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Research Article

Approximate n-Lie Homomorphisms and Jordan n-Lie Homomorphisms on n-Lie Algebras

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Using fixed point methods, we establish the stability of n-Lie homomorphisms and Jordan n-Lie homomorphisms on n-Lie algebras associated to the following generalized Jensen functional equation $\mu f(\sum_{i=1}^{n} x_i/n) + \mu \sum_{j=2}^{n} f(\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j/n) = f(\mu x_1)(n \geq 2)$.

1. Introduction

Let n be a natural number greater or equal to 3. The notion of an n-Lie algebra was introduced by Filippov in 1985 [1]. The Lie product is taken between n elements of the algebra instead of two. This new bracket is n-linear, antisymmetric and satisfies a generalization of the Jacobi identity. For $n = 3$ this product is a special case of the Nambu bracket, well known in physics, which was introduced by Nambu [2] in 1973, as a generalization of the Poisson bracket in Hamiltonian mechanics.

An n-Lie algebra is a natural generalization of a Lie algebra. Namely, a vector space V together with a multilinear, antisymmetric n-ary operation $[]:\Lambda^n V \to V$ is called an n-Lie algebra, $n \geq 3$, if the n-ary bracket is a derivation with respect to itself, that is,

$$[[x_1, \ldots, x_n], x_{n+1}, \ldots, x_{2n-1}] = \sum_{i=1}^{n} [x_1, \ldots, x_{i-1}[x_i, x_{n+1}, \ldots, x_{2n-1}], \ldots, x_n], \tag{1.1}$$

where $x_1, x_2, \ldots, x_{2n-1} \in V$. Equation (1.1) is called the generalized Jacobi identity. The meaning of this identity is similar to that of the usual Jacobi identity for a Lie algebra (which is a 2-Lie algebra).

In [1] and several subsequent papers, [3–5] a structure theory of finite-dimensional n-Lie algebras over a field $\mathbb{F}$ of characteristic 0 was developed.

n-ary algebras have been considered in physics in the context of Nambu mechanics [2, 6] and, recently (for $n = 3$), in the search for the effective action of coincident $M2$-branes in M-theory initiated by the Bagger-Lambert-Gustavsson (BLG) model [7, 8] (further references on the physical applications of n-ary algebras are given in [9]).

From now on, we only consider n-Lie algebras over the field of complex numbers. An n-Lie algebra A is a normed n-Lie algebra if there exists a norm $\|\|$ on A such that $\|[x_1, x_2, \dots, x_n]\| \le \|x_1\|\|x_2\| \cdots \|x_n\|$ for all $x_1, x_2, \dots, x_n \in A$. A normed n-Lie algebra A is called a Banach n-Lie algebra, if $(A, \|\|)$ is a Banach space.

Let $(A, []_A)$ and $(B, []_B)$ be two Banach n-Lie algebras. A $\mathbb{C}$-linear mapping $H : (A, []_A) \to (B, []_B)$ is called an n-Lie homomorphism if

$$H([x_1 x_2 \cdots x_n]_A) = [H(x_1)H(x_2) \cdots H(x_n)]_B \tag{1.2}$$

for all $x_1, x_2, \dots, x_n \in A$. A $\mathbb{C}$-linear mapping $H : (A, []_A) \to (B, []_B)$ is called a Jordan n-Lie homomorphism if

$$H([xx \cdots x]_A) = [H(x)H(x) \cdots H(x)]_B \tag{1.3}$$

for all $x \in A$.

The study of stability problems had been formulated by Ulam [10] during a talk in 1940. Under what condition does there exist a homomorphism near an approximate homomorphism? In the following year, Hyers [11] answered affirmatively the question of Ulam for Banach spaces, which states that if $\varepsilon > 0$ and $f : X \to Y$ is a map with X a normed space, Y a Banach spaces such that

$$\|f(x + y) - f(x) - f(y)\| \le \varepsilon \tag{1.4}$$

for all $x, y \in X$, then there exists a unique additive map $T : X \to Y$ such that

$$\|f(x) - T(x)\| \le \varepsilon \tag{1.5}$$

for all $x \in X$. A generalized version of the theorem of Hyers for approximately linear mappings was presented by Rassias [12] in 1978 by considering the case when inequality (1.4) is unbounded. Due to that fact, the additive functional equation $f(x+y) = f(x)+f(y)$ is said to have the generalized Hyers-Ulam-Rassias stability property. A large list of references concerning the stability of functional equations can be found in [13–32].

In 1982–1994, Rassias (see [26–28]) solved the Ulam problem for different mappings and for many Euler-Lagrange type quadratic mappings, by involving a product of different powers of norms. In addition, Rassias considered the mixed product sum of powers of norms control function. For more details see [33–57].

In 2003 Cădariu and Radu applied the fixed-point method to the investigation of the Jensen functional equation [58]. They could present a short and a simple proof (different of the "*direct method*", initiated by Hyers in 1941) for the generalized Hyers-Ulam stability of Jensen functional equation [58] and for quadratic functional equation.

Park and Rassias [59] proved the stability of homomorphisms in C^*-algebras and Lie C^*-algebras and also of derivations on C^*-algebras and Lie C^*-algebras for the Jensen-type functional equation

$$\mu f\left(\frac{x+y}{2}\right)+\mu f\left(\frac{x-y}{2}\right)-f(\mu x)=0 \tag{1.6}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C}; |\lambda| = 1\}$.

In this paper, by using the fixed-point methods, we establish the stability of n-Lie homomorphisms and Jordan n-Lie homomorphisms on n-Lie Banach algebras associated to the following generalized Jensen type functional equation:

$$\mu f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)+\mu \sum_{j=2}^{n} f\left(\frac{\sum_{i=1, i\neq j}^{n} x_i-(n-1)x_j}{n}\right)-f(\mu x_1)=0 \tag{1.7}$$

for all $\mu \in (\mathbb{T}^1_{1/n_o} := \{e^{i\theta}; 0 \le \theta \le 2\pi/n_o\} \cup \{1\})$, where $n \ge 2$.

Throughout this paper, assume that $(A, []_A)$, $(B, []_B)$ are two n-Lie Banach algebras.

2. Main Results

Before proceeding to the main results, we recall a fundamental result in fixed point theory.

Theorem 2.1 (see [60]). *Let (Ω, d) be a complete generalized metric space, and let $T : \Omega \to \Omega$ be a strictly contractive function with Lipschitz constant L. Then for each given $x \in \Omega$, either*

$$d\left(T^m x, T^{m+1} x\right)=\infty \quad \forall m \ge 0, \tag{2.1}$$

or other exists a natural number m_0 such that

(i) *$d(T^m x, T^{m+1} x) < \infty$ for all $m \ge m_0$;*

(ii) *the sequence $\{T^m x\}$ is convergent to a fixed point y^* of T;*

(iii) *y^* is the unique fixed point of T in $\Lambda = \{y \in \Omega : d(T^{m_0} x, y) < \infty\}$;*

(iv) *$d(y, y^*) \le (1/(1-L)) d(y, Ty)$ for all $y \in \Lambda$.*

We start our work with the main theorem of the our paper.

Theorem 2.2. *Let $n_0 \in \mathbb{N}$ be a fixed positive integer number. Let $f : A \to B$ be a function for which there exists a function $\phi : A^n \to [0, \infty)$ such that*

$$\left\|\mu f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)+\mu \sum_{j=2}^{n} f\left(\frac{\sum_{i=1, i\neq j}^{n} x_i-(n-1)x_j}{n}\right)-f(\mu x_1)\right\|_B \le \phi(x_1, x_2, \ldots, x_n) \tag{2.2}$$

for all $\mu \in (T^1_{1/n_o} := \{e^{i\theta};\ 0 \leq \theta \leq 2\pi/n_o\} \cup \{1\})$ *and all* $x_1, \ldots, x_n \in A$*, and that*

$$\left\| f([x_1 x_2 \cdots x_n]_A) - \left[f(x_1) f(x_2) \cdots f(x_n)\right]_B \right\|_B \leq \phi(x_1, x_2, \ldots, x_n) \tag{2.3}$$

for all $x_1, \ldots, x_n \in A$*. If there exists an* $L < 1$ *such that*

$$\phi(x_1, x_2, \ldots, x_n) \leq nL\phi\left(\frac{x_1}{n}, \frac{x_2}{n}, \ldots, \frac{x_n}{n}\right) \tag{2.4}$$

for all $x_1, \ldots, x_n \in A$*, then there exists a unique* n*-Lie homomorphism* $H : A \to B$ *such that*

$$\|f(x) - H(x)\| \leq \frac{L}{1-L}\phi(x, 0, 0, \ldots, 0) \tag{2.5}$$

for all $x \in A$*.*

Proof. Let Ω be the set of all functions from A into B and let

$$d(g, h) := \inf\{C \in \mathbb{R}^+ : \|g(x) - h(x)\|_B \leq C\phi(x, 0, \ldots, 0), \forall x \in A\}. \tag{2.6}$$

It is easy to show that (Ω, d) is a generalized complete metric space [61].

Now we define the mapping $J : \Omega \to \Omega$ by $J(h)(x) = (1/n)h(nx)$ for all $x \in A$.

Note that for all $g, h \in \Omega$,

$$\begin{aligned} d(g, h) < C &\Longrightarrow \|g(x) - h(x)\| \leq C\phi(x, 0, \ldots, 0), \quad \forall x \in A, \\ &\Longrightarrow \left\|\frac{1}{n}g(nx) - \frac{1}{n}h(nx)\right\| \leq \frac{1}{|n|^{\ell}}C\phi(nx, 0, \ldots, 0), \quad \forall x \in A, \\ &\Longrightarrow \left\|\frac{1}{n}g(nx) - \frac{1}{n}h(nx)\right\| \leq LC\phi(x, 0, \ldots, 0), \quad \forall x \in A, \\ &\Longrightarrow d(J(g), J(h)) \leq LC. \end{aligned} \tag{2.7}$$

Hence we see that

$$d(J(g), J(h)) \leq Ld(g, h) \tag{2.8}$$

for all $g, h \in \Omega$. It follows from (2.4) that

$$\lim_{m\to\infty} \frac{1}{n^m}\phi(n^m x_1, n^m x_2, \ldots, n^m x_n) = 0 \tag{2.9}$$

for all $x_1, \ldots, x_n \in A$. Putting $\mu = 1$, $x_1 = x$, and $x_j = 0$ $(j = 2, \ldots, n)$ in (2.2), we obtain

$$\left\|nf\left(\frac{x}{n}\right) - f(x)\right\|_B \leq \phi(x, 0, \ldots, 0) \tag{2.10}$$

for all $x \in A$. Thus by using (2.4), we obtain that

$$\left\| \frac{1}{n} f(nx) - f(x) \right\|_B \leq \frac{1}{n} \phi(nx, 0, \ldots, 0) \leq L\phi(x, 0, \ldots, 0) \tag{2.11}$$

for all $x \in A$, that is,

$$d(f, J(f)) \leq L < \infty. \tag{2.12}$$

By Theorem 2.1, J has a unique fixed point in the set $X_1 := \{h \in \Omega : d(f, h) < \infty\}$. Let H be the fixed point of J. H is the unique mapping with

$$H(nx) = nH(x) \tag{2.13}$$

for all $x \in A$, such that there exists $C \in (0, \infty)$ satisfying

$$\| f(x) - H(x) \|_B \leq C\phi(x, 0, \ldots, 0) \tag{2.14}$$

for all $x \in A$. On the other hand we have $\lim_{m \to \infty} d(J^m(f), H) = 0$, so

$$\lim_{m \to \infty} \frac{1}{n^m} f(n^m x) = H(x) \tag{2.15}$$

for all $x \in A$. Also by Theorem 2.1, we have

$$d(f, H) \leq \frac{1}{1 - L} d(f, J(f)). \tag{2.16}$$

It follows from (2.12) and (2.16) that

$$d(f, H) \leq \frac{L}{1 - L}. \tag{2.17}$$

This implies the inequality (2.5). By (2.21), we have

$$\begin{aligned}
&\| H([x_1 x_2 \cdots x_n]_A) - [H(x_1)H(x_2)H(x_3) \cdots H(x_n)]_B \|_B \\
&= \lim_{m \to \infty} \left\| \frac{1}{n^{nm}} H([n^m x_1 n^m x_2 \cdots n^m x_n]_A) - \frac{1}{n^{nm}} ([H(n^m x_1)H(n^m x_2)H(n^m x_3) \cdots H(n^m x_n)]_B) \right\| \\
&\leq \lim_{m \to \infty} \frac{1}{n^{nm}} \phi(n^m x_1, n^m x_2, \ldots, n^m x_n) = 0
\end{aligned} \tag{2.18}$$

for all $x_1, \ldots, x_n \in A$. Hence

$$H([x_1 x_2 \cdots x_n]_A) = [H(x_1)H(x_2)H(x_3)\cdots H(x_n)]_B \tag{2.19}$$

for all $x_1, \ldots, x_n \in A$.

On the other hand, it follows from (2.2), (2.9), and (2.15) that

$$\begin{aligned}
&\left\| H\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) + \sum_{j=2}^{n} H\left(\frac{\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j}{n}\right) - H(x_1)\right\|_B \\
&\quad = \lim_{m\to\infty} \frac{1}{n^m}\left\| f\left(n^{m-1}\sum_{i=1}^{n} x_i\right) + \sum_{j=2}^{n}\left(f\left(n^{m-1}\left(\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j\right)\right)\right) - f(n^m x_1)\right\|_B \\
&\quad \le \lim_{m\to\infty} \frac{1}{n^m}\phi(n^m x_1, n^m x_2, \ldots, n^m x_n) = 0
\end{aligned} \tag{2.20}$$

for all $x_1, \ldots, x_n \in A$. Then

$$H\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) + \sum_{j=2}^{n} H\left(\frac{\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j}{n}\right) = H(x_1) \tag{2.21}$$

for all $x_1, \ldots, x_n \in A$. Putting $s_1 = \sum_{i=1}^{n} x_i / n$ and $s_j = \sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j / n$ $(j = 2, 3, \ldots, n)$ in (2.21), we obtain

$$H\left(\sum_{j=1}^{n} s_j\right) = \sum_{j=1}^{n} H(s_j) \tag{2.22}$$

for all $s_1, \ldots, s_n \in A$. Setting $s_j = 0$ $(j = 3, 4, \ldots, n)$ in (2.22) to get

$$H(s_1 + s_2) = H(s_1) + H(s_2) \tag{2.23}$$

hence H is cauchy additive. Letting $x_i = x$ for all $i = 1, 2, \ldots, n$ in (2.2), we obtain

$$\|\mu f(x) - f(\mu x)\|_B \le \phi(x, x, \ldots, x) \tag{2.24}$$

for all $x \in A$. It follows that

$$\begin{aligned}
\|H(\mu x) - \mu H(x)\| &= \lim_{m\to\infty} \frac{1}{n^m}\|f(\mu n^m x) - \mu f(n^m x)\|_B \\
&\le \lim_{m\to\infty} \frac{1}{n^m}\phi(n^m x, n^m x, \ldots, n^m x) = 0
\end{aligned} \tag{2.25}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$, and all $x \in A$. One can show that the mapping $H : A \to B$ is $\mathbb{C}$-linear. Hence, $H : A \to B$ is an n-Lie homomorphism satisfying (2.5), as desired. □

Corollary 2.3. *Let θ and p be nonnegative real numbers such that $p < 1$. Suppose that a function $f : A \to B$ satisfies*

$$\left\| \mu f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) + \mu \sum_{j=2}^{n} f\left(\frac{\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j}{n}\right) - f(\mu x_1) \right\|_B \leq \theta \sum_{i=1}^{n} \left(\|x_i\|_A^p\right) \tag{2.26}$$

for all $\mu \in \mathbb{T}^1$ and all $x_1, \ldots, x_n \in A$ and

$$\left\| f([x_1 x_2 \cdots x_n]_A) - [f(x_1) f(x_2) \cdots f(x_n)]_B \right\|_B \leq \theta \sum_{i=1}^{n} \left(\|x_i\|_A^p\right) \tag{2.27}$$

for all $x_1, \ldots, x_n \in A$. Then there exists a unique n-Lie homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\|_B \leq \frac{2^p}{\ell(2 - 2^p)} \theta \|x\|_A^p \tag{2.28}$$

for all $x \in A$.

Proof. Put $\phi(x_1, x_2, \ldots, x_n) := \theta \sum_{i=1}^{n} (\|x_i\|_A^p)$ for all $x_1, \ldots, x_n \in A$ in Theorem 2.2. Then (2.9) holds for $p < 1$, and (2.28) holds when $L = 2^{(p-1)}$. □

Theorem 2.4. *Let $n_0 \in \mathbb{N}$ be a fixed positive integer number. Let $f : A \to B$ be a function for which there exists a function $\phi : A^n \to [0, \infty)$ such that*

$$\left\| \mu f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) + \mu \sum_{j=2}^{n} f\left(\frac{\sum_{i=1, i\neq j}^{n} x_i - (n-1)x_j}{n}\right) - f(\mu x_1) \right\|_B \leq \phi(x_1, x_2, \ldots, x_n) \tag{2.29}$$

for all $\mu \in (\mathbb{T}^1_{1/n_o} := \{e^{i\theta};\ 0 \leq \theta \leq 2\pi/n_o\} \cup \{1\})$ and all $x_1, \ldots, x_n \in A$, and that

$$\left\| f([x x \cdots x]_A) - [f(x) f(x) \cdots f(x)]_B \right\|_B \leq \phi(x, x, \ldots, x) \tag{2.30}$$

for all $x \in A$. If there exists an $L < 1$ such that

$$\phi(x_1, x_2, \ldots, x_n) \leq nL\phi\left(\frac{x_1}{n}, \frac{x_2}{n}, \ldots, \frac{x_n}{n}\right) \tag{2.31}$$

for all $x_1, \ldots, x_n \in A$, then there exists a unique Jordan n-Lie homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \leq \frac{L}{1 - L} \phi(x, 0, 0, \ldots, 0) \tag{2.32}$$

for all $x \in A$.

Proof. By the same reasoning as the proof of Theorem 2.2, we can define the mapping

$$H(x) = \lim_{m\to\infty} \frac{1}{n^{m\ell}} f\left(n^{m\ell} x\right) \tag{2.33}$$

for all $x \in A$. Moreover, we can show that H is $\mathbb{C}$-linear. The inequality (2.30) follows that

$$\begin{aligned} &\|H([xx\cdots x]_A) - [H(x)H(x)\cdots H(x)]_B\|_B \\ &\quad = \lim_{m\to\infty} \left\| \frac{1}{n^{nm}} H([n^m x \cdots n^m x]_A) - \frac{1}{n^{nm}} ([H(n^m x)H(n^m x)\cdots H(n^m x)]_B) \right\|_B \\ &\quad \le \lim_{m\to\infty} \frac{1}{n^{nm}} \phi(n^m x, n^m x, \ldots, n^m x) = 0 \end{aligned} \tag{2.34}$$

for all $x \in A$. So

$$H([xx\cdots x]_A) = [H(x)H(x)\cdots H(x)]_B \tag{2.35}$$

for all $x \in A$. Hence $H : A \to B$ is a Jordan n-Lie homomorphism satisfying (2.32). □

Corollary 2.5. *Let θ and p be nonnegative real numbers such that $p < 1$. Suppose that a function $f : A \to B$ satisfies*

$$\left\| \mu f\left(\frac{\sum_{i=1}^n x_i}{n} \right) + \mu \sum_{j=2}^n f\left(\frac{\sum_{i=1, i\ne j}^n x_i - (n-1)x_j}{n} \right) - f(\mu x_1) \right\|_B \le \theta \sum_{i=1}^n \left(\|x_i\|_A^p \right) \tag{2.36}$$

for all $\mu \in \mathbb{T}^1$ and all $x_1, \ldots, x_n \in A$ and

$$\left\| f([xx\cdots x]_A) - \left[f(x)f(x)\cdots f(x)\right]_B \right\|_B \le n\theta\left(\|x\|_A^p \right) \tag{2.37}$$

for all $x \in A$. Then there exists a unique Jordan n-Lie homomorphism $H : A \to B$ such that

$$\left\| f(x) - H(x) \right\|_B \le \frac{2^p}{\ell(2 - 2^p)} \theta \|x\|_A^p \tag{2.38}$$

for all $x \in A$.

Proof. It follows by Theorem 2.4 by putting $\phi(x_1, x_2, \ldots, x_n) := \theta \sum_{i=1}^n (\|x_i\|_A^p)$ for all $x_1, \ldots, x_n \in A$ and $L = 2^{(p-1)}$. □

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