Research Article

Application of the Local Fractional Series Expansion Method and the Variational Iteration Method to the Helmholtz Equation Involving Local Fractional Derivative Operators

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We investigate solutions of the Helmholtz equation involving local fractional derivative operators. We make use of the series expansion method and the variational iteration method, which are based upon the local fractional derivative operators. The nondifferentiable solution of the problem is obtained by using these methods.

1. Introduction

The Helmholtz equation is known to arise in several physical problems such as electromagnetic radiation, seismology, and acoustics. It is a partial differential equation, which models the normal and nonfractal physical phenomena in both time and space [1]. It is an important differential equation, which is usually investigated by means of some analytical and numerical methods (see [2–11] and the references therein). For example, the FEM solution for the Helmholtz equation in one, two, and three dimensions was investigated in [2, 3]. The variational iteration method was used to solve the Helmholtz equation in [4]. The explicit solution for the Helmholtz equation was considered in [5] by using the homotopy perturbation method. The domain decomposition method for the Helmholtz equation was presented in [6]. The boundary element method for the Helmholtz equation was considered in [7, 8]. The modified Fourier-Galerkin method for the Helmholtz equations was applied in [9]. The Green's function for the two-dimensional Helmholtz equation in periodic domains was suggested in [10, 11].

Fractional calculus theory [12–26] has been applied to deal with the differentiable models from the practical engineering discipline, which are the anomalous and fractal physical phenomena. The fractional Helmholtz equations were considered in [27–29]. In this work, there are two methods to deal with such problems. For example, an analytic solution for the fractional Helmholtz equation in terms of the Mittag-Leffler function was investigated in [28]. The homotopy perturbation method for multidimensional fractional Helmholtz equation was considered in [29].

Local fractional calculus theory [30–44] has been used to process the nondifferentiable problems in natural phenomena. Taking an example, the local fractional Fokker-Planck equation was proposed in [30]. The mechanics of quasi-brittle materials with a fractal microstructure with the local fractional derivative was presented in [31]. The anomalous diffusion modeling by fractal and fractional derivatives was considered in [35]. The local fractional wave and heat equations were discussed in [36, 37]. Newtonian mechanics on fractals subset of real-line was investigated in [38]. In [39], the Helmholtz equation on the Cantor sets involving local

fractional derivative operators was proposed. There are some other methods to handle the local fractional differential equations, such as local fractional series expansion method [40] and variational iteration method [41–44].

The main objective of the present paper is to solve the Helmholtz equation involving the local fractional derivative operators by means of the local fractional series expansion method and the variational iteration method. The structure of the paper is as follows. In Section 2, we describe the Helmholtz equation involving the local fractional derivative operators. In Section 3, we give analysis of the methods used. In Section 4, we apply the local fractional series expansion method to deal with the Helmholtz equation. In Section 5, we apply the local fractional variational iteration method to deal with the Helmholtz equation. Finally, in Section 6, we present our conclusions.

2. Helmholtz Equations within Local Fractional Derivative Operators

The Helmholtz equation involving local fractional derivative operators was proposed.

Let us denote the local fractional derivative as follows [36, 37, 39–44]:

$$f^{(\alpha)}(x_0) = \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \quad (1)$$

where $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha)\Delta(f(x) - f(x_0))$.

Using separation of variables in nondifferentiable functions, the three-dimensional Helmholtz equation involving local fractional derivative operators was suggested by the following expression [39]:

$$\frac{\partial^{2\alpha} M(x, y, z)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y, z)}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y, z)}{\partial z^{2\alpha}}$$
$$+ \omega^{2\alpha} M(x, y, z) = 0, \quad (2)$$

where the operator involved is a local fractional derivative operator.

In this case, the two-dimensional Helmholtz equation involving local fractional derivative operators is expressed as follows (see [39]):

$$\frac{\partial^{2\alpha} M(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y)}{\partial y^{2\alpha}} + \omega^{2\alpha} M(x, y) = 0. \quad (3)$$

The three-dimensional inhomogeneous Helmholtz equation is given by (see [39])

$$\frac{\partial^{2\alpha} M(x, y, z)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y, z)}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y, z)}{\partial z^{2\alpha}}$$
$$+ \omega^{2\alpha} M(x, y, z) = f(x, y, z), \quad (4)$$

where $f(x, y, z)$ is a local fractional continuous function.

The two-dimensional local fractional inhomogeneous Helmholtz equation is considered as follows (see [39]):

$$\frac{\partial^{2\alpha} M(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} M(x, y)}{\partial y^{2\alpha}} + \omega^{2\alpha} M(x, y) = f(x, y), \quad (5)$$

where $f(x, y)$ is a local fractional continuous function.

The previous local fractional Helmholtz equations with local fractional derivative operators are applied to describe the governing equations in fractal electromagnetic radiation, seismology, and acoustics.

3. Analysis of the Methods Used

3.1. The Local Fractional Series Expansion Method. Let us consider a given local fractional differential equation

$$u_t^{2\alpha} = L_\alpha u, \quad (6)$$

where L is a linear local fractional derivative operator of order 2α with respect to x.

By the local fractional series expansion method [40], a multiterm separated function of independent variables t and x reads as

$$u(x, t) = \sum_{i=0}^{\infty} T_i(t) X_i(x), \quad (7)$$

where $T_i(t)$ and $X_i(x)$ are local fractional continuous functions.

In view of (7), we have

$$T_i(t) = \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)}, \quad (8)$$

so that

$$u(x, t) = \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)} X_i(x). \quad (9)$$

Making use of (9), we get

$$u_t^{2\alpha} = \sum_{i=0}^{\infty} \frac{1}{\Gamma(1 + i\alpha)} t^{i\alpha} X_{i+2}(x),$$

$$L_\alpha u = L_\alpha \left[\sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)} X_i(x) \right] = \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)} (L_\alpha X_i)(x). \quad (10)$$

In view of (10), we have

$$\sum_{i=0}^{\infty} \frac{1}{\Gamma(1 + i\alpha)} t^{i\alpha} X_{i+2}(x) = \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)} (L_\alpha X_i)(x). \quad (11)$$

Hence, from (11), the recursion reads as follows:

$$X_{i+2}(x) = (L_\alpha X_i)(x). \quad (12)$$

By using (12), we arrive at the following result:

$$u(x, t) = \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1 + i\alpha)} X_i(x). \quad (13)$$

3.2. The Local Fractional Variational Iteration Method. Let us consider the following local fractional operator equation:

$$L_\alpha u + R_\alpha u = g(t), \tag{14}$$

where L_α is linear local fractional derivative operator of order 2α, R_α is a lower-order local fractional derivative operator, and $g(t)$ is the inhomogeneous source term.

By using the local fractional variational iteration method [41–44], we can construct a correctional local fractional functional as follows:

$$
u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)} \\
\times \{\eta(s)[L_\alpha u_n(s) + R_\alpha \tilde{u}_n(s) - \tilde{g}(s)]\}, \tag{15}
$$

where the local fractional operator is defined as follows [36, 37, 41–44]:

$$
{}_aI_b^{(\alpha)} f(x) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\
= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha \tag{16}
$$

and a partition of the interval $[a, b]$ is $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max\{\Delta t_1, \Delta t_2, \Delta t_j, \ldots\}$, and $j = 0, \ldots, N-1$, $t_0 = a, t_N = b$.

Following (15), we have

$$
\delta^\alpha u_{n+1}(x) = \delta^\alpha u_n(x) + {}_0I_x^{(\alpha)} \delta^\alpha \\
\times \{\eta(s)[L_\alpha u_n(s) + R_\alpha \tilde{u}_n(s) - \tilde{g}(s)]\}. \tag{17}
$$

The extremum condition of u_{n+1} is given by [37, 41, 42]

$$\delta^\alpha u_{n+1} = 0. \tag{18}$$

In view of (18), we have the following stationary conditions:

$$
1 - \eta(s)^{(\alpha)}\big|_{s=x} = 0, \qquad \eta(s)\big|_{s=x} = 0, \\
\eta(s)^{(2\alpha)}\big|_{s=x} = 0. \tag{19}
$$

So, from (19), we get

$$\eta(s) = \frac{(s-x)^\alpha}{\Gamma(1+\alpha)}. \tag{20}$$

The initial value $u_0(x)$ is given by

$$u_0(x) = u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0). \tag{21}$$

In view of (20), we have

$$
u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)} \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \\
\times \{L_\alpha u_n(s) + R_\alpha \tilde{u}_n(s) - \tilde{g}(s)\}. \tag{22}
$$

Finally, from (22), we obtain the solution of (14) as follows:

$$u = \lim_{n\to\infty} u_n. \tag{23}$$

4. Local Fractional Series Expansion Method for the Helmholtz Equation

Let us consider the following Helmholtz equation involving local fractional derivative operators:

$$\frac{\partial^{2\alpha} u(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x, y)}{\partial y^{2\alpha}} = u(x, y). \tag{24}$$

We now present the initial value conditions as follows:

$$
u(0, y) = 0, \\
\frac{\partial}{\partial x^\alpha} u(0, y) = E_\alpha(y^\alpha). \tag{25}
$$

Using relation (12), we have

$$
u_{i+2}(y) = (L_\alpha u_i)(y), \\
u_0(y) = u(0, y) = 0, \\
u_1(y) = \frac{\partial}{\partial x^\alpha} u(0, y) = E_\alpha(y^\alpha), \tag{26}
$$

where

$$L_\alpha u_i = u_i - \frac{\partial^{2\alpha} u_i}{\partial y^{2\alpha}}. \tag{27}$$

Hence, we get the following iterative relations:

$$
u_{i+2}(y) = \left(u_i - \frac{\partial^{2\alpha} u_i}{\partial y^{2\alpha}}\right)(y), \\
u_0(y) = u(0, y) = 0, \tag{28}
$$

$$
u_{i+2}(y) = \left(u_i - \frac{\partial^{2\alpha} u_i}{\partial y^{2\alpha}}\right)(y), \\
u_1(y) = E_\alpha(y^\alpha). \tag{29}
$$

From (28), we have

$$u_0(y) = u_2(y) = u_4(y) = \cdots = 0. \tag{30}$$

From (29), we get the following terms:

$$
u_1(y) = E_\alpha(y^\alpha), \\
u_3(y) = \left(u_1 - \frac{\partial^{2\alpha} u_1}{\partial y^{2\alpha}}\right)(y) \\
= [E_\alpha(y^\alpha) - E_\alpha(y^\alpha)] = 0,
$$

$$u_5(y) = 0,$$

$$u_7(y) = \cdots = 0.$$

(31)

Hence, we obtain

$$u(x, y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(32)

5. Local Fractional Variational Iteration Method for the Helmholtz Equation

We now consider (24) with the initial and boundary conditions in (25) by using the local fractional variational iteration method.

Applying the iterative relation equation (22), we get

$$u_{n+1}(x, y) = u_n(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_n(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_n(x, y)}{\partial y^{2\alpha}} - u_n(x, y) \right\},$$

(33)

where the initial value is given by

$$u_0(x, y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(34)

Therefore, from (34) we have

$$u_1(x, y) = u_0(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_0(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_0(x, y)}{\partial y^{2\alpha}} - u_0(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(35)

The second approximate term reads as follows:

$$u_2(x, y) = u_1(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_1(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_1(x, y)}{\partial y^{2\alpha}} - u_1(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(36)

The third approximate term reads as follows:

$$u_3(x, y) = u_2(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_2(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_2(x, y)}{\partial y^{2\alpha}} - u_2(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(37)

Other approximate terms are presented as follows:

$$u_4(x, y) = u_3(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_3(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_3(x, y)}{\partial y^{2\alpha}} - u_3(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha),$$

$$u_5(x, y) = u_4(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_4(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_4(x, y)}{\partial y^{2\alpha}} - u_4(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha)$$

$$\vdots$$

$$u_n(x, y) = u_{n-1}(x, y) + {_0I_y^{(\alpha)}} \frac{(s-y)^\alpha}{\Gamma(1+\alpha)}$$

$$\times \left\{ \frac{\partial^{2\alpha} u_{n-1}(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u_{n-1}(x, y)}{\partial y^{2\alpha}} \right.$$

$$\left. - u_{n-1}(x, y) \right\}$$

$$= \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha)$$

(38)

and so on.

So, we get

$$u(x, y) = \lim_{n \to \infty} u_n(x, y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha).$$

(39)

The result is the same as the one which is obtained by the local fractional series expansion method. The nondifferentiable solution is shown in Figure 1.

6. Conclusions

In this work, the nondifferentiable solution for the Helmholtz equation involving local fractional derivative operators is
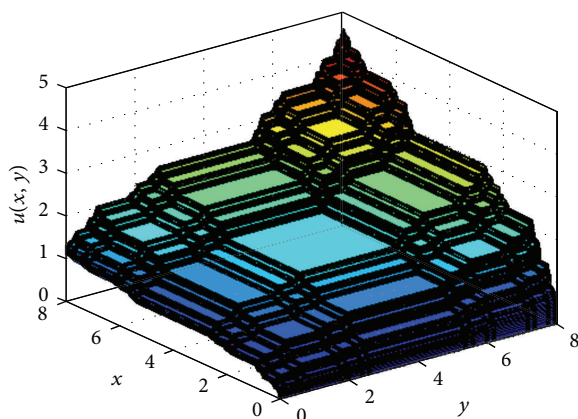
FIGURE 1: Graph of $u(x, y)$ for $\alpha = \ln 2/\ln 3$.

investigated by using the local fractional series expansion method and the variational iteration method. By using these two markedly different methods, the same solution is obtained. These two approaches are remarkably efficient to process other linear local fractional differential equations as well.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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