|}{dt} &\le \varepsilon_3|u_5| - \varepsilon_3 b_3|u_6|. \end{aligned} \quad (11)$$

Noting that system (11) is the first-order linear system of equations with constant coefficients, the eigenvalues of system (11) are $\lambda_{i1,i2} = (-(k_i + \varepsilon_i b_i) \pm \sqrt{(k_i + \varepsilon_i b_i)^2 - 4\varepsilon_i(k_i b_i + 1)})/2$ $(i = 1, 2, 3)$. Since $k_i > 0$, $\varepsilon_i > 0$, $b_i > 0$ $(i = 1, 2, 3)$, $\lambda_{i1,i2} < 0$ if $(k_i + \varepsilon_i b_i)^2 - 4\varepsilon_i(k_i b_i + 1) > 0$ or $\lambda_{i1,i2}$ are complex numbers with $\operatorname{Re}\lambda_{i1,i2} < 0$ if $(k_i + \varepsilon_i b_i)^2 - 4\varepsilon_i(k_i b_i + 1) < 0$ $(i = 1, 2, 3)$. This implies that all solutions of system (11), as well as the system, (6) are bounded according to the theory of the first-order linear system of equations with constant coefficients.

According to [10], there is the same oscillatory behavior for systems (8) and (6). So, in order to investigate the periodic oscillatory behavior of system (6), we only need to deal with system (8). □

Lemma 2. *Suppose that matrix* C $(= A + B)$ *is a nonsingular matrix. Then, system* (9) *has a unique equilibrium point.*

Proof. An equilibrium point $u^* = [u_1^*, u_2^*, \ldots, u_6^*]^T$ is the solution of the following algebraic equation:

$$AU^* + BU^* = (A + B)U^* = 0. \quad (12)$$

Assume that U^* and V^* are two equilibrium points of system (9); then we have

$$(A + B)(U^* - V^*) = C(U^* - V^*) = 0. \quad (13)$$

Since C is a nonsingular matrix, implying that $U^* - V^* = 0$ and $U^* = V^*$ system (9) has a unique equilibrium point. Obviously, this equilibrium point is exactly the zero point. □

3. Periodic Oscillation

Theorem 3. *Suppose that* $b_i > 0$, $0 < \varepsilon_i \ll 1$, $d_i < 0$, $(\alpha_i + c_i)^2 + (4/3)d_i < 0$ $(i = 1, 2, 3)$, *and* C *is a nonsingular matrix. Let* $\tau_* = \min\{\tau_1, \tau_3, \tau_5\}$, $\tau^* = \max\{\tau_1, \tau_3, \tau_5\}$, $\rho_1, \rho_2, \ldots, \rho_6$, *and* $\omega_1, \omega_2, \ldots, \omega_6$ *denote the eigenvalues of matrices A and B, respectively. Assume that there is at least one* $\rho_i > 0$, $i \in (1, 2, \ldots, 6)$, *and the following inequalities hold:*

$$|\omega_i|\tau_* e^{-\rho_i\tau_*} > 1, \qquad |\omega_i|\tau^* e^{-\rho_i\tau^*} > 1. \quad (14)$$

Then, the trivial solution of system (8) *is unstable, implying that there is a periodic oscillatory solution of system* (6).

Proof. From the assumptions, we know that system (8) has a unique equilibrium point and all solutions are bounded. We will prove that the unique equilibrium point is unstable. We first discuss the case that $\tau_1 = \tau_3 = \tau_5 = \tau_*$ in system (8). The characteristic equation of system (8) is as follows:

$$\det\left(\lambda I - A - Be^{-\lambda\tau_*}\right) = 0. \quad (15)$$

Equation (15) is equal to

$$\prod_{k=1}^{6}\left(\lambda-\rho_k-\omega_k e^{-\lambda\tau_*}\right)=0. \tag{16}$$

Therefore, we are led to an investigation of the nature of the roots for the following equations:

$$\lambda-\rho_k-\omega_k e^{-\lambda\tau_*}=0,\quad k=1,2,\ldots,6. \tag{17}$$

For some $\rho_i>0$, consider equation

$$\lambda-\rho_i-\omega_i e^{-\lambda\tau_*}=0. \tag{18}$$

If $\lambda<0$ is a solution of (18), then $|\lambda|=-\lambda$; from (18) we have

$$|\lambda|\geq|\omega_i|\,e^{|\lambda|\tau_*}-\rho_i. \tag{19}$$

Using the formula $ex\leq e^x(x>0)$, leads to the fact that

$$\begin{aligned}1&\geq\frac{|\omega_i|\,e^{|\lambda|\tau_*}}{|\lambda|+\rho_i}=\frac{|\omega_i|\,\tau_* e^{-\rho_i\tau_*}\cdot e^{(|\lambda|+\rho_i)\tau_*}}{(|\lambda|+\rho_i)\,\tau_*}\\&\geq|\omega_i|\,\tau_* e^{-\rho_i\tau_*}.\end{aligned} \tag{20}$$

Equation (20) contradicts the first inequality of assumption (14). Then, we discuss the case that $\tau_1=\tau_3=\tau_5=\tau^*$ in system (8). Similarly, if $\lambda<0$ is a solution of the equation $\lambda-\rho_i-\omega_i e^{-\lambda\tau^*}=0$, we also have a contradiction with the second inequality of assumption (14). Since $\tau_*\leq\tau_i\leq\tau^*$ $(i=1,3,5)$, one can conclude that there exists a positive real part of the eigenvalue of system (8) for any τ_i $(i=1,3,5)$ under the assumptions. This means that the trivial solution of system (8) is unstable, implying that there is a periodic oscillatory solution of system (6) based on Chafee's criterion. □

Theorem 4. *Suppose that $b_i>0$, $0<\varepsilon_i\ll 1$, $d_i<0$, $(\alpha_i+c_i)^2+(4/3)d_i<0$ $(i=1,2,3)$, and C is a nonsingular matrix. Let $\rho_k=\rho_{k1}+i\rho_{k2}$ (ρ_{k2} may equal zero) and $\omega_k=\omega_{k1}+i\omega_{k2}$ (ω_{k2} may equal zero) $(k=1,2,\ldots,6)$ denote the eigenvalues of matrices A and B, respectively. If, for some ρ_i, $|\rho_{i1}|<\omega_{i1}$ as $\rho_{i1}<0$, then the trivial solution of system* (8) *is unstable, implying that system* (6) *has a periodic oscillatory solution.*

Proof. The assumptions guarantee that system (8) has a unique equilibrium point and all solutions are bounded. In this case, we first consider $\tau_1=\tau_3=\tau_5=\tau_*$ in system (8). Then, for some ρ_i, let $\lambda=\lambda_1+i\lambda_2$; from (18) we have

$$\begin{aligned}&\lambda_1-\rho_{i1}-\omega_{i1}e^{-\lambda_1\tau_*}\cos(\lambda_2\tau_*)=0,\\&\lambda_2-\rho_{i2}+\omega_{i2}e^{-\lambda_1\tau_*}\sin(\lambda_2\tau_*)=0.\end{aligned} \tag{21}$$

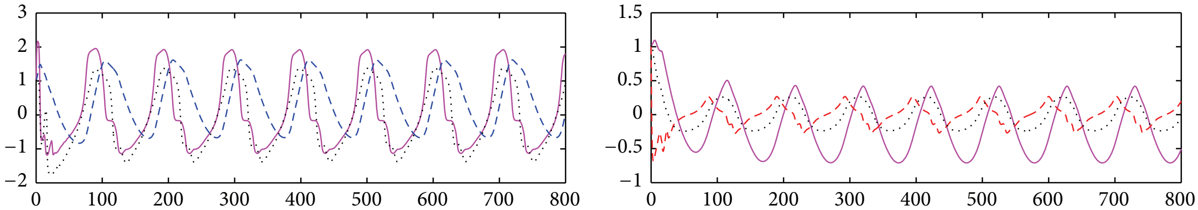
We will show that $\lambda_1>0$ and there is an eigenvalue which has positive real part of system (18). Let $f(\lambda_1)=\lambda_1-\rho_{i1}-\omega_{i1}e^{-\lambda_1\tau_*}\cos(\lambda_2\tau_*)$; then $f(\lambda_1)$ is a continuous function of λ_1. If $\rho_{i1}>0$, then select suitable delay τ_* such that $\omega_{i1}\cos(\lambda_2\tau_*)>-\rho_{i1}$. Therefore, $f(0)=-\rho_{i1}-\omega_{i1}\cos(\lambda_2\tau_*)<0$. Noting that $e^{-\lambda_1\tau_*}\to 0$ as $\lambda_1\to+\infty$, obviously, there exists a suitably large $\lambda_1(>0)$ such that $f(\lambda_1)=\lambda_1-\rho_{i1}-\omega_{i1}e^{-\lambda_1\tau_*}\cos(\lambda_2\tau_*)>0$. By the continuity of $f(\lambda_1)$, there exists a positive ${\lambda_1}^*\in(0,\lambda_1)$ such that $f({\lambda_1}^*)=0$. If $\rho_{i1}<0$, since $|\rho_{i1}|<\omega_{i1}$ $(\omega_{i1}\neq 0)$, then there exists a suitable delay τ_* and a positive $\overline{\lambda}_1$ such that $\omega_{i1}\cos(\lambda_2\tau_*)<-\rho_{i1}$ and $\overline{\lambda}_1-\omega_{i1}e^{-\overline{\lambda}_1\tau_*}\cos(\lambda_2\tau_*)<0$ both hold. Then, $f(0)=-\rho_{i1}-\omega_{i1}\cos(\lambda_2\tau_*)>0$ and $f(\overline{\lambda}_1)=\overline{\lambda}_1-\omega_{i1}e^{-\overline{\lambda}_1\tau_*}\cos(\lambda_2\tau_*)<0$. Again, from the continuity of $f(\lambda_1)$, there exists a positive ${\lambda_1}^{**}\in(0,\overline{\lambda}_1)$ such that $f({\lambda_1}^{**})=0$. Thus, there is an eigenvalue of system (18) that has positive real part. Implying that the trivial solution of system (8) is unstable. Thus, the trivial solution of system (6) is also unstable. Based on the theory of delay differential equation, the oscillatory behavior of the solution will maintain as time delay increasing. Therefore, for any $\tau_i\geq\tau_*$ $(i=1,3,5)$, system (8), as well as system, (6) generates a periodic oscillatory solution. We select a suitable delay τ_* such that system (6) has a periodic oscillatory solution. This oscillation is said to be induced by time delay. □

4. Simulation Result

The parameter values are selected as $\alpha_1=-1.5$, $\alpha_2=-1.5$, $\alpha_3=-1.2$; $b_1=0.16$, $b_2=0.25$, $b_3=0.12$; $c_1=1.3$, $c_2=1.302$, $c_3=1.305$; $d_1=-0.705$, $d_2=-0.706$, $d_3=-0.707$; $\beta_1=1.5$, $\beta_2=1.5$, $\beta_3=0.15$; $\varepsilon_1=0.05$, $\varepsilon_2=0.025$, $\varepsilon_3=0.085$, respectively. It is easy to check that the conditions of Lemmas 1 and 2 hold. The activation functions are selected as $\arctan(u)$ and $\tanh(u)$, respectively. In this case, $\gamma_1=\gamma_3=\gamma_5=1$, and eigenvalues of matrices A and B are $\rho_1=-0.6238$, $\rho_2=-0.0892$, $\rho_3=-0.6682$, $\rho_4=-0.0440$, $\rho_5=-0.5493$, and $\rho_6=-0.1679$, and $\omega_1=1.7562$, $\omega_2=-1.5000$, $\omega_3=-0.2562$, $\omega_4=0$, and $\omega_5=0$, $\omega_6=0$, respectively. Since $|\rho_1|=0.6238<\omega_1$, there is a periodic oscillatory solution based on Theorem 4. Both in Figures 1 and 2, the time delays are selected as $\tau_1=10$, $\tau_2=8$, and $\tau_3=4$. Then, we change delays as $\tau_1=1$, $\tau_2=2$, $\tau_3=3$; activation function is kept as $\tanh(u)$; periodic oscillatory solution also occurred (Figure 3). In Figure 4, the parameter values are selected as $\alpha_1=-0.95$, $\alpha_2=-1.2$, $\alpha_3=-1.25$; $b_1=0.18$, $b_2=0.2$, $b_3=0.16$; $c_1=1.4$, $c_2=1.42$, $c_3=1.45$; $d_1=-0.7$, $d_2=-0.72$, $d_3=-0.75$; $\beta_1=1.25$, $\beta_2=1.2$, $\beta_3=1.15$; $\varepsilon_1=0.05$, $\varepsilon_2=0.045$, and $\varepsilon_3=0.065$, respectively. The activation function is $\tanh(u)$. The eigenvalues of matrices A and B are $\rho_1=-0.6179$, $\rho_2=-0.0911$, $\rho_3=-0.6498$, $\rho_4=-0.0792$, $\rho_5=-0.6481$, $\rho_6=-0.1123$ and $\omega_1=2.3654$, $\omega_2=-1.2154$, $\omega_3=-1.1500$, $\omega_4=0$, $\omega_5=0$, and $\omega_6=0$, respectively. We see that periodic oscillatory solution appeared.

5. Conclusion

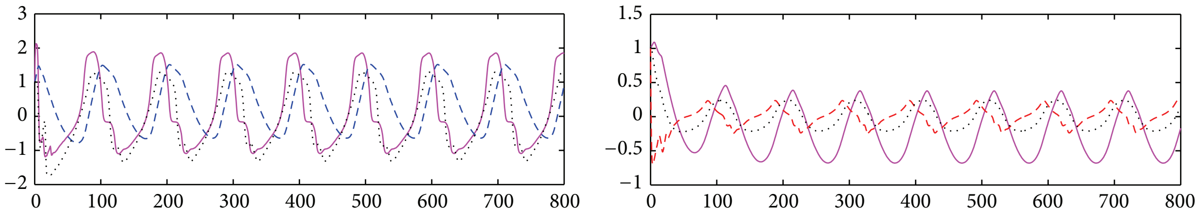
This paper discusses a three coupled FHN neurons model in which the synaptic strength of self-connection, neighborhoodinteraction for each neuron, and the time delays are



(a) Solid line: $u_1(t)$, dashed line: $u_2(t)$, dotted line: $u_3(t)$

(b) Solid line: $u_4(t)$, dashed line: $u_5(t)$, dotted line: $u_6(t)$

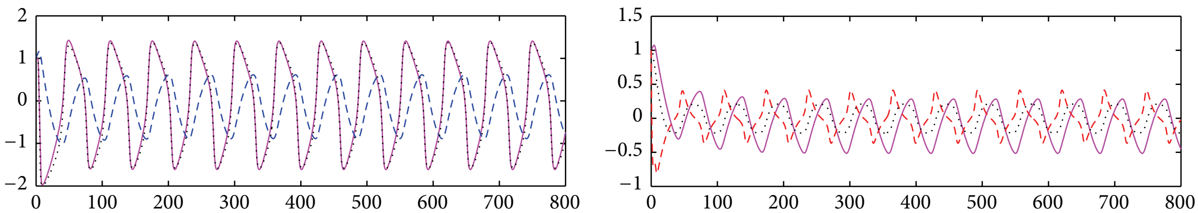
FIGURE 1: Periodic oscillatory behavior, activation function: arctan(u), and delays: (10, 8, 4).



(a) Solid line: $u_1(t)$, dashed line: $u_2(t)$, dotted line: $u_3(t)$

(b) Solid line: $u_4(t)$, dashed line: $u_5(t)$, dotted line: $u_6(t)$

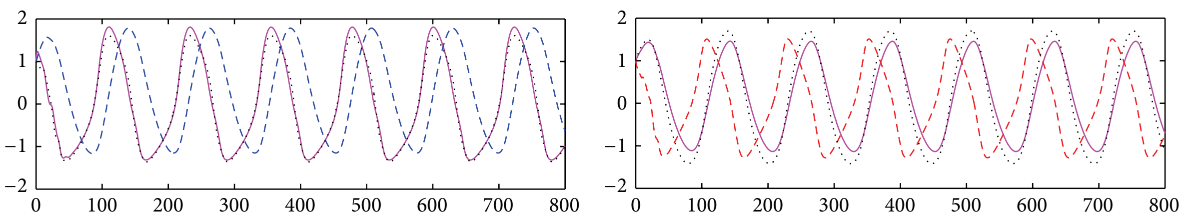
FIGURE 2: Periodic oscillatory behavior, activation function: tanh(u), and delays: (10, 8, 4).



(a) Solid line: $u_1(t)$, dashed line: $u_2(t)$, dotted line: $u_3(t)$

(b) Solid line: $u_4(t)$, dashed line: $u_5(t)$, dotted line: $u_6(t)$

FIGURE 3: Periodic oscillatory behavior, activation function: tanh(u), and delays: (1, 2, 3).



(a) Solid line: $u_1(t)$, dashed line: $u_2(t)$, dotted line: $u_3(t)$

(b) Solid line: $u_4(t)$, dashed line: $u_5(t)$, dotted line: $u_6(t)$

FIGURE 4: Periodic oscillatory behavior, activation function: tanh(u), and delays: (9, 10, 12).

different. Two theorems are provided to determine the periodic oscillatory behavior of the solutions based on Chafee's criterion of limit cycle. Computer simulation suggested that those theorems only are sufficient conditions.

Acknowledgments

This research was supported by NNSF of China (11361010) and Scientific Research Foundation of the Education Department of Guangxi Province (201010LX463).

References

[1] Q. Wang, Q. Lu, G. Chen, Z. Feng, and L. Duan, "Bifurcation and synchronization of synaptically coupled FHN models with time delay," *Chaos, Solitons & Fractals*, vol. 49, pp. 918–925, 2009.

[2] D. Fan and L. Hong, "Hopf bifurcation analysis in a synaptically coupled FHN neuron model with delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 7, pp. 1873–1886, 2010.

[3] U. Tetsushi, M. Hisayo, K. Takuji, and K. Hiroshi, “Bifurcation and chaos in coupled BVP oscillators,” *International Journal of Bifurcation and Chaos*, vol. 4, no. 14, pp. 1305–1324, 2004.

[4] U. Tetsushi and K. Hiroshi, “Bifurcation in asymmetrically coupled BVP oscillators,” *International Journal of Bifurcation and Chaos*, vol. 5, no. 13, pp. 1319–1327, 2003.

[5] T. Kunichika, A. Kazuyuki, and K. Hiroshi, “Bifurcations in synaptically coupled BVP neurons,” *International Journal of Bifurcation and Chaos*, vol. 4, no. 11, pp. 1053–1064, 2001.

[6] Q. Wang, Q. Lu, G. Chen, Z. Feng, and L. Duan, “Bifurcation and synchronization of synaptically coupled FHN models with time delay,” *Chaos, Solitons & Fractals*, vol. 39, pp. 918–925, 2009.

[7] D. Fan and L. Hong, “Hopf bifurcation analysis in a synaptically coupled FHN neuron model with delays,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 7, pp. 1873–1886, 2010.

[8] B. Zhen and J. Xu, “Bautin bifurcation analysis for synchronous solution of a coupled FHN neural system with delay,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 2, pp. 442–458, 2010.

[9] N. Chafee, “A bifurcation problem for a functional differential equation of finitely retarded type,” *Journal of Mathematical Analysis and Applications*, vol. 35, pp. 312–348, 1971.

[10] I. Győri and G. Ladas, *Oscillation Theory of Delay Differential Equations*, Clarendon Press, Oxford, UK, 1991.