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Research Article

Positive Solutions and Iterative Approximations for a Nonlinear Two-Dimensional Difference System with Multiple Delays

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This paper studies the nonlinear two-dimensional difference system with multiple delays $\Delta(x_n + p_{1n}x_{n-\tau_1}) + f_1(n, x_{a_{1n}}, \dots, x_{a_{hn}}, y_{b_{1n}}, \dots, y_{b_{kn}}) = q_{1n}, \Delta(y_n + p_{2n}y_{n-\tau_2}) + f_2(n, x_{c_{1n}}, \dots, x_{c_{hn}}, y_{d_{1n}}, \dots, y_{d_{kn}}) = q_{2n}, n \ge n_0$. Using the Banach fixed point theorem and a few new analysis techniques, we show the existence of uncountably many bounded positive solutions for the system, suggest Mann iterative algorithms with errors, and discuss the error estimates between the positive solutions and iterative sequences generated by the Mann iterative algorithms. Examples to illustrates the results are included.

1. Introduction and Preliminaries

In recent years, there has been an increasing interest in the study of oscillation, nonoscillation, asymptotic behavior, existence and multiplicity of solutions, positive solutions, nonoscillatory solutions, and periodic solutions, respectively, for various difference equations and systems, see for example, [1–34] and the references cited therein.

Jiang and Tang [18] and Graef and Thandapani [14] studied the oscillation of the linear two-dimensional difference system

$$\begin{aligned} \Delta x_n &= p_n y_n, \\ \Delta y_{n-1} &= -q_n x_n, \quad n \ge n_0, \end{aligned} \tag{1.1}$$

and nonlinear two-dimensional difference system

$$\begin{aligned}\Delta x_n &= b_n g(y_n),\\ \Delta y_{n-1} &= -a_n f(x_n), \quad n \geq n_0,\end{aligned} \tag{1.2}$$

where $n_0 \in \mathbb{N}$, $\{p_n\}_{n\geq n_0}$, $\{q_n\}_{n\geq n_0}$, $\{a_n\}_{n\geq n_0}$, and $\{b_n\}_{n\geq n_0}$ are nonnegative sequences, $f, g \in C(\mathbb{R}, \mathbb{R})$ with $uf(u) > 0$ and $ug(u) > 0$ for all $u \neq 0$. Jiang and Tang [19] also gave some necessary and sufficient conditions for all solutions of System (1.2) to be oscillatory. Agarwal et al. [2] discussed the two-dimensional nonlinear difference system of the form

$$\begin{aligned}\Delta x_n &= a_n f(y_n),\\ \Delta y_{n-1} &= b_n g(x_n), \quad n \geq n_0,\end{aligned} \tag{1.3}$$

where $n_0 \in \mathbb{N}$, $\{a_n\}_{n\geq n_0}$ and $\{b_n\}_{n\geq n_0}$ are positive sequences, $f, g \in C(\mathbb{R}, \mathbb{R})$ are increasing with $uf(u) > 0$ and $ug(u) > 0$ for all $u \neq 0$, and they provided a classification scheme of positive solutions for System (1.3) and established conditions for the existence of solutions with designated asymptotic behavior. Li [21] introduced the two-dimensional nonlinear difference system:

$$\begin{aligned}\Delta x_n &= a_n f(y_n),\\ \Delta y_n &= -b_n g(x_n), \quad n \geq n_0,\end{aligned} \tag{1.4}$$

where $n_0 \in \mathbb{N}$, $\{a_n\}_{n\geq n_0}$ and $\{b_n\}_{n\geq n_0}$ are nonnegative sequences, $f, g \in C(\mathbb{R}, \mathbb{R})$ with $uf(u) > 0$, and $ug(u) > 0$ for all $u \neq 0$, he showed both classification schemes for nonoscillatory solutions of System (1.4) and gave necessary and sufficient conditions for the existence of these solutions. Huo and Li [15, 16] considered the nonlinear two-dimensional difference system

$$\begin{aligned}\Delta x_n &= b_n g(y_n),\\ \Delta y_n &= -a_n f(x_n) + r_n, \quad n \geq n_0,\end{aligned} \tag{1.5}$$

and the Emden-Fowler difference system

$$\begin{aligned}\Delta x_n &= b_n g(y_n),\\ \Delta y_{n-1} &= -a_n f(x_n) + r_n, \quad n \geq 1,\end{aligned} \tag{1.6}$$

where $n_0 \in \mathbb{N}$, $\{a_n\}_{n\geq n_0}$ is a real sequence, $\{b_n\}_{n\geq n_0}$ is a nonnegative sequence, $\sum_{i=1}^{\infty} |r_i| < +\infty$, $f, g \in C(\mathbb{R}, \mathbb{R})$ with $uf(u) > 0$, and $ug(u) > 0$ for all $u \neq 0$, they proved some oscillation results for Systems (1.5) and (1.6). Jiang and Li [17] investigated the nonlinear two-dimensional difference system:

$$\begin{aligned}\Delta x_n &= a_n g(y_n),\\ \Delta y_{n-1} &= f(n, x_n), \quad n \geq n_0,\end{aligned} \tag{1.7}$$

where $n_0 \in \mathbb{N}$, $\{a_n\}_{n\geq n_0}$ is a nonnegative sequence, $f \in C(\mathbb{N}_{n_0} \times \mathbb{R}, \mathbb{R})$, $g \in C(\mathbb{R}, \mathbb{R})$ with $uf(n,u) > 0$ for all $n \in \mathbb{N}_{n_0}$, and $u \neq 0$ and $ug(u) > 0$ for all $u \neq 0$, they obtained some necessary and sufficient conditions for all solutions of System (1.7) to be oscillatory. Thandapani and Kumar [31] studied the oscillation for the nonlinear two-dimensional difference system of the neutral type

$$\begin{aligned} \Delta(x_n - a_n x_{\sigma_n}) &= p_n g(y_n), \\ \Delta y_n &= \delta q_n f(x_{\tau_n}), \quad n \geq n_0, \end{aligned} \tag{1.8}$$

where $\delta = \pm 1$, $n_0 \in \mathbb{N}$, $\{a_n\}_{n\geq n_0}$ is a positive sequence, $\{p_n\}_{n\geq n_0}$ and $\{q_n\}_{n\geq n_0}$ are nonnegative sequences with $\sum_{i=n_0}^{\infty} p_i = +\infty$, $\lim_{n\to\infty}\sigma_n = \lim_{n\to\infty}\tau_n = +\infty$, $f, g \in C(\mathbb{R}, \mathbb{R})$ are nondecreasing with $uf(u) > 0$ and $ug(u) > 0$ for all $u \neq 0$. Wu and Liu [33] established the existence and multiplicity of periodic solutions for the first-order neutral difference system

$$\begin{aligned} \Delta(x_n - cx_{n-\tau}) &= a_{1n} g_1(x_n) x_n - \lambda b_{1n} f_1(x_{n-\tau_{1n}}, y_{n-\rho_{1n}}), \\ \Delta(y_n - cy_{n-\tau}) &= a_{2n} g_2(y_n) y_n - \mu b_{2n} f_2(x_{n-\tau_{2n}}, y_{n-\rho_{2n}}), \end{aligned} \tag{1.9}$$

where $\tau \in \mathbb{N}$, $\lambda, \mu \in \mathbb{R}^+ \setminus \{0\}$, $c \in \mathbb{R}$ with $|c| \neq 1$, $\{a_{1n}\}$, $\{a_{2n}\}$, $\{b_{1n}\}$, and $\{b_{2n}\}$ are positive T-periodic sequences, $\{\tau_{1n}\}, \{\tau_{2n}\}, \{\rho_{1n}\}$, and $\{\rho_{2n}\}$ are positive T-periodic integer sequences. Tang [30] proved the existence results of a bounded nonoscillatory solution for the second-order linear delay difference equation

$$\Delta^2 x_n = p_n x_{n-k}, \quad n \geq 0, \tag{1.10}$$

where $k \in \mathbb{N}$ and $\{p_n\}_{n\geq 0}$ is a nonnegative sequence. Cheng [20] utilized the Banach fixed point theorem to discuss the existence of a nonoscillatory solution for the second-order neutral delay difference equation with positive and negative coefficients:

$$\Delta^2(x_n + px_{n-m}) + p_n x_{n-k} - q_n x_{n-l} = 0, \quad n \geq n_0, \tag{1.11}$$

where $m, k, l \in \mathbb{N}$, $p \in \mathbb{R} \setminus \{-1\}$, $\{p_n\}_{n\geq 0}$ and $\{q_n\}_{n\geq 0}$ are nonnegative sequences with $\sum_{n=n_0}^{\infty} n \max\{p_n, q_n\} < +\infty$. M. Migda and J. Migda [29] obtained the asymptotic behavior of the second-order neutral difference equation:

$$\Delta^2(x_n + px_{n-k}) + f(n, x_n) = 0, \quad n \geq 1, \tag{1.12}$$

where $p \in \mathbb{R}$, $k \in \mathbb{N}_0$, and $f \in C(\mathbb{N} \times \mathbb{R}, \mathbb{R})$. Liu et al. [27] studied the global existence of uncountably many bounded nonoscillatory solutions for the second-order nonlinear neutral delay difference equation:

$$\Delta(a_n \Delta(x_n + bx_{n-\tau})) + f(n, x_{n-d_{1n}}, x_{n-d_{2n}}, \ldots, x_{n-d_{kn}}) = c_n, \quad n \geq n_0, \tag{1.13}$$

where $b \in \mathbb{R}$, $\tau, k \in \mathbb{N}$, $n_0 \in \mathbb{N}_0$, $\{a_n\}_{n\in\mathbb{N}_{n_0}}$ is a positive sequence, $\{c_n\}_{n\in\mathbb{N}_{n_0}}$ is a real sequence, $\bigcup_{l=1}^{k}\{d_{ln}\}_{n\in\mathbb{N}_{n_0}} \subseteq \mathbb{Z}$ with $\lim_{n\to\infty}(n-d_{ln}) = +\infty$ for $l \in \{1,2,\ldots,k\}$, and $f : \mathbb{N}_{n_0} \times \mathbb{R}^k \to \mathbb{R}$ is a mapping.

The purpose of this paper is to study the below nonlinear two-dimensional difference system with multiple delays

$$\begin{aligned} &\Delta\left(x_n + p_{1n}x_{n-\tau_1}\right) + f_1\left(n, x_{a_{1n}}, \ldots, x_{a_{hn}}, y_{b_{1n}}, \ldots, y_{b_{kn}}\right) = q_{1n}, \\ &\Delta\left(y_n + p_{2n}y_{n-\tau_2}\right) + f_2\left(n, x_{c_{1n}}, \ldots, x_{c_{hn}}, y_{d_{1n}}, \ldots, y_{d_{kn}}\right) = q_{2n}, \quad n \ge n_0, \end{aligned} \tag{1.14}$$

where $h, k, \tau_i \in \mathbb{N}$, $n_0 \in \mathbb{N}_0$, $\{p_{in}\}_{n\in\mathbb{N}_{n_0}}, \{q_{in}\}_{n\in\mathbb{N}_{n_0}} \subset \mathbb{R}$, and $f_i \in C(\mathbb{N}_{n_0} \times \mathbb{R}^{h+k}, \mathbb{R})$ for $i \in \Lambda_2$, $\{a_{ln}\}_{n\in\mathbb{N}_{n_0}}, \{c_{ln}\}_{n\in\mathbb{N}_{n_0}} \subset \mathbb{Z}$ with $\lim_{n\to\infty}a_{ln} = \lim_{n\to\infty}c_{ln} = +\infty$ for $l \in \Lambda_h$ and $\{b_{jn}\}_{n\in\mathbb{N}_{n_0}}$, $\{d_{jn}\}_{n\in\mathbb{N}_{n_0}} \subset \mathbb{Z}$ with $\lim_{n\to\infty}b_{jn} = \lim_{n\to\infty}d_{jn} = +\infty$ for $j \in \Lambda_k$. It is easy to see that the system (1.14) includes Systems (1.1)–(1.9), (1.10)–(1.13), and a lot of the first- and second-order nonlinear, half-linear, and quasilinear difference equations as special cases. Using the Banach fixed point theorem and some analysis techniques, we prove the existence of uncountably many bounded positive solutions for System (1.14), establish their iterative approximations, and discuss the error estimates between the positive solutions and iterative approximations. Our results sharp, and improve [14, Theorem 1] and [27, Theorems 2.1–2.7]. To illustrate our results, fifteen examples are also included.

Throughout this paper, we assume that Δ is the forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$, $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{Z}$, $\mathbb{N}$, and $\mathbb{N}_0$ stand for the sets of all integers, positive integers, and nonnegative integers, respectively,

$$\begin{gathered} \mathbb{N}_{n_0} = \{n : n \in \mathbb{N}_0 \text{ with } n \ge n_0\}, \quad \Lambda_n = \{1, \ldots, n\}, \quad \forall n \in \mathbb{N}, \\ \alpha = \inf\left\{n_0 - \tau_1, n_0 - \tau_2, a_{ln}, c_{ln}, b_{jn}, d_{jn} : n \in \mathbb{N}_{n_0},\ l \in \Lambda_h,\ j \in \Lambda_k\right\}, \\ \mathbb{Z}_\alpha = \{n : n \in \mathbb{Z} \text{ with } n \ge \alpha\}, \end{gathered} \tag{1.15}$$

l_α^∞ denotes the Banach space of all bounded sequences on $\mathbb{Z}_\alpha$ with norm

$$\|x\| = \sup_{n\in\mathbb{Z}_\alpha}|x_n| \quad \text{for } x = \{x_n\}_{n\in\mathbb{Z}_\alpha} \in l_\alpha^\infty. \tag{1.16}$$

Let

$$\begin{aligned} &A(N, M) \\ &\quad = \left\{(x, y) = \left(\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}\right) \in l_\alpha^\infty \times l_\alpha^\infty : N \le x_n \le M,\ N \le y_n \le M,\ n \in \mathbb{Z}_\alpha\right\}, \\ &A(N, M, N_0, M_0) \\ &\quad = \left\{(x, y) = \left(\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}\right) \in l_\alpha^\infty \times l_\alpha^\infty : N \le x_n \le M,\ N_0 \le y_n \le M_0,\ n \in \mathbb{Z}_\alpha\right\}, \end{aligned} \tag{1.17}$$

for any $M > N > 0$ and $M_0 > N_0 > 0$. It is easy to see that $A(N, M)$ and $A(N, M, N_0, M_0)$ are bounded closed and convex subsets of the Banach space $l_\alpha^\infty \times l_\alpha^\infty$ with norm

$$\|(x, y)\|_1 = \max\{\|x\|, \|y\|\} \quad \text{for } (x, y) \in l_\alpha^\infty \times l_\alpha^\infty. \tag{1.18}$$

By a solution of System (1.14), we mean a sequence $(\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha})$ with a positive integer $T \geq n_0 + \tau_1 + \tau_2 + |\alpha|$ such that System (1.14) is satisfied for all $n \geq T$. A solution $(\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha})$ of System (1.14) is said to be positive if both components are positive.

Lemma 1.1 (see [35]). *Let $\{A_n\}_{n\geq 0}$, $\{B_n\}_{n\geq 0}$, $\{C_n\}_{n\geq 0}$, and $\{D_n\}_{n\geq 0}$ be four nonnegative real sequences satisfying the inequality*

$$A_{n+1} \leq (1 - D_n)A_n + D_n B_n + C_n, \quad \forall n \geq 0, \tag{1.19}$$

where $\{D_n\}_{n\geq 0} \subset [0,1]$, $\sum_{n=0}^\infty D_n = +\infty$, $\lim_{n\to\infty} B_n = 0$, and $\sum_{n=0}^\infty C_n < +\infty$. Then $\lim_{n\to\infty} A_n = 0$.

Lemma 1.2 (see [36]). *Let $\tau \in \mathbb{N}$, $n_0 \in \mathbb{N}_0$, and $\{B_n\}_{n\in\mathbb{N}_{n_0}}$ be a nonnegative sequence. Then*

$$\sum_{i=0}^{\infty} \sum_{n=n_0+i\tau}^{\infty} B_n < +\infty \Longleftrightarrow \sum_{n=n_0}^{\infty} nB_n < +\infty. \tag{1.20}$$

2. Main Results

In this section, we investigate the existence of uncountably many bounded positive solutions for System (1.14) and their iterative approximations and the error estimates between the positive solutions and iterative approximations.

Theorem 2.1. *Assume that there exist constants $M, N \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$, and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying*

$$\left|f_i(n, u_1, \ldots, u_h, v_1, \ldots, v_k)\right| \leq r_{in}, \quad \forall (n, u_l, v_j) \in \mathbb{N}_{n_1} \times [N, M]^2,\ l \in \Lambda_h,\ j \in \Lambda_k,\ i \in \Lambda_2, \tag{2.1}$$

$$\begin{gathered} \left|f_i(n, u_1, \ldots, u_h, v_1, \ldots, v_k) - f_i(n, w_1, \ldots, w_h, z_1, \ldots, z_k)\right| \\ \leq t_{in} \max\{|u_l - w_l|, |v_j - z_j| : l \in \Lambda_h, j \in \Lambda_k\}, \\ \forall (n, u_l, w_l, v_j, z_j) \in \mathbb{N}_{n_1} \times [N, M]^4,\ l \in \Lambda_h,\ j \in \Lambda_k,\ i \in \Lambda_2, \end{gathered} \tag{2.2}$$

$$\sum_{n=n_0}^{\infty} n \max\{r_{in}, t_{in}, |q_{in}| : i \in \Lambda_2\} < +\infty, \tag{2.3}$$

$$N < M, \qquad p_{in} = -1, \quad \forall n \geq n_1,\ i \in \Lambda_2. \tag{2.4}$$

Then,

(a) *for each* $L \in (N, M)$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \ge \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by the schemes*

$$x_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)x_n^\mu \\ \quad +\beta_\mu\left\{L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ \quad +\gamma_\mu\delta_{1n}^\mu, & \mu \ge 0,\ n \ge T_L, \\ (1-\beta_\mu-\gamma_\mu)x_{T_L}^\mu \\ \quad +\beta_\mu\left\{L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_1}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ \quad +\gamma_\mu\delta_{1T_L}^\mu, & \mu \ge 0,\ \alpha \le n < T_L, \end{cases} \tag{2.5}$$

$$y_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)y_n^\mu \\ \quad +\beta_\mu\left\{L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu\delta_{2n}^\mu, & \mu \ge 0,\ n \ge T_L, \\ (1-\beta_\mu-\gamma_\mu)y_{T_L}^\mu \\ \quad +\beta_\mu\left\{L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu\delta_{2T_L}^\mu, & \mu \ge 0,\ \alpha \le n < T_L \end{cases} \tag{2.6}$$

converges to a bounded positive solution $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate:*

$$\left\|\left(x^{\mu+1}, y^{\mu+1}\right) - (x, y)\right\|_1 \le (1 - \beta_\mu(1-\theta_L))\left\|(x^\mu, y^\mu) - (x, y)\right\|_1 + 2M\gamma_\mu, \quad \forall \mu \ge 0, \tag{2.7}$$

where $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$ *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N,M)$ *are arbitrary sequences with*

$$\sum_{\mu=0}^{\infty}\beta_\mu = +\infty, \tag{2.8}$$

$$\begin{gathered}\sum_{\mu=0}^{\infty}\gamma_\mu < +\infty \text{ or there exists a sequence } \{\eta_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,+\infty) \text{ satisfying} \\ \gamma_\mu = \beta_\mu\eta_\mu, \quad \forall\mu\in\mathbb{N}_0, \quad \lim_{\mu\to\infty}\eta_\mu = 0,\end{gathered} \tag{2.9}$$

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N,M)$.

Proof. (a) Let $L \in (N,M)$. Now we construct a contraction mapping $S_L : A(N,M) \to A(N,M)$ and prove that its fixed point is a bounded positive solution of System (1.14). It follows from (2.3) and (2.4) that there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1,\tau_2\}+n_1+|\alpha|$ satisfying

$$\theta_L = \sum_{m=T_L}^{\infty}(1+m)\max\{t_{im} : i\in\Lambda_2\}, \tag{2.10}$$

$$\sum_{m=T_L}^{\infty}(1+m)\max\{r_{im}+|q_{im}| : i\in\Lambda_2\} < \min\{M-L, L-N\}. \tag{2.11}$$

Define three mappings $S_L : A(N,M) \to l_\alpha^\infty \times l_\alpha^\infty$, $S_{1L}, S_{2L} : A(N,M) \to l_\alpha^\infty$ by

$$S_{1L}(x_n,y_n) = \begin{cases} L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left[f_1(m, x_{a_{1m}},\ldots,x_{a_{hm}}, y_{b_{1m}},\ldots,y_{b_{km}}) - q_{1m}\right], & n\geq T_L, \\ S_{1L}(x_{T_L}, y_{T_L}), & \alpha\leq n< T_L, \end{cases} \tag{2.12}$$

$$S_{2L}(x_n,y_n) = \begin{cases} L - \displaystyle\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left[f_2(m, x_{c_{1m}},\ldots,x_{c_{hm}}, y_{d_{1m}},\ldots,y_{d_{km}}) - q_{2m}\right], & n\geq T_L, \\ S_{2L}(x_{T_L}, y_{T_L}), & \alpha\leq n< T_L, \end{cases} \tag{2.13}$$

$$S_L(x_n,y_n) = (S_{1L}(x_n,y_n), S_{2L}(x_n,y_n)), \quad n\in\mathbb{Z}_\alpha, \tag{2.14}$$

for all $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$. It follows from (2.1), (2.3), (2.12), (2.13), and Lemma 1.2 that S_{1L} and S_{2L} are well defined.

In view of (2.1), (2.2), and (2.10)–(2.14), we know that for any $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$ and $(u,v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$,

$$
\begin{aligned}
&\left\|S_L(x,y) - S_L(u,v)\right\|_1 \\
&\quad = \max\left\{\left\|S_{iL}(x,y) - S_{iL}(u,v)\right\| : i \in \Lambda_2\right\} \\
&\quad = \max\left\{\sup_{n\ge T_L}\left|S_{iL}(x_n,y_n) - S_{iL}(u_n,v_n)\right| : i \in \Lambda_2\right\} \\
&\quad \le \max\left\{\sup_{n\ge T_L}\left\{\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left|f_1(m,x_{a_{1m}},\dots,x_{a_{hm}},y_{b_{1m}},\dots,y_{b_{km}})\right.\right.\right. \\
&\qquad\qquad\qquad\qquad \left.\left. - f_1(m,u_{a_{1m}},\dots,u_{a_{hm}},v_{b_{1m}},\dots,v_{b_{km}})\right|\right\}, \\
&\qquad\quad \sup_{n\ge T_L}\left\{\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left|f_2(m,x_{c_{1m}},\dots,x_{c_{hm}},y_{db_{1m}},\dots,y_{d_{km}})\right.\right. \\
&\qquad\qquad\qquad\qquad \left.\left.\left. - f_2(m,u_{c_{1m}},\dots,u_{c_{hm}},v_{d_{1m}},\dots,v_{d_{km}})\right|\right\}\right\} \\
&\quad \le \max\left\{\sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_1}^{\infty} t_{1m}\max\left\{\left|x_{a_{im}} - u_{a_{im}}\right|, \left|y_{b_{jm}} - v_{b_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_k\right\},\right. \\
&\qquad\quad \left.\sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_2}^{\infty} t_{2m}\max\left\{\left|x_{c_{im}} - u_{c_{im}}\right|, \left|y_{d_{jm}} - v_{d_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_k\right\}\right\} \\
&\quad \le \max\left\{\sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_1}^{\infty} t_{1m}, \sum_{l=0}^{\infty}\sum_{m=T_L+(l+1)\tau_2}^{\infty} t_{2m}\right\}\max\{\|x-u\|, \|y-v\|\} \\
&\quad \le \max\left\{\sum_{m=T_L}^{\infty}\left(1+\frac{m}{\tau_1}\right)t_{1m}, \sum_{m=T_L}^{\infty}\left(1+\frac{m}{\tau_2}\right)t_{2m}\right\}\left\|(x,y)-(u,v)\right\|_1 \\
&\quad \le \left(\sum_{m=T_L}^{\infty}(1+m)\max\{t_{1m},t_{2m}\}\right)\left\|(x,y)-(u,v)\right\|_1 \\
&\quad = \theta_L\left\|(x,y)-(u,v)\right\|_1, \\
&\max\left\{\left|S_{1L}(x_n,y_n) - L\right|, \left|S_{2L}(x_n,y_n) - L\right|\right\} \\
&\quad \le \max\left\{\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left|f_1(m,x_{a_{1m}},\dots,x_{a_{hm}},y_{b_{1m}},\dots,y_{b_{km}}) - q_{1m}\right|,\right. \\
&\qquad\quad \left.\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left|f_2(m,x_{c_{1m}},\dots,x_{c_{hm}},y_{d_{1m}},\dots,y_{d_{km}}) - q_{2m}\right|\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\le \max\left\{\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left(r_{1m}+\left|q_{1m}\right|\right), \sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left(r_{2m}+\left|q_{2m}\right|\right)\right\} \\
&\le \max\left\{\sum_{m=T_L}^{\infty}\left(1+\frac{m}{\tau_1}\right)\left(r_{1m}+\left|q_{1m}\right|\right), \sum_{m=T_L}^{\infty}\left(1+\frac{m}{\tau_2}\right)\left(r_{2m}+\left|q_{2m}\right|\right)\right\} \\
&\le \sum_{m=T_L}^{\infty}(1+m)\max\left\{r_{im}+\left|q_{im}\right| : i\in\Lambda_2\right\} \\
&< \min\{M-L, L-N\}, \quad \forall n \ge T_L,
\end{aligned}
\tag{2.15}
$$

which give that

$$
\left\|S_L(x,y)-S_L(u,v)\right\|_1 \le \theta_L\left\|(x,y)-(u,v)\right\|_1, \quad \forall (x,y),(u,v)\in A(N,M), \tag{2.16}
$$

$$
N \le S_{1L}(x_n,y_n) \le M, \quad N \le S_{2L}(x_n,y_n) \le M, \quad \forall\left(\{x_n\}_{n\in\mathbb{Z}_\alpha},\{y_n\}_{n\in\mathbb{Z}_\alpha}\right)\in A(N,M),\ n\ge T_L. \tag{2.17}
$$

Observe that $\theta_L \in (0,1)$. It is easy to see that (2.12)–(2.17) give that $S_L : A(N,M) \to A(N,M)$ is a contraction. Consequently, S_L possesses a unique fixed point $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha},\{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$, which implies that

$$
\begin{aligned}
x_n &= L-\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left[f_1\left(m,x_{a_{1m}},\ldots,x_{a_{hm}},y_{b_{1m}},\ldots,y_{b_{km}}\right)-q_{1m}\right], \quad n\ge T_L, \\
y_n &= L-\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_2}^{\infty}\left[f_2\left(m,x_{c_{1m}},\ldots,x_{c_{hm}},y_{d_{1m}},\ldots,y_{d_{km}}\right)-q_{2m}\right], \quad n\ge T_L,
\end{aligned}
\tag{2.18}
$$

which yield that

$$
\begin{aligned}
x_n - x_{n-\tau_1} &= L-\sum_{l=0}^{\infty}\sum_{m=n+(l+1)\tau_1}^{\infty}\left[f_1\left(m,x_{a_{1m}},\ldots,x_{a_{hm}},y_{b_{1m}},\ldots,y_{b_{km}}\right)-q_{1m}\right] \\
&\quad - L+\sum_{l=0}^{\infty}\sum_{m=n+l\tau_1}^{\infty}\left[f_1\left(m,x_{a_{1m}},\ldots,x_{a_{hm}},y_{b_{1m}},\ldots,y_{b_{km}}\right)-q_{1m}\right] \\
&= \sum_{m=n}^{\infty}\left[f_1\left(m,x_{a_{1m}},\ldots,x_{a_{hm}},y_{b_{1m}},\ldots,y_{b_{km}}\right)-q_{1m}\right], \quad \forall n\ge T_L+\tau_1,
\end{aligned}
$$

$$
\begin{aligned}
y_n - y_{n-\tau_2} &= L - \sum_{l=0}^{\infty} \sum_{m=n+(l+1)\tau_2}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right] \\
&\quad - L + \sum_{l=0}^{\infty} \sum_{m=n+l\tau_2}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right] \\
&= \sum_{m=n}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right], \quad \forall n \geq T_L + \tau_2,
\end{aligned}
\tag{2.19}
$$

which give that

$$
\begin{aligned}
\Delta\left(x_n - x_{n-\tau_1}\right) &= -f_1\left(n, x_{a_{1n}}, \ldots, x_{a_{hn}}, y_{b_{1n}}, \ldots, y_{b_{kn}}\right) + q_{1n}, \quad \forall n \geq T_L + \tau_1, \\
\Delta\left(y_n - y_{n-\tau_2}\right) &= -f_2\left(n, x_{c_{1n}}, \ldots, x_{c_{hn}}, y_{d_{1n}}, \ldots, y_{d_{kn}}\right) + q_{2n}, \quad \forall n \geq T_L + \tau_2,
\end{aligned}
\tag{2.20}
$$

that is, (x, y) is a bounded positive solution of System (1.14) in $A(N, M)$.

In light of (2.5), (2.6), and (2.11)–(2.16), we arrive at

$$
\begin{aligned}
&\left\| \left(x^{\mu+1}, y^{\mu+1}\right) - (x, y) \right\|_1 \\
&= \max\left\{ \sup_{n \geq T_L} \left| x_n^{\mu+1} - x_n \right|, \sup_{n \geq T_L} \left| y_n^{\mu+1} - y_n \right| \right\} \\
&\leq \max\Bigg\{ \sup_{n \geq T_L} \Bigg[\left(1 - \beta_\mu - \gamma_\mu\right) \left| x_n^\mu - x_n \right| \\
&\qquad + \beta_\mu \left| L - \sum_{l=0}^{\infty} \sum_{m=n+(l+1)\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}^\mu, \ldots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \ldots, y_{b_{km}}^\mu\right) - q_{1m} \right] - x_n \right| \\
&\qquad + \gamma_\mu \left| \delta_{1n}^\mu - x_n \right| \Bigg], \\
&\qquad \sup_{n \geq T_L} \Bigg[\left(1 - \beta_\mu - \gamma_\mu\right) \left| y_n^\mu - y_n \right| \\
&\qquad + \beta_\mu \left| L - \sum_{l=0}^{\infty} \sum_{m=n+(l+1)\tau_2}^{\infty} \left[f_2\left(m, x_{c_{1m}}^\mu, \ldots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \ldots, y_{d_{km}}^\mu\right) - q_{2m} \right] - y_n \right| \\
&\qquad + \gamma_\mu \left| \delta_{2n}^\mu - y_n \right| \Bigg] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\leq \max\left\{\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|x_n^\mu - x_n\right| + \beta_\mu\left|S_{1L}\left(x_n^\mu, y_n^\mu\right) - S_{1L}(x_n, y_n)\right| + 2M\gamma_\mu\right],\right.\\
&\qquad\left.\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|y_n^\mu - y_n\right| + \beta_\mu\left|S_{2L}\left(x_n^\mu, y_n^\mu\right) - S_{2L}(x_n, y_n)\right| + 2M\gamma_\mu\right]\right\}\\
&\leq \max\{(1-\beta_\mu-\gamma_\mu)\|x^\mu - x\| + \theta_L\beta_\mu\|(x^\mu, y^\mu) - (x,y)\|_1 + 2M\gamma_\mu,\\
&\qquad (1-\beta_\mu-\gamma_\mu)\|y^\mu - y\| + \theta_L\beta_\mu\|(x^\mu, y^\mu) - (x,y)\|_1 + 2M\gamma_\mu\}\\
&\leq (1-\beta_\mu(1-\theta_L))\|(x^\mu, y^\mu) - (x,y)\|_1 + 2M\gamma_\mu, \quad \forall \mu \geq 0,
\end{aligned}
\tag{2.21}
$$

that is, (2.7) holds. It follows from (2.7)–(2.9) and Lemma 1.1 that $\lim_{\mu\to\infty}\|(x^\mu, y^\mu)-(x,y)\|_1 = 0$.

(b) Let $L_1, L_2 \in (N, M)$, and $L_1 \neq L_2$. As in the proof of (a), we similarly infer that for each $i \in \Lambda_2$, there exist a constant $\theta_{L_i} \in (0,1)$, a positive integer $T_{L_i} \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$, and mappings $S_{L_i}, S_{1L_i}, S_{2L_i}$ satisfying (2.10)–(2.14), where L, θ_L and T_L are replaced by L_i, θ_{L_i} and T_{L_i}, respectively, and the contraction mappings S_{L_1} and S_{L_2} have the unique fixed points $(u,v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha})$, $(w,z) = (\{w_n\}_{n\in\mathbb{Z}_\alpha}, \{z_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$, respectively, (u,v) and (w,z) are bounded positive solutions of System (1.14) in $A(N,M)$. In order to show that System (1.14) possesses uncountably many bounded positive solutions in $A(N,M)$, we prove only that $(u,v) \neq (w,z)$. By means of (2.10) and (2.12)–(2.16), we conclude that

$$
\begin{aligned}
&\|(u,v)-(w,z)\|_1\\
&= \max\{\|u-w\|, \|v-z\|\}\\
&\geq \max\left\{\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|u_n - w_n|, \sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|v_n - z_n|\right\}\\
&\geq \max\left\{|L_1 - L_2| - \sum_{l=0}^{\infty}\sum_{m=\max\{T_{L_1},T_{L_2}\}+(l+1)\tau_1}^{\infty}\Big|f_1(m, u_{a_{1m}}, \ldots, u_{a_{hm}}, v_{b_{1m}}, \ldots, v_{b_{km}})\right.\\
&\qquad\qquad - f_1(m, w_{a_{1m}}, \ldots, w_{a_{hm}}, z_{b_{1m}}, \ldots, z_{b_{km}})\Big|,\\
&\qquad |L_1 - L_2| - \sum_{l=0}^{\infty}\sum_{m=\max\{T_{L_1},T_{L_2}\}+(l+1)\tau_2}^{\infty}\Big|f_2(m, u_{c_{1m}}, \ldots, u_{c_{hm}}, v_{d_{1m}}, \ldots, v_{d_{km}})\\
&\qquad\qquad \left. - f_2(m, w_{c_{1m}}, \ldots, w_{c_{hm}}, z_{d_{1m}}, \ldots, z_{d_{km}})\Big|\right\}\\
&\geq \max\left\{|L_1 - L_2| - \sum_{l=0}^{\infty}\sum_{m=\max\{T_{L_1},T_{L_2}\}+(l+1)\tau_1}^{\infty} t_{1m}\max\left\{|u_{a_{im}} - w_{a_{im}}|, \left|v_{b_{jm}} - z_{b_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_K\right\},\right.
\end{aligned}
$$

$$
\begin{aligned}
&\qquad |L_1 - L_2| - \sum_{l=0}^{\infty} \sum_{m=\max\{T_{L_1}, T_{L_2}\}+(l+1)\tau_2}^{\infty} t_{2m} \\
&\qquad \times \max\left\{|u_{c_{im}} - w_{c_{im}}|, \left|v_{d_{jm}} - z_{d_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_K\right\}\Bigg\} \\
&\geq \max\Bigg\{|L_1 - L_2| - \sum_{l=0}^{\infty} \sum_{m=\max\{T_{L_1}, T_{L_2}\}+(l+1)\tau_1}^{\infty} t_{1m} \max\{\|u - w\|, \|v - z\|\}, \\
&\qquad |L_1 - L_2| - \sum_{l=0}^{\infty} \sum_{m=\max\{T_{L_1}, T_{L_2}\}+(l+1)\tau_2}^{\infty} t_{2m} \max\{\|u - w\|, \|v - z\|\}\Bigg\} \\
&\geq |L_1 - L_2| - \left(\sum_{m=\max\{T_{L_1}, T_{L_2}\}}^{\infty} (1+m) \max\{t_{im} : i \in \Lambda_2\}\right) \|(u,v) - (w,z)\|_1 \\
&\geq |L_1 - L_2| - \max\{\theta_{L_1}, \theta_{L_2}\} \|(u,v) - (w,z)\|_1,
\end{aligned}
\tag{2.22}
$$

which implies that

$$
\|(u,v) - (w,z)\|_1 \geq \frac{|L_1 - L_2|}{1 + \max\{\theta_{L_1}, \theta_{L_2}\}} > 0, \tag{2.23}
$$

that is, $(u,v) \neq (w,z)$. This completes the proof. □

Theorem 2.2. *Assume that there exist constants $M, N \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$, and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying* (2.1), (2.2),

$$
\sum_{n=n_0}^{\infty} \max\{r_{in}, t_{in}, |q_{in}| : i \in \Lambda_2\} < +\infty, \tag{2.24}
$$

$$
N < M, \qquad p_{in} = 1, \quad \forall n \geq n_1,\ i \in \Lambda_2. \tag{2.25}
$$

Then,

(a) *for each* $L \in (N, M)$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by the schemes*

$$x_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)x_n^\mu \\ +\beta_\mu\left\{L+\displaystyle\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ +\gamma_\mu\delta_{1n}^\mu, & \mu \geq 0,\ n \geq T_L, \\ (1-\beta_\mu-\gamma_\mu)x_{T_L}^\mu \\ +\beta_\mu\left\{L+\displaystyle\sum_{l=1}^{\infty}\sum_{m=T_L+(2l-1)\tau_1}^{T+2l\tau_1-1}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ +\gamma_\mu\delta_{1T_L}^\mu, & \mu \geq 0,\ \alpha \leq n < T_L, \end{cases} \tag{2.26}$$

$$y_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)y_n^\mu \\ +\beta_\mu\left\{L+\displaystyle\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ +\gamma_\mu\delta_{2n}^\mu, & \mu \geq 0,\ n \geq T_L, \\ (1-\beta_\mu-\gamma_\mu)y_{T_L}^\mu \\ +\beta_\mu\left\{L+\displaystyle\sum_{l=1}^{\infty}\sum_{m=T_L+(2l-1)\tau_2}^{T_L+2l\tau_2-1}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ +\gamma_\mu\delta_{2T_L}^\mu, & \mu \geq 0,\ \alpha \leq n < T_L \end{cases} \tag{2.27}$$

converges to a bounded positive solution $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Proof. (a) Let $L \in (N, M)$. Now we construct a contraction mapping $S_L : A(N, M) \to A(N, M)$ and prove that its fixed point is a bounded positive solution of System (1.14). Note that (2.24) and (2.25) guarantee that there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ satisfying

$$\theta_L = \sum_{m=T_L}^{\infty} \max\{t_{im} : i \in \Lambda_2\}, \tag{2.28}$$

$$\sum_{m=T_L}^{\infty} \max\{r_{im} + |q_{im}| : i \in \Lambda_2\} < \min\{M - L, L - N\}. \tag{2.29}$$

Define three mappings $S_L : A(N,M) \to l_\alpha^\infty \times l_\alpha^\infty$, $S_{1L}, S_{2L} : A(N,M) \to l_\alpha^\infty$ by (2.14),

$$S_{1L}(x_n, y_n) = \begin{cases} L + \displaystyle\sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1} \left[f_1\left(m, x_{a_{1m}}, \dots, x_{a_{hm}}, y_{b_{1m}}, \dots, y_{b_{km}}\right) - q_{1m} \right], & n \geq T_L, \\ S_{1L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.30}$$

$$S_{2L}(x_n, y_n) = \begin{cases} L + \displaystyle\sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1} \left[f_2\left(m, x_{c_{1m}}, \dots, x_{c_{hm}}, y_{d_{1m}}, \dots, y_{d_{km}}\right) - q_{2m} \right], & n \geq T_L, \\ S_{2L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.31}$$

for all $(x, y) = (\{x_n\}_{n \in \mathbb{Z}_\alpha}, \{y_n\}_{n \in \mathbb{Z}_\alpha}) \in A(N,M)$.

Using (2.1), (2.2), (2.14), and (2.28)–(2.31), we deduce that for any $(x, y) = (\{x_n\}_{n \in \mathbb{Z}_\alpha}, \{y_n\}_{n \in \mathbb{Z}_\alpha})$ and $(u, v) = (\{u_n\}_{n \in \mathbb{Z}_\alpha}, \{v_n\}_{n \in \mathbb{Z}_\alpha}) \in A(N,M)$,

$$\begin{aligned}
&\left\| S_L(x,y) - S_L(u,v) \right\|_1 \\
&\quad = \max\left\{ \left\| S_{iL}(x,y) - S_{iL}(u,v) \right\| : i \in \Lambda_2 \right\} \\
&\quad = \max\left\{ \sup_{n \geq T_L} \left| S_{iL}(x_n, y_n) - S_{iL}(u_n, v_n) \right| : i \in \Lambda_2 \right\} \\
&\quad \leq \max\left\{ \sup_{n \geq T_L} \left\{ \sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_1}^{m=n+2l\tau_1-1} \left| f_1\left(m, x_{a_{1m}}, \dots, x_{a_{hm}}, y_{b_{1m}}, \dots, y_{b_{km}}\right) \right.\right.\right. \\
&\qquad\qquad \left.\left. - f_1\left(m, u_{a_{1m}}, \dots, u_{a_{hm}}, v_{b_{1m}}, \dots, v_{b_{km}}\right) \right| \right\}, \\
&\qquad\quad \sup_{n \geq T_L} \left\{ \sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_2}^{m=n+2l\tau_2-1} \left| f_2\left(m, x_{c_{1m}}, \dots, x_{c_{hm}}, y_{d_{1m}}, \dots, y_{d_{km}}\right) \right.\right. \\
&\qquad\qquad \left.\left.\left. - f_2\left(m, u_{c_{1m}}, \dots, u_{c_{hm}}, v_{d_{1m}}, \dots, v_{d_{km}}\right) \right| \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
&\le \max\Bigg\{\sum_{m=T_L+\tau_1}^{\infty} t_{1m}\max\Big\{|x_{a_{im}}-u_{a_{im}}|,\Big|y_{b_{jm}}-v_{b_{jm}}\Big| : i\in\Lambda_h, j\in\Lambda_k\Big\},\\
&\qquad\sum_{m=T_L+\tau_2}^{\infty} t_{2m}\max\Big\{|x_{c_{im}}-u_{c_{im}}|,\Big|y_{d_{jm}}-v_{d_{jm}}\Big| : i\in\Lambda_h, j\in\Lambda_k\Big\}\Bigg\}\\
&\le \max\Bigg\{\sum_{m=T_L+\tau_1}^{\infty} t_{1m},\sum_{m=T_L+\tau_2}^{\infty} t_{2m}\Bigg\}\max\{\|x-u\|,\|y-v\|\}\\
&\le \Bigg(\sum_{m=T_L}^{\infty}\max\{t_{im} : i\in\Lambda_2\}\Bigg)\|(x,y)-(u,v)\|_1\\
&= \theta_L\|(x,y)-(u,v)\|_1,\\
\max\{&|S_{1L}(x_n,y_n)-L|,|S_{2L}(x_n,y_n)-L|\}\\
&\le \max\Bigg\{\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1}\left|f_1(m,x_{a_{1m}},\dots,x_{a_{hm}},y_{b_{1m}},\dots,y_{b_{km}})-q_{1m}\right|,\\
&\qquad\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1}\left|f_2(m,x_{c_{1m}},\dots,x_{c_{hm}},y_{d_{1m}},\dots,y_{d_{km}})-q_{2m}\right|\Bigg\}\\
&\le \max\Bigg\{\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1}(r_{1m}+|q_{1m}|),\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1}(r_{2m}+|q_{2m}|)\Bigg\}\\
&\le \max\Bigg\{\sum_{m=T_L+\tau_1}^{\infty}(r_{1m}+|q_{1m}|),\sum_{m=T_L+\tau_2}^{\infty}(r_{2m}+|q_{2m}|)\Bigg\}\\
&\le \sum_{m=T_L}^{\infty}\max\{r_{im}+|q_{im}| : i\in\Lambda_2\}\\
&< \min\{M-L,L-N\},\quad \forall n\ge T_L,
\end{aligned} \tag{2.32}$$

which imply (2.16) and (2.17) and which ensure that S_L is a self-mapping from $A(N,M)$ into itself and is a contraction by $\theta_L\in(0,1)$. The Banach fixed point theorem means that S_L has a unique fixed point $(x,y)=(\{x_n\}_{n\in\mathbb{Z}_\alpha},\{y_n\}_{n\in\mathbb{Z}_\alpha})\in A(N,M)$, that is,

$$\begin{aligned}
x_n &= L+\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1}\left[f_1(m,x_{a_{1m}},\dots,x_{a_{hm}},y_{b_{1m}},\dots,y_{b_{km}})-q_{1m}\right],\quad n\ge T_L,\\
y_n &= L+\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1}\left[f_2(m,x_{c_{1m}},\dots,x_{c_{hm}},y_{d_{1m}},\dots,y_{d_{km}})-q_{2m}\right],\quad n\ge T_L,
\end{aligned} \tag{2.33}$$

which reveal that

$$
\begin{aligned}
x_n + x_{n-\tau_1} &= L + \sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&\quad + L + \sum_{l=1}^{\infty} \sum_{m=n+2(l-1)\tau_1}^{n+(2l-1)\tau_1-1} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&= 2L + \sum_{m=n}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right], \quad \forall n \geq T_L + \tau_1, \\
y_n + y_{n-\tau_2} &= L + \sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right] \\
&\quad + L + \sum_{l=1}^{\infty} \sum_{m=n+2(l-1)\tau_2}^{n+(2l-1)\tau_2-1} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right] \\
&= 2L + \sum_{m=n}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right], \quad \forall n \geq T_L + \tau_2,
\end{aligned}
\tag{2.34}
$$

which yield that

$$
\begin{aligned}
\Delta\left(x_n - x_{n-\tau_1}\right) &= -f_1\left(n, x_{a_{1n}}, \ldots, x_{a_{hn}}, y_{b_{1n}}, \ldots, y_{b_{kn}}\right) + q_{1n}, \quad \forall n \geq T_L + \tau_1, \\
\Delta\left(y_n - y_{n-\tau_2}\right) &= -f_2\left(n, x_{c_{1n}}, \ldots, x_{c_{hn}}, y_{d_{1n}}, \ldots, y_{d_{kn}}\right) + q_{2n}, \quad \forall n \geq T_L + \tau_2,
\end{aligned}
\tag{2.35}
$$

that is, (x, y) is a bounded positive solution of System (1.14) in $A(N, M)$.

It follows from (2.14), (2.16), (2.26)–(2.28), (2.30), and (2.31) that

$$
\begin{aligned}
&\left\| \left(x^{\mu+1}, y^{\mu+1}\right) - (x, y) \right\|_1 \\
&= \max\left\{ \sup_{n \geq T_L} \left| x_n^{\mu+1} - x_n \right|, \sup_{n \geq T_L} \left| y_n^{\mu+1} - y_n \right| \right\} \\
&\leq \max\left\{ \sup_{n \geq T_L} \left[\left(1 - \beta_\mu - \gamma_\mu\right) \left| x_n^\mu - x_n \right| \right.\right. \\
&\qquad + \beta_\mu \left| L + \sum_{l=1}^{\infty} \sum_{m=n+(2l-1)\tau_1}^{n+2l\tau_1-1} \left[f_1\left(m, x_{a_{1m}}^\mu, \ldots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \ldots, y_{b_{km}}^\mu\right) - q_{1m} \right] - x_n \right| \\
&\qquad \left. + \gamma_\mu \left| \delta_{1n}^\mu - x_n \right| \right],
\end{aligned}
$$

$$
\begin{aligned}
&\sup_{n\geq T_L}\Bigg[(1-\beta_\mu-\gamma_\mu)\left|y_n^\mu-y_n\right| \\
&\qquad+\beta_\mu\left|L+\sum_{l=1}^{\infty}\sum_{m=n+(2l-1)\tau_2}^{n+2l\tau_2-1}\left[f_2\left(m,x^\mu_{c_{1m}},\dots,x^\mu_{c_{hm}},y^\mu_{d_{1m}},\dots,y^\mu_{d_{km}}\right)-q_{2m}\right]-y_n\right| \\
&\qquad+\ \gamma_\mu\left|\delta^\mu_{2n}-y_n\right|\Bigg]\Bigg]\Bigg\} \\
&\leq\max\left\{\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|x_n^\mu-x_n\right|+\beta_\mu\left|S_{1L}\left(x_n^\mu,y_n^\mu\right)-S_{1L}(x_n,y_n)\right|+2M\gamma_\mu\right],\right. \\
&\qquad\left.\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|y_n^\mu-y_n\right|+\beta_\mu\left|S_{2L}\left(x_n^\mu,y_n^\mu\right)-S_{2L}(x_n,y_n)\right|+2M\gamma_\mu\right]\right\} \\
&\leq(1-\beta_\mu(1-\theta_L))\left\|(x^\mu,y^\mu)-(x,y)\right\|_1+2M\gamma_\mu,\quad\forall\mu\geq0,
\end{aligned}
\tag{2.36}
$$

which implies (2.7). Thus Lemma 1.1 and (2.7)–(2.9) mean that $\lim_{\mu\to\infty}\|(x^\mu,y^\mu)-(x,y)\|_1=0$.

(b) Let $L_1,L_2\in(N,M)$ and $L_1\neq L_2$. As in the proof of (a), we conclude that for each $i\in\Lambda_2$, there exist a constant $\theta_{L_i}\in(0,1)$, a positive integer $T_{L_i}\geq\max\{\tau_1,\tau_2\}+n_1+|\alpha|$, and mappings S_{L_i}, S_{1L_i}, S_{2L_i} satisfying (2.14) and (2.28)–(2.31), where L, θ_L, and T_L are replaced by L_i, θ_{L_i}, and T_{L_i}, respectively, and the contraction mappings S_{L_1} and S_{L_2} have the unique fixed points $(u,v)=(\{u_n\}_{n\in\mathbb{Z}_\alpha},\{v_n\}_{n\in\mathbb{Z}_\alpha})$, $(w,z)=(\{w_n\}_{n\in\mathbb{Z}_\alpha},\{z_n\}_{n\in\mathbb{Z}_\alpha})\in A(N,M)$, respectively, (u,v) and (w,z) are bounded positive solutions of the system (1.14) in $A(N,M)$. In order to show that System (1.14) possesses uncountably many bounded positive solutions in $A(N,M)$, we prove only that $(u,v)\neq(w,z)$. By virtue of (2.14) and (2.28)–(2.31), we infer that

$$
\begin{aligned}
&\|(u,v)-(w,z)\|_1 \\
&=\max\{\|u-w\|,\|v-z\|\} \\
&\geq\max\left\{\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|u_n-w_n|,\ \sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|v_n-z_n|\right\} \\
&\geq\max\left\{|L_1-L_2|-\sum_{l=1}^{\infty}\sum_{m=\max\{T_{L_1},T_{L_2}\}+(2l-1)\tau_1}^{\max\{T_{L_1},T_{L_2}\}+2l\tau_1-1}\left|f_1(m,u_{a_{1m}},\dots,u_{a_{hm}},v_{b_{1m}},\dots,v_{b_{km}})\right.\right. \\
&\qquad\left.-f_1\left(m,w_{a_{1m}},\dots,w_{a_{hm}},z_{b_{1m}},\dots,z_{b_{km}}\right)\right|,
\end{aligned}
$$

$$
\begin{aligned}
&|L_1 - L_2| - \sum_{l=1}^{\infty} \sum_{m=\max\{T_{L_1},T_{L_2}\}+(2l-1)\tau_2}^{\max\{T_{L_1},T_{L_2}\}+2l\tau_2-1} \big| f_2(m, u_{c_{1m}}, \dots, u_{c_{hm}}, v_{d_{1m}}, \dots, v_{d_{km}}) \\
&\qquad\qquad\qquad\qquad - f_2\left(m, w_{c_{1m}}, \dots, w_{c_{hm}}, z_{d_{1m}}, \dots, z_{d_{km}}\right)\big| \Bigg\} \\
&\geq \max\Bigg\{ |L_1 - L_2| - \sum_{m=\max\{T_{L_1},T_{L_2}\}+\tau_1}^{\infty} t_{1m} \max\left\{ |u_{a_{im}} - w_{a_{im}}|, \left| v_{b_{jm}} - z_{b_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_K \right\}, \\
&\qquad |L_1 - L_2| - \sum_{m=\max\{T_{L_1},T_{L_2}\}+\tau_2}^{\infty} t_{2m} \max\left\{ |u_{c_{im}} - w_{c_{im}}|, \left| v_{d_{jm}} - z_{d_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_K \right\} \Bigg\} \\
&\geq \max\Bigg\{ |L_1 - L_2| - \sum_{m=\max\{T_{L_1},T_{L_2}\}+\tau_1}^{\infty} t_{1m} \max\{\|u - w\|, \|v - z\|\}, \\
&\qquad |L_1 - L_2| - \sum_{m=\max\{T_{L_1},T_{L_2}\}+\tau_2}^{\infty} t_{2m} \max\{\|u - w\|, \|v - z\|\} \Bigg\} \\
&\geq |L_1 - L_2| - \left(\sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} \max\{t_{im} : i \in \Lambda_2\} \right) \|(u, v) - (w, z)\|_1 \\
&\geq |L_1 - L_2| - \max\{\theta_{L_1}, \theta_{L_2}\} \|(u, v) - (w, z)\|_1,
\end{aligned}
\tag{2.37}
$$

which yields that

$$
\|(u, v) - (w, z)\|_1 \geq \frac{|L_1 - L_2|}{1 + \max\{\theta_{L_1}, \theta_{L_2}\}} > 0, \tag{2.38}
$$

that is, $(u, v) \neq (w, z)$. This completes the proof. □

Theorem 2.3. *Assume that there exist constants $M, N \in \mathbb{R}^+ \setminus \{0\}$, $\overline{p}, \underline{p} \in \mathbb{R}^+$, $n_1 \in \mathbb{N}_{n_0}$, and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying* (2.1), (2.2), (2.24), *and*

$$
N < \left(1 - \underline{p} - \overline{p}\right) M, \quad \underline{p} + \overline{p} < 1, \quad -\underline{p} \leq p_{in} \leq \overline{p}, \quad \forall n \geq n_1,\ i \in \Lambda_2. \tag{2.39}
$$

Then,

(a) *for each* $L \in (N + \overline{p}M, M(1 - \underline{p}))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n \in \mathbb{Z}_\alpha}, \{y_n^0\}_{n \in \mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu \in \mathbb{N}_0} = \{(\{x_n^\mu\}_{n \in \mathbb{Z}_\alpha}, \{y_n^\mu\}_{n \in \mathbb{Z}_\alpha})\}_{\mu \in \mathbb{N}_0}$ *generated by the schemes*

$$x_n^{\mu+1} = \begin{cases} (1 - \beta_\mu - \gamma_\mu) x_n^\mu \\ \quad + \beta_\mu \Big\{ L - p_{1\mu} x_{n-\tau_1}^\mu \\ \qquad + \displaystyle\sum_{m=n}^{\infty} \Big[f_1\Big(m, x_{a_{1m}}^\mu, \ldots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \ldots, y_{b_{km}}^\mu\Big) - q_{1m}\Big] \Big\} \\ \quad + \gamma_\mu \delta_{1n}^\mu, & \mu \geq 0, \ n \geq T_L, \\ (1 - \beta_\mu - \gamma_\mu) x_{T_L}^\mu \\ \quad + \beta_\mu \Big\{ L - p_{1\mu} x_{T_L-\tau_1}^\mu \\ \qquad + \displaystyle\sum_{m=T_L}^{\infty} \Big[f_1\Big(m, x_{a_{1m}}^\mu, \ldots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \ldots, y_{b_{km}}^\mu\Big) - q_{1m}\Big] \Big\} \\ \quad + \gamma_\mu \delta_{1T_L}^\mu, & \mu \geq 0, \ \alpha \leq n < T_L, \end{cases} \tag{2.40}$$

$$y_n^{\mu+1} = \begin{cases} (1 - \beta_\mu - \gamma_\mu) y_n^\mu \\ \quad + \beta_\mu \Big\{ L - p_{2\mu} y_{n-\tau_2}^\mu \\ \qquad + \displaystyle\sum_{m=n}^{\infty} \Big[f_2\Big(m, x_{c_{1m}}^\mu, \ldots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \ldots, y_{d_{km}}^\mu\Big) - q_{2m}\Big] \Big\} \\ \quad + \gamma_\mu \delta_{2n}^\mu, & \mu \geq 0, \ n \geq T_L, \\ (1 - \beta_\mu - \gamma_\mu) y_{T_L}^\mu \\ \quad + \beta_\mu \Big\{ L - p_{2\mu} y_{T_L-\tau_2}^\mu \\ \qquad + \displaystyle\sum_{m=T_L}^{\infty} \Big[f_2\Big(m, x_{c_{1m}}^\mu, \ldots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \ldots, y_{d_{km}}^\mu\Big) - q_{2m}\Big] \Big\} \\ \quad + \gamma_\mu \delta_{2T_L}^\mu, & \mu \geq 0, \ \alpha \leq n < T_L \end{cases} \tag{2.41}$$

converges to a bounded positive solution $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu \in \mathbb{N}_0}, \{\gamma_\mu\}_{\mu \in \mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu \in \mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n \in \mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n \in \mathbb{Z}_\alpha})\}_{\mu \in \mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Proof. (a) Let $L \in (N + \overline{p}M, M(1 - \underline{p}))$. Now we construct a contraction mapping $S_L : A(N, M) \to A(N, M)$ and prove that its fixed point is a bounded positive solution of the system (1.14). Notice that (2.24) and (2.39) mean that there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ satisfying

$$\theta_L = \underline{p} + \overline{p} + \sum_{m=T_L}^{\infty} \max\{t_{im} : i \in \Lambda_2\}, \tag{2.42}$$

$$\sum_{m=T_L}^{\infty} \max\{r_{im} + |q_{im}| : i \in \Lambda_2\} < \min\Big\{M\Big(1-\underline{p}\Big) - L, L - N - \overline{p}M\Big\}. \tag{2.43}$$

Define three mappings $S_L : A(N,M) \to l_\alpha^\infty \times l_\alpha^\infty$, $S_{1L}, S_{2L} : A(N,M) \to l_\alpha^\infty$ by (2.14),

$$S_{1L}(x_n, y_n) = \begin{cases} L - p_{1n}x_{n-\tau_1} + \sum_{m=n}^{\infty}\left[f_1(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}) - q_{1m}\right], & n \geq T_L, \\ S_{1L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.44}$$

$$S_{2L}(x_n, y_n) = \begin{cases} L - p_{2n}y_{n-\tau_2} + \sum_{m=n}^{\infty}\left[f_2(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}) - q_{2m}\right], & n \geq T_L, \\ S_{2L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.45}$$

for all $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$.

It follows from (2.1), (2.2), (2.14), and (2.42)–(2.45) that for any $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha})$ and $(u,v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$,

$$\begin{aligned}
&\|S_L(x,y) - S_L(u,v)\|_1 \\
&\quad = \max\{\|S_{iL}(x,y) - S_{iL}(u,v)\| : i \in \Lambda_2\} \\
&\quad = \max\left\{\sup_{n\geq T_L}|S_{iL}(x_n,y_n) - S_{iL}(u_n,v_n)| : i \in \Lambda_2\right\} \\
&\quad \leq \max\left\{\sup_{n\geq T_L}\left\{|p_{1n}(x_{n-\tau_1} - u_{n-\tau_1})| + \sum_{m=n}^{\infty}\left|f_1(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}})\right.\right.\right. \\
&\qquad\qquad \left.\left. - f_1\ (m, u_{a_{1m}}, \ldots, u_{a_{hm}}, v_{b_{1m}}, \ldots, v_{b_{km}})\right|\right\}, \\
&\qquad \sup_{n\geq T_L}\left\{|p_{2n}(y_{n-\tau_2} - v_{n-\tau_2})| + \sum_{m=n}^{\infty}\left|f_2(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}})\right.\right. \\
&\qquad\qquad \left.\left.\left. - f_2\ (m, u_{c_{1m}}, \ldots, u_{c_{hm}}, v_{d_{1m}}, \ldots, v_{d_{km}})\right|\right\}\right\} \\
&\quad \leq \max\left\{\Big(\underline{p} + \overline{p}\Big)\|x-u\| + \sum_{m=T_L}^{\infty} t_{1m}\max\Big\{|x_{a_{lm}} - u_{a_{lm}}|, \Big|y_{b_{jm}} - v_{b_{jm}}\Big| : l \in \Lambda_h, j \in \Lambda_k\Big\},\right.
\end{aligned}$$

$$
\begin{aligned}
&\qquad \left(\underline{p}+\overline{p}\right)\|y-v\| + \sum_{m=T_L}^{\infty} t_{2m}\max\left\{|x_{c_{lm}}-u_{c_{lm}}|, \left|y_{d_{jm}}-v_{d_{jm}}\right| : l\in\Lambda_h, j\in\Lambda_k\right\}\Bigg\} \\
&\le \max\left\{\left(\underline{p}+\overline{p}\right)\|x-u\| + \sum_{m=T_L}^{\infty} t_{1m}\max\{\|x-u\|, \|y-v\|\},\right. \\
&\qquad \left.\left(\underline{p}+\overline{p}\right)\|y-v\| + \sum_{m=T_L}^{\infty} t_{2m}\max\{\|x-u\|, \|y-v\|\}\right\} \\
&\le \left(\underline{p}+\overline{p}+\sum_{m=T_L}^{\infty}\max\{t_{im} : i\in\Lambda_2\}\right)\|(x,y)-(u,v)\|_1 \\
&= \theta_L\|(x,y)-(u,v)\|_1, \\
&S_{1L}(x_n,y_n) \\
&\quad = L - p_{1n}x_{n-\tau_1} + \sum_{m=n}^{\infty}\left[f_1(m, x_{a_{1m}},\ldots,x_{a_{hm}}, y_{b_{1m}},\ldots,y_{b_{km}}) - q_{1m}\right] \\
&\quad \le L + \underline{p}M + \sum_{m=T_L}^{\infty}(r_{1m}+|q_{1m}|) \\
&\quad \le L + \underline{p}M + \min\left\{M\left(1-\underline{p}\right)-L, L-N-\overline{p}M\right\} \\
&\quad \le M, \quad \forall n\ge T_L, \\
&S_{1L}(x_n,y_n) \\
&\quad = L - p_{1n}x_{n-\tau_1} + \sum_{m=n}^{\infty}\left[f_1(m, x_{a_{1m}},\ldots,x_{a_{hm}}, y_{b_{1m}},\ldots,y_{b_{km}}) - q_{1m}\right] \\
&\quad \ge L - \overline{p}M - \sum_{m=T_L}^{\infty}(r_{1m}+|q_{1m}|) \\
&\quad \ge L - \overline{p}M - \min\left\{M\left(1-\underline{p}\right)-L, L-N-\overline{p}M\right\} \\
&\quad \ge N, \quad \forall n\ge T_L, \\
&N \le S_{2L}(x_n,y_n)\le M, \quad \forall n\ge T_L,
\end{aligned} \tag{2.46}
$$

which give (2.16) and (2.17), which together with $\theta_L\in(0,1)$ guarantee that $S_L : A(N,M)\to A(N,M)$ is a contraction. Thus the Banach fixed point theorem ensures that S_L has a unique fixed point $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha})\in A(N,M)$, that is,

$$
\begin{aligned}
x_n &= L - p_{1n}x_{n-\tau_1} + \sum_{m=n}^{\infty}\left[f_1(m, x_{a_{1m}},\ldots,x_{a_{hm}}, y_{b_{1m}},\ldots,y_{b_{km}}) - q_{1m}\right], \\
y_n &= L - p_{2n}y_{n-\tau_2} + \sum_{m=n}^{\infty}\left[f_2(m, x_{c_{1m}},\ldots,x_{c_{hm}}, y_{d_{1m}},\ldots,y_{d_{km}}) - q_{2m}\right], \quad n\ge T_L,
\end{aligned} \tag{2.47}
$$

which yield that

$$\Delta\left(x_n + p_{1n}x_{n-\tau_1}\right) = -f_1\left(n, x_{a_{1n}}, \ldots, x_{a_{hn}}, y_{b_{1n}}, \ldots, y_{b_{kn}}\right) + q_{1n},$$
$$\Delta\left(y_n + p_{2n}y_{n-\tau_2}\right) = f_2\left(n, x_{c_{1n}}, \ldots, x_{c_{hn}}, y_{d_{1n}}, \ldots, y_{d_{kn}}\right) + q_{2n}, \quad \forall n \geq T_L, \tag{2.48}$$

that is, (x, y) is a bounded positive solution of System (1.14) in $A(N, M)$.

In light of (2.14), (2.16), (2.40), (2.41), (2.44), and (2.45), we deduce that

$$\begin{aligned}
&\left\|\left(x^{\mu+1}, y^{\mu+1}\right) - (x, y)\right\|_1 \\
&= \max\left\{\sup_{n\geq T_L}\left|x_n^{\mu+1} - x_n\right|, \sup_{n\geq T_L}\left|y_n^{\mu+1} - y_n\right|\right\} \\
&\leq \max\left\{\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|x_n^\mu - x_n\right|\right.\right. \\
&\qquad\qquad + \beta_\mu\left|L - p_{1n}x_{n-\tau_1} + \sum_{m=n}^{\infty}\left[f_1\left(m, x^\mu_{a_{1m}}, \ldots, x^\mu_{a_{hm}}, y^\mu_{b_{1m}}, \ldots, y^\mu_{b_{km}}\right) - q_{1m}\right] - x_n\right| \\
&\qquad\qquad \left. + \gamma_\mu\left|\delta^\mu_{1n} - x_n\right|\right], \\
&\qquad\quad \sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|y_n^\mu - y_n\right|\right. \\
&\qquad\qquad\quad + \beta_\mu\left|L - p_{2n}y_{n-\tau_2} + \sum_{m=n}^{\infty}\left[f_2\left(m, x^\mu_{c_{1m}}, \ldots, x^\mu_{c_{hm}}, y^\mu_{d_{1m}}, \ldots, y^\mu_{d_{km}}\right) - q_{2m}\right] - y_n\right| \\
&\qquad\qquad\quad \left.\left. + \gamma_\mu\left|\delta^\mu_{2n} - y_n\right|\right]\right\} \\
&\leq \max\left\{\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\|x^\mu - x\| + \beta_\mu\left|S_{1L}\left(x_n^\mu, y_n^\mu\right) - S_{1L}(x_n, y_n)\right| + 2M\gamma_\mu\right],\right. \\
&\qquad\quad \left.\sup_{n\geq T_L}\left[(1-\beta_\mu-\gamma_\mu)\|y^\mu - y\| + \beta_\mu\left|S_{2L}\left(x_n^\mu, y_n^\mu\right) - S_{2L}(x_n, y_n)\right| + 2M\gamma_\mu\right]\right\} \\
&\leq \left(1 - \beta_\mu(1-\theta_L)\right)\left\|(x^\mu, y^\mu) - (x, y)\right\|_1 + 2M\gamma_\mu, \quad \forall \mu \geq 0,
\end{aligned} \tag{2.49}$$

which implies (2.7). Thus Lemma 1.1 and (2.7)–(2.9) imply that $\lim_{\mu\to\infty}\|(x^\mu, y^\mu) - (x, y)\|_1 = 0$.

(b) Let $L_1, L_2 \in (N, M)$ and $L_1 \neq L_2$. Similar to the proof of (a), we know that for each $i \in \Lambda_2$, there exist a constant $\theta_{L_i} \in (0,1)$, a positive integer $T_{L_i} \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ and mappings S_{L_i}, S_{1L_i}, S_{2L_i} satisfying (2.14) and (2.42)–(2.45), where L, θ_L, and T_L are replaced by L_i, θ_{L_i} and T_{L_i}, respectively, and the contraction mappings S_{L_1} and S_{L_2} have the unique fixed points $(u,v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha})$, $(w,z) = (\{w_n\}_{n\in\mathbb{Z}_\alpha}, \{z_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$, which are bounded positive solutions of System (1.14) in $A(N,M)$, respectively. In order to show that System (1.14) possesses uncountably many bounded positive solutions in $A(N,M)$, we prove only that $(u,v) \neq (w,z)$. In terms of (2.14) and (2.42)–(2.45), we get that

$$
\begin{aligned}
&\|(u,v)-(w,z)\|_1 \\
&= \max\{\|u-w\|, \|v-z\|\} \\
&\geq \max\left\{ \sup_{n\geq \max\{T_{L_1},T_{L_2}\}} |u_n - w_n|, \sup_{n\geq \max\{T_{L_1},T_{L_2}\}} |v_n - z_n| \right\} \\
&\geq \max\left\{ \sup_{n\geq \max\{T_{L_1},T_{L_2}\}} \left\{ |L_1 - L_2| - \left|p_{1n}\right| |u_{n-\tau_1} - w_{n-\tau_1}| \right.\right. \\
&\qquad\qquad - \sum_{m=n}^{\infty} \left| f_1(m, u_{a_{1m}}, \dots, u_{a_{hm}}, v_{b_{1m}}, \dots, v_{b_{km}}) \right. \\
&\qquad\qquad\qquad \left.\left. - f_1(m, w_{a_{1m}}, \dots, w_{a_{hm}}, z_{b_{1m}}, \dots, z_{b_{km}}) \right| \right\}, \\
&\qquad \sup_{n\geq \max\{T_{L_1},T_{L_2}\}} \left\{ |L_1 - L_2| - \left|p_{2n}\right| |v_{n-\tau_2} - z_{n-\tau_2}| \right. \\
&\qquad\qquad - \sum_{m=n}^{\infty} \left| f_2(m, u_{c_{1m}}, \dots, u_{c_{hm}}, v_{d_{1m}}, \dots, v_{d_{km}}) \right. \\
&\qquad\qquad\qquad \left.\left.\left. - f_2(m, w_{c_{1m}}, \dots, w_{c_{hm}}, z_{d_{1m}}, \dots, z_{d_{km}}) \right| \right\} \right\} \\
&\geq \max\left\{ |L_1 - L_2| - \left(\underline{p} + \overline{p}\right) \|u - w\| \right. \\
&\qquad - \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{1m} \max\left\{ |u_{a_{im}} - w_{a_{im}}|, \left| v_{b_{jm}} - z_{b_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_K \right\}, \\
&\qquad |L_1 - L_2| - \left(\underline{p} + \overline{p}\right) \|v - z\|
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{2m} \max\left\{ |u_{c_{im}} - w_{c_{im}}|, \left| v_{d_{jm}} - z_{d_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_K \right\} \Bigg\} \\
&\geq \max\Bigg\{ |L_1 - L_2| - \left(\underline{p} + \overline{p}\right) \|u - w\| - \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{1m} \max\{\|u - w\|, \|v - z\|\}, \\
&\qquad |L_1 - L_2| - \left(\underline{p} + \overline{p}\right) \|v - z\| - \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{2m} \max\{\|u - w\|, \|v - z\|\} \Bigg\} \\
&\geq |L_1 - L_2| - \left(\underline{p} + \overline{p} + \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} \max\{t_{im} : i \in \Lambda_2\} \right) \|(u,v) - (w,z)\|_1 \\
&\geq |L_1 - L_2| - \max\{\theta_{L_1}, \theta_{L_2}\} \|(u,v) - (w,z)\|_1,
\end{aligned}
\tag{2.50}
$$

which yields that

$$
\|(u,v) - (w,z)\|_1 \geq \frac{|L_1 - L_2|}{1 + \max\{\theta_{L_1}, \theta_{L_2}\}} > 0, \tag{2.51}
$$

that is, $(u,v) \neq (w,z)$. This completes the proof. □

Theorem 2.4. *Assume that there exist constants $M, N, \overline{p}, \underline{p} \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying* (2.1), (2.2), (2.24), *and*

$$
0 < N\left(\underline{p} - 1\right) < M(\overline{p} - 1), \quad -\underline{p} \leq p_{in} \leq -\overline{p}, \quad \forall n \geq n_1,\ i \in \Lambda_2. \tag{2.52}
$$

Then,

(a) *for each $L \in (N(\underline{p} - 1), M(\overline{p} - 1))$, there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ such that for any $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, the Mann iterative* *sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by the schemes*

$$x_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)x_n^\mu \\ \quad +\beta_\mu\left\{\dfrac{-L}{p_{1n+\tau_1}} - \dfrac{x_{n+\tau_1}^\mu}{p_{1n+\tau_1}} + \dfrac{1}{p_{1n+\tau_1}} \right. \\ \qquad \left. \times \displaystyle\sum_{m=n+\tau_1}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ \quad +\gamma_\mu \delta_{1n}^\mu, & \mu \ge 0,\ n \ge T_L, \\ (1-\beta_\mu-\gamma_\mu)x_{T_L}^\mu \\ \quad +\beta_\mu\left\{\dfrac{-L}{p_{1T_L+\tau_1}} - \dfrac{x_{T_L+\tau_1}^\mu}{p_{1T_L+\tau_1}} + \dfrac{1}{p_{1T_L+\tau_1}} \right. \\ \qquad \left. \times \displaystyle\sum_{m=T_L+\tau_1}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right]\right\} \\ \quad +\gamma_\mu \delta_{1T_L}^\mu, & \mu \ge 0,\ \alpha \le n < T_L, \end{cases} \tag{2.53}$$

$$y_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)y_n^\mu \\ \quad +\beta_\mu\left\{\dfrac{-L}{p_{2n+\tau_2}} - \dfrac{y_{n+\tau_2}^\mu}{p_{2n+\tau_2}} + \dfrac{1}{p_{2n+\tau_2}} \right. \\ \qquad \left. \times \displaystyle\sum_{m=n+\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu \delta_{2n}^\mu, & \mu \ge 0,\ n \ge T_L, \\ (1-\beta_\mu-\gamma_\mu)y_{T_L}^\mu \\ \quad +\beta_\mu\left\{\dfrac{-L}{p_{2T_L+\tau_2}} - \dfrac{y_{T_L+\tau_2}^\mu}{p_{2T_L+\tau_2}} + \dfrac{1}{p_{2T_L+\tau_2}} \right. \\ \qquad \left. \times \displaystyle\sum_{m=T_L+\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu \delta_{2T_L}^\mu, & \mu \ge 0,\ \alpha \le n < T_L \end{cases} \tag{2.54}$$

converges to a bounded positive solution $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7)*, where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$ *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9)*,*

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$*.*

Proof. (a) Let $L \in (N(\underline{p} - 1), M(\overline{p} - 1))$. Now we construct a contraction mapping $S_L : A(N, M) \to A(N, M)$ and prove that its fixed point is a bounded positive solution of the system (1.14). Observe that (2.24) and (2.52) guarantee that there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ satisfying

$$\theta_L = \frac{1}{\overline{p}}\left(1 + \sum_{m=\mathrm{T}_L}^{\infty} \max\{t_{im} : i \in \Lambda_2\}\right), \tag{2.55}$$

$$\sum_{m=T_L}^{\infty} \max\{r_{im} + |q_{im}| : i \in \Lambda_2\} < \min\left\{M(\overline{p} - 1) - L, \overline{p}\left(\frac{L+N}{\underline{p}} - N\right)\right\}. \tag{2.56}$$

Define three mappings $S_L : A(N,M) \to l_\alpha^\infty \times l_\alpha^\infty$, $S_{1L}, S_{2L} : A(N,M) \to l_\alpha^\infty$ by (2.14),

$$S_{1L}(x_n, y_n) = \begin{cases} \dfrac{-L}{p_{1n+\tau_1}} - \dfrac{x_{n+\tau_1}}{p_{1n+\tau_1}} \\ \quad + \dfrac{1}{p_{1n+\tau_1}} \displaystyle\sum_{m=n+\tau_1}^{\infty} \left[f_1(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}) - q_{1m}\right], & n \geq T_L, \\ S_{1L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.57}$$

$$S_{2L}(x_n, y_n) = \begin{cases} \dfrac{-L}{p_{2n+\tau_2}} - \dfrac{y_{n+\tau_2}}{p_{2n+\tau_2}} \\ \quad + \dfrac{1}{p_{2n+\tau_2}} \displaystyle\sum_{m=n+\tau_2}^{\infty} \left[f_2(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}) - q_{2m}\right], & n \geq T_L, \\ S_{2L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.58}$$

for all $(x, y) = (\{x_n\}_{n \in \mathbb{Z}_\alpha}, \{y_n\}_{n \in \mathbb{Z}_\alpha}) \in A(N,M)$.

It follows from (2.1), (2.2), (2.14), and (2.55)–(2.58) that for any $(x,y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha})$ and $(u,v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N,M)$

$$\begin{aligned}
&\left\|S_L(x,y) - S_L(u,v)\right\|_1 \\
&= \max\left\{\left\|S_{iL}(x,y) - S_{iL}(u,v)\right\| : i \in \Lambda_2\right\} \\
&= \max\left\{\sup_{n\ge T_L}\left|S_{iL}(x_n,y_n) - S_{iL}(u_n,v_n)\right| : i \in \Lambda_2\right\} \\
&\le \max\left\{\sup_{n\ge T_L}\left\{\frac{1}{\left|p_{1n+\tau_1}\right|}\left(|x_{n+\tau_1} - u_{n+\tau_1}|\right.\right.\right. \\
&\qquad\qquad + \sum_{m=n+\tau_1}^{\infty}\left|f_1(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}})\right. \\
&\qquad\qquad\qquad \left.\left.\left. - f_1\ (m, u_{a_{1m}}, \ldots, u_{a_{hm}}, v_{b_{1m}}, \ldots, v_{b_{km}})\right|\right)\right\}, \\
&\qquad \sup_{n\ge T_L}\left\{\frac{1}{\left|p_{2n+\tau_2}\right|}\left(\left|y_{n+\tau_2} - v_{n+\tau_2}\right|\right.\right. \\
&\qquad\qquad + \sum_{m=n+\tau_2}^{\infty}\left|f_2(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}})\right. \\
&\qquad\qquad\qquad \left.\left.\left.\left. - f_2\ (m, u_{c_{1m}}, \ldots, u_{c_{hm}}, v_{d_{1m}}, \ldots, v_{d_{km}})\right|\right)\right\}\right\} \\
&\le \frac{1}{p}\max\left\{\|x-u\| + \sum_{m=T_L}^{\infty} t_{1m}\max\left\{|x_{a_{im}} - u_{a_{im}}|, \left|y_{b_{jm}} - v_{b_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_k\right\},\right. \\
&\qquad \left.\|y-v\| + \sum_{m=T_L}^{\infty} t_{2m}\max\left\{|x_{c_{im}} - u_{c_{im}}|, \left|y_{d_{jm}} - v_{d_{jm}}\right| : i \in \Lambda_h, j \in \Lambda_k\right\}\right\} \\
&\le \frac{1}{p}\max\left\{\|x-u\| + \sum_{m=T_L}^{\infty} t_{1m}\max\{\|x-u\|, \|y-v\|\},\right. \\
&\qquad \left.\|y-v\| + \sum_{m=T_L}^{\infty} t_{2m}\max\{\|x-u\|, \|y-v\|\}\right\} \\
&\le \frac{1}{p}\left(1 + \sum_{m=T_L}^{\infty}\max\{t_{im} : i \in \Lambda_2\}\right)\left\|(x,y) - (u,v)\right\|_1 \\
&= \theta_L\left\|(x,y) - (u,v)\right\|_1,
\end{aligned}$$

$$
\begin{aligned}
&S_{1L}(x_n, y_n) \\
&\quad = \frac{-L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&\quad \le \frac{L}{\overline{p}} + \frac{M}{\overline{p}} + \frac{1}{\overline{p}} \sum_{m=T_L+\tau_1}^{\infty} \left(r_{1m} + |q_{1m}| \right) \\
&\quad \le \frac{L}{\overline{p}} + \frac{M}{\overline{p}} + \frac{1}{\overline{p}} \min\left\{ M(\overline{p}-1) - L, \overline{p}\left(\frac{L+N}{\underline{p}} - N \right) \right\} \\
&\quad \le M, \quad \forall n \ge T_L, \\
&S_{1L}(x_n, y_n) \\
&\quad = \frac{-L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&\quad \ge \frac{L}{\underline{p}} + \frac{N}{\underline{p}} - \frac{1}{\overline{p}} \sum_{m=T_L+\tau_1}^{\infty} \left(r_{1m} + |q_{1m}| \right) \\
&\quad \ge \frac{L}{\underline{p}} + \frac{N}{\underline{p}} - \frac{1}{\overline{p}} \min\left\{ M(\overline{p}-1) - L, \overline{p}\left(\frac{L+N}{\underline{p}} - N \right) \right\} \\
&\quad \ge N, \quad \forall n \ge T_L, \\
&N \le S_{2L}(x_n, y_n) \le M, \quad \forall n \ge T_L,
\end{aligned}
\tag{2.59}
$$

which yield (2.16) and (2.17), which together with $\theta_L \in (0,1)$ ensure that S_L is a contraction in $A(N, M)$. By the Banach fixed point theorem, we deduce that S_L has a unique fixed point $(x, y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, that is,

$$
\begin{aligned}
x_n &= \frac{-L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right], \quad n \ge T_L, \\
y_n &= \frac{-L}{p_{2n+\tau_2}} - \frac{y_{n+\tau_2}}{p_{2n+\tau_2}} + \frac{1}{p_{2n+\tau_2}} \sum_{m=n+\tau_2}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right], \quad n \ge T_L,
\end{aligned}
\tag{2.60}
$$

which yield that

$$
\begin{aligned}
x_n + p_{1n} x_{n-\tau_1} &= -L + \sum_{m=n}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right], \quad n \ge T_L + \tau_1, \\
y_n + p_{2n} y_{n-\tau_2} &= -L + \sum_{m=n}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right], \quad n \ge T_L + \tau_2,
\end{aligned}
$$

$$\Delta\left(x_n + p_{1n}x_{n-\tau_1}\right) = -f_1\left(n, x_{a_{1n}}, \dots, x_{a_{hn}}, y_{b_{1n}}, \dots, y_{b_{kn}}\right) + q_{1n}, \quad \forall n \ge T_L + \tau_1,$$
$$\Delta\left(y_n + p_{2n}y_{n-\tau_2}\right) = -f_2\left(n, x_{c_{1n}}, \dots, x_{c_{hn}}, y_{d_{1n}}, \dots, y_{d_{kn}}\right) + q_{2n}, \quad \forall n \ge T_L + \tau_2, \tag{2.61}$$

that is, (x, y) is a bounded positive solution of the system (1.14) in $A(N, M)$.

In light of (2.14), (2.16), (2.53)–(2.55), (2.57), and (2.58), we get that

$$\begin{aligned}
&\left\|\left(x^{\mu+1}, y^{\mu+1}\right) - (x, y)\right\|_1 \\
&= \max\left\{\sup_{n\ge T_L}\left|x_n^{\mu+1} - x_n\right|, \sup_{n\ge T_L}\left|y_n^{\mu+1} - y_n\right|\right\} \\
&\le \max\left\{\sup_{n\ge T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|x_n^\mu - x_n\right| + \beta_\mu\right.\right. \\
&\qquad\qquad \times\left|\frac{-L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}^\mu}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}}\sum_{m=n}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m}\right] - x_n\right| \\
&\qquad\qquad \left.+ \gamma_\mu\left|\delta_{1n}^\mu - x_n\right|\right], \\
&\qquad \sup_{n\ge T_L}\left[(1-\beta_\mu-\gamma_\mu)\left|y_n^\mu - y_n\right| + \beta_\mu\right. \\
&\qquad\qquad \times\left|\frac{-L}{p_{2n+\tau_2}} - \frac{y_{n+\tau_2}^\mu}{p_{2n+\tau_2}} + \frac{1}{p_{2n+\tau_2}}\sum_{m=n}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m}\right] - y_n\right| \\
&\qquad\qquad \left.\left.+ \gamma_\mu\left|\delta_{2n}^\mu - y_n\right|\right]\right\} \\
&\le \max\left\{\sup_{n\ge T_L}\left[(1-\beta_\mu-\gamma_\mu)\|x^\mu - x\| + \beta_\mu\left|S_{1L}\left(x_n^\mu, y_n^\mu\right) - S_{1L}(x_n, y_n)\right| + 2M\gamma_\mu\right],\right. \\
&\qquad \left.\sup_{n\ge T_L}\left[(1-\beta_\mu-\gamma_\mu)\|y^\mu - y\| + \beta_\mu\left|S_{2L}\left(x_n^\mu, y_n^\mu\right) - S_{2L}(x_n, y_n)\right| + 2M\gamma_\mu\right]\right\} \\
&\le (1-\beta_\mu(1-\theta_L))\left\|(x^\mu, y^\mu) - (x, y)\right\|_1 + 2M\gamma_\mu, \quad \forall \mu \ge 0,
\end{aligned} \tag{2.62}$$

which implies (2.7). Thus Lemma 1.1 and (2.7)–(2.9) imply that $\lim_{\mu\to\infty}\|(x^\mu, y^\mu) - (x, y)\|_1 = 0$.

(b) Let $L_1, L_2 \in (N, M)$ and $L_1 \ne L_2$. Similar to the proof of (a), we conclude that for each $i \in \Lambda_2$, there exist a constant $\theta_{L_i} \in (0, 1)$, a positive integer $T_{L_i} \ge \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$, and mappings S_{L_i}, S_{1L_i}, S_{2L_i} satisfying (2.14) and (2.55)–(2.58), where L, θ_L, and T_L are replaced by L_i, θ_{L_i}, and T_{L_i}, respectively, and the contraction mappings S_{L_1} and S_{L_2} have the unique fixed points $(u, v) = (\{u_n\}_{n\in\mathbb{Z}_\alpha}, \{v_n\}_{n\in\mathbb{Z}_\alpha})$, $(w, z) = (\{w_n\}_{n\in\mathbb{Z}_\alpha}, \{z_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, which are bounded positive solutions of System (1.14) in $A(N,M)$, respectively. In order to show that System (1.14) possesses uncountably many bounded positive solutions in $A(N,M)$, we prove only that $(u,v)\neq(w,z)$. By virtue of (2.14) and (2.55)–(2.58), we get that

$$\begin{aligned}
&\|(u,v)-(w,z)\|_1\\
&=\max\{\|u-w\|,\|v-z\|\}\\
&\geq\max\left\{\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|u_n-w_n|,\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}|v_n-z_n|\right\}\\
&\geq\max\left\{\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}\left\{\frac{|L_1-L_2|}{|p_{1n+\tau_1}|}-\frac{|u_{n+\tau_1}-w_{n+\tau_1}|}{|p_{1n+\tau_1}|}\right.\right.\\
&\qquad\qquad-\frac{1}{|p_{1n+\tau_1}|}\sum_{m=n+\tau_1}^{\infty}\left|f_1(m,u_{a_{1m}},\ldots,u_{a_{hm}},v_{b_{1m}},\ldots,v_{b_{km}})\right.\\
&\qquad\qquad\qquad\left.\left.-f_1\left(m,w_{a_{1m}},\ldots,w_{a_{hm}},z_{b_{1m}},\ldots,z_{b_{km}}\right)\right|\right\},\\
&\qquad\sup_{n\geq\max\{T_{L_1},T_{L_2}\}}\left\{\frac{|L_1-L_2|}{|p_{2n+\tau_2}|}-\frac{|u_{n+\tau_2}-w_{n+\tau_2}|}{|p_{2n+\tau_2}|}-\frac{1}{|p_{2n+\tau_2}|}\right.\\
&\qquad\qquad\times\sum_{m=n+\tau_2}^{\infty}\left|f_2(m,u_{c_{1m}},\ldots,u_{c_{hm}},v_{d_{1m}},\ldots,v_{d_{km}})\right.\\
&\qquad\qquad\qquad\left.\left.\left.-f_2\left(m,w_{c_{1m}},\ldots,w_{c_{hm}},z_{d_{1m}},\ldots,z_{d_{km}}\right)\right|\right\}\right\}\\
&\geq\max\left\{\frac{|L_1-L_2|}{\underline{p}}-\frac{\|u-w\|}{\overline{p}}\right.\\
&\qquad-\frac{1}{\overline{p}}\sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty}t_{1m}\max\left\{|u_{a_{im}}-w_{a_{im}}|,\left|v_{b_{jm}}-z_{b_{jm}}\right|:i\in\Lambda_h,j\in\Lambda_K\right\},\\
&\qquad\frac{|L_1-L_2|}{\underline{p}}-\frac{\|v-z\|}{\overline{p}}\\
&\qquad\left.-\frac{1}{\overline{p}}\sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty}t_{2m}\max\left\{|u_{c_{im}}-w_{c_{im}}|,\left|v_{d_{jm}}-z_{d_{jm}}\right|:i\in\Lambda_h,j\in\Lambda_K\right\}\right\}\\
&\geq\max\left\{\frac{|L_1-L_2|}{\underline{p}}-\frac{\|u-w\|}{\overline{p}}-\frac{1}{\overline{p}}\sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty}t_{1m}\max\{\|u-w\|,\|v-z\|\},\right.
\end{aligned}$$

$$\begin{aligned}
&\left.\frac{|L_1-L_2|}{\underline{p}}-\frac{\|v-z\|}{\overline{p}}-\frac{1}{\overline{p}}\sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty}t_{2m}\max\{\|u-w\|,\|v-z\|\}\right\}\\
&\geq\frac{|L_1-L_2|}{\underline{p}}-\frac{1}{\overline{p}}(1+\max\{\theta_{L_1},\theta_{L_2}\})\|(u,v)-(w,z)\|_1,
\end{aligned}\tag{2.63}$$

which yields that

$$\|(u,v)-(w,z)\|_1\geq\frac{\overline{p}|L_1-L_2|}{\underline{p}(\overline{p}+\max\{\theta_{L_1},\theta_{L_2}\})}>0,\tag{2.64}$$

that is, $(u,v)\neq(w,z)$. This completes the proof. □

Theorem 2.5. *Assume that there exist constants* $M,N,\overline{p},\underline{p}\in\mathbb{R}^+\setminus\{0\}$, $n_1\in\mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2), (2.24), *and*

$$\frac{N\left(\overline{p}^2-\underline{p}\right)}{\overline{p}}<\frac{M\left(\underline{p}^2-\overline{p}\right)}{\underline{p}},\quad 1<\underline{p}\leq p_{in}\leq\overline{p}<\underline{p}^2,\quad\forall n\geq n_1,\ i\in\Lambda_2.\tag{2.65}$$

Then,

(a) *for each* $L\in(\overline{p}(M/\underline{p}+N),\underline{p}(N/\overline{p}+M))$, *there exist* $\theta_L\in(0,1)$ *and* $T_L\geq\max\{\tau_1,\tau_2\}+n_1+|\alpha|$ *such that for any* $(x^0,y^0)=(\{x_n^0\}_{n\in\mathbb{Z}_\alpha},\{y_n^0\}_{n\in\mathbb{Z}_\alpha})\in A(N,M)$, *the Mann iterative sequence with errors* $\{(x^\mu,y^\mu)\}_{\mu\in\mathbb{N}_0}=\{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha},\{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by the schemes*

$$x_n^{\mu+1}=\begin{cases}(1-\beta_\mu-\gamma_\mu)x_n^\mu\\
\quad+\beta_\mu\left\{\dfrac{L}{p_{1n+\tau_1}}-\dfrac{x_{n+\tau_1}^\mu}{p_{1n+\tau_1}}+\dfrac{1}{p_{1n+\tau_1}}\right.\\
\qquad\left.\times\displaystyle\sum_{m=n+\tau_1}^{\infty}\left[f_1\left(m,x_{a_{1m}}^\mu,\ldots,x_{a_{hm}}^\mu,y_{b_{1m}}^\mu,\ldots,y_{b_{km}}^\mu\right)-q_{1m}\right]\right\}\\
\quad+\gamma_\mu\delta_{1n}^\mu, & \mu\geq0,\ n\geq T_L,\\
(1-\beta_\mu-\gamma_\mu)x_{T_L}^\mu\\
\quad+\beta_\mu\left\{\dfrac{L}{p_{1T_L+\tau_1}}-\dfrac{x_{T_L+\tau_1}^\mu}{p_{1T_L+\tau_1}}+\dfrac{1}{p_{1T_L+\tau_1}}\right.\\
\qquad\left.\times\displaystyle\sum_{m=T_L+\tau_1}^{\infty}\left[f_1\left(m,x_{a_{1m}}^\mu,\ldots,x_{a_{hm}}^\mu,y_{b_{1m}}^\mu,\ldots,y_{b_{km}}^\mu\right)-q_{1m}\right]\right\}\\
\quad+\gamma_\mu\delta_{1T_L}^\mu, & \mu\geq0,\ \alpha\leq n<T_L,\end{cases}\tag{2.66}$$

$$y_n^{\mu+1} = \begin{cases} (1-\beta_\mu-\gamma_\mu)y_n^\mu \\ \quad +\beta_\mu\left\{\dfrac{L}{p_{2n+\tau_2}} - \dfrac{y_{n+\tau_2}^\mu}{p_{2n+\tau_2}} + \dfrac{1}{p_{2n+\tau_2}}\right. \\ \qquad \left.\times \displaystyle\sum_{m=n+\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \ldots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \ldots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu \delta_{2n}^\mu, & \mu \ge 0,\ n \ge T_L, \\ (1-\beta_\mu-\gamma_\mu)y_{T_L}^\mu \\ \quad +\beta_\mu\left\{\dfrac{L}{p_{2T_L+\tau_2}} - \dfrac{y_{T_L+\tau_2}^\mu}{p_{2T_L+\tau_2}} + \dfrac{1}{p_{2T_L+\tau_2}}\right. \\ \qquad \left.\times \displaystyle\sum_{m=T_L+\tau_2}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \ldots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \ldots, y_{d_{km}}^\mu\right) - q_{2m}\right]\right\} \\ \quad +\gamma_\mu \delta_{2T_L}^\mu, & \mu \ge 0,\ \alpha \le n < T_L \end{cases} \tag{2.67}$$

converges to a bounded positive solution $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Proof. (a) Let $L \in (\overline{p}(M/\underline{p} + N), \underline{p}(N/\overline{p} + M))$. Now we construct a contraction mapping $S_L : A(N, M) \to A(N, M)$ and prove that its fixed point is a bounded positive solution of System (1.14). Observe that (2.24) and (2.65) imply that there exist $\theta_L \in (0,1)$ and $T_L \ge \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ satisfying

$$\theta_L = \frac{1}{\underline{p}}\left(1 + \sum_{m=T_L}^{\infty} \max\{t_{im} : i \in \Lambda_2\}\right), \tag{2.68}$$

$$\sum_{m=T_L}^{\infty} \max\{r_{im} + |q_{im}| : i \in \Lambda_2\} < \min\left\{\frac{N\underline{p}}{\overline{\overline{p}}} + M\underline{p} - L, \frac{L\underline{p}}{\overline{\overline{p}}} - M - N\underline{p}\right\}. \tag{2.69}$$

Define three mappings $S_L : A(N, M) \to l_\alpha^\infty \times l_\alpha^\infty$, $S_{1L}, S_{2L} : A(N, M) \to l_\alpha^\infty$ by (2.14),

$$S_{1L}(x_n, y_n) = \begin{cases} \dfrac{L}{p_{1n+\tau_1}} - \dfrac{x_{n+\tau_1}}{p_{1n+\tau_1}} \\ \quad + \dfrac{1}{p_{1n+\tau_1}} \displaystyle\sum_{m=n+\tau_1}^{\infty} \left[f_1(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}) - q_{1m}\right], & n \ge T_L, \\ S_{1L}(x_{T_L}, y_{T_L}), & \alpha \le n < T_L, \end{cases} \tag{2.70}$$

$$S_{2L}(x_n, y_n) = \begin{cases} \dfrac{L}{p_{2n+\tau_2}} - \dfrac{y_{n+\tau_2}}{p_{2n+\tau_2}} \\ \quad + \dfrac{1}{p_{2n+\tau_2}} \displaystyle\sum_{m=n+\tau_2}^{\infty} \left[f_2(m, x_{c_{1m}}, \dots, x_{c_{hm}}, y_{d_{1m}}, \dots, y_{d_{km}}) - q_{2m} \right], & n \geq T_L, \\ S_{2L}(x_{T_L}, y_{T_L}), & \alpha \leq n < T_L, \end{cases} \tag{2.71}$$

for all $(x, y) = (\{x_n\}_{n \in \mathbb{Z}_\alpha}, \{y_n\}_{n \in \mathbb{Z}_\alpha}) \in A(N, M)$.

Using (2.1), (2.2), (2.14), (2.65), and (2.68)–(2.71), we deduce that for any $(x, y) = (\{x_n\}_{n \in \mathbb{Z}_\alpha}, \{y_n\}_{n \in \mathbb{Z}_\alpha})$ and $(u, v) = (\{u_n\}_{n \in \mathbb{Z}_\alpha}, \{v_n\}_{n \in \mathbb{Z}_\alpha}) \in A(N, M)$,

$$\begin{aligned}
&\left\| S_L(x, y) - S_L(u, v) \right\|_1 \\
&= \max\left\{ \left\| S_{iL}(x, y) - S_{iL}(u, v) \right\| : i \in \Lambda_2 \right\} \\
&= \max\left\{ \sup_{n \geq T_L} \left| S_{iL}(x_n, y_n) - S_{iL}(u_n, v_n) \right| : i \in \Lambda_2 \right\} \\
&\leq \max\left\{ \sup_{n \geq T_L} \left\{ \frac{1}{|p_{1n+\tau_1}|} \left(|x_{n+\tau_1} - u_{n+\tau_1}| + \sum_{m=n+\tau_1}^{\infty} \left| f_1(m, x_{a_{1m}}, \dots, x_{a_{hm}}, y_{b_{1m}}, \dots, y_{b_{km}}) \right. \right. \right. \right. \\
&\qquad\qquad \left. \left. \left. - f_1\ (m, u_{a_{1m}}, \dots, u_{a_{hm}}, v_{b_{1m}}, \dots, v_{b_{km}}) \right| \right) \right\}, \\
&\qquad \sup_{n \geq T_L} \left\{ \frac{1}{|p_{2n+\tau_2}|} \left(\left| y_{n+\tau_2} - v_{n+\tau_2} \right| \right. \right. \\
&\qquad\qquad + \sum_{m=n+\tau_2}^{\infty} \left| f_2(m, x_{c_{1m}}, \dots, x_{c_{hm}}, y_{d_{1m}}, \dots, y_{d_{km}}) \right. \\
&\qquad\qquad \left. \left. \left. \left. - f_2\ (m, u_{c_{1m}}, \dots, u_{c_{hm}}, v_{d_{1m}}, \dots, v_{d_{km}}) \right| \right) \right\} \right\} \\
&\leq \frac{1}{\underline{p}} \max\left\{ \|x - u\| + \sum_{m=T_L}^{\infty} t_{1m} \max\left\{ |x_{a_{im}} - u_{a_{im}}|, \left| y_{b_{jm}} - v_{b_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_k \right\}, \right. \\
&\qquad \left. \|y - v\| + \sum_{m=T_L}^{\infty} t_{2m} \max\left\{ |x_{c_{im}} - u_{c_{im}}|, \left| y_{d_{jm}} - v_{d_{jm}} \right| : i \in \Lambda_h, j \in \Lambda_k \right\} \right\} \\
&\leq \frac{1}{\underline{p}} \max\left\{ \|x - u\| + \sum_{m=T_L}^{\infty} t_{1m} \max\{ \|x - u\|, \|y - v\| \}, \|y - v\| + \sum_{m=T_L}^{\infty} t_{2m} \max\{ \|x - u\|, \|y - v\| \} \right\} \\
&\leq \frac{1}{\underline{p}} \left(1 + \sum_{m=T_L}^{\infty} \max\{ t_{1m}, t_{2m} \} \right) \left\| (x, y) - (u, v) \right\|_1 \\
&= \theta_L \left\| (x, y) - (u, v) \right\|_1,
\end{aligned}$$

$$
\begin{aligned}
S_{1L}&(x_n, y_n) \\
&= \frac{L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&\leq \frac{L}{\underline{p}} - \frac{N}{\overline{p}} + \frac{1}{\underline{p}} \sum_{m=T_L+\tau_1}^{\infty} \left(r_{1m} + |q_{1m}|\right) \\
&\leq \frac{L}{\underline{p}} - \frac{N}{\overline{p}} + \frac{1}{\underline{p}} \min\left\{ \frac{N\underline{p}}{\overline{p}} + M\underline{p} - L, \frac{L\underline{p}}{\overline{p}} - M - N\underline{p} \right\} \\
&\leq M, \quad \forall n \geq T_L,
\end{aligned}
$$

$$
\begin{aligned}
S_{1L}&(x_n, y_n) \\
&= \frac{L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right] \\
&\geq \frac{L}{\overline{p}} - \frac{M}{\underline{p}} - \frac{1}{\underline{p}} \sum_{m=T_L+\tau_1}^{\infty} \left(r_{1m} + |q_{1m}|\right) \\
&\geq \frac{L}{\overline{p}} - \frac{M}{\underline{p}} - \frac{1}{\underline{p}} \min\left\{ \frac{N\underline{p}}{\overline{p}} + M\underline{p} - L, \frac{L\underline{p}}{\overline{p}} - M - N\underline{p} \right\} \\
&\geq N, \quad \forall n \geq T_L,
\end{aligned}
$$

$$
N \leq S_{2L}(x_n, y_n) \leq M, \quad \forall n \geq T_L, \tag{2.72}
$$

which yield (2.16) and (2.17), and which guarantee that S_L is a self-mapping from $A(N, M)$ into itself and is a contraction by $\theta_L \in (0, 1)$. Thus the Banach fixed point theorem means that S_L has a unique fixed point $(x, y) = (\{x_n\}_{n\in\mathbb{Z}_\alpha}, \{y_n\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, that is,

$$
\begin{aligned}
x_n &= \frac{L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n+\tau_1}^{\infty} \left[f_1\left(m, x_{a_{1m}}, \ldots, x_{a_{hm}}, y_{b_{1m}}, \ldots, y_{b_{km}}\right) - q_{1m} \right], \quad n \geq T_L, \\
y_n &= \frac{L}{p_{2n+\tau_2}} - \frac{y_{n+\tau_2}}{p_{2n+\tau_2}} + \frac{1}{p_{2n+\tau_2}} \sum_{m=n+\tau_2}^{\infty} \left[f_2\left(m, x_{c_{1m}}, \ldots, x_{c_{hm}}, y_{d_{1m}}, \ldots, y_{d_{km}}\right) - q_{2m} \right], \quad n \geq T_L,
\end{aligned} \tag{2.73}
$$

which yield that

$$\begin{gathered}
x_n + p_{1n}x_{n-\tau_1} = L + \sum_{m=n}^{\infty}\left[f_1(m, x_{a_{1m}}, \dots, x_{a_{hm}}, y_{b_{1m}}, \dots, y_{b_{km}}) - q_{1m}\right], \quad n \ge T_L + \tau_1, \\
y_n + p_{2n}y_{n-\tau_2} = L + \sum_{m=n}^{\infty}\left[f_2(m, x_{c_{1m}}, \dots, x_{c_{hm}}, y_{d_{1m}}, \dots, y_{d_{km}}) - q_{2m}\right], \quad n \ge T_L + \tau_2, \\
\Delta(x_n + p_{1n}x_{n-\tau_1}) = -f_1(n, x_{a_{1n}}, \dots, x_{a_{hn}}, y_{b_{1n}}, \dots, y_{b_{kn}}) + q_{1n}, \quad \forall n \ge T_L + \tau_1, \\
\Delta(y_n + p_{2n}y_{n-\tau_2}) = -f_2(n, x_{c_{1n}}, \dots, x_{c_{hn}}, y_{d_{1n}}, \dots, y_{d_{kn}}) + q_{2n}, \quad \forall n \ge T_L + \tau_2,
\end{gathered} \tag{2.74}$$

that is, (x, y) is a bounded positive solution of the system (1.14) in $A(N, M)$.

In view of (2.14), (2.16), (2.65)–(2.67), (2.70), and (2.71), we gain that

$$\begin{aligned}
&\left\| \left(x^{\mu+1}, y^{\mu+1}\right) - (x, y) \right\|_1 \\
&= \max\left\{ \sup_{n \ge T_L}\left| x_n^{\mu+1} - x_n \right|, \sup_{n \ge T_L}\left| y_n^{\mu+1} - y_n \right| \right\} \\
&\le \max\Bigg\{ \sup_{n \ge T_L}\Bigg[(1 - \beta_\mu - \gamma_\mu)\left| x_n^\mu - x_n \right| + \beta_\mu \\
&\qquad \times \left| \frac{L}{p_{1n+\tau_1}} - \frac{x_{n+\tau_1}^\mu}{p_{1n+\tau_1}} + \frac{1}{p_{1n+\tau_1}} \sum_{m=n}^{\infty}\left[f_1\left(m, x_{a_{1m}}^\mu, \dots, x_{a_{hm}}^\mu, y_{b_{1m}}^\mu, \dots, y_{b_{km}}^\mu\right) - q_{1m} \right] - x_n \right| \\
&\qquad + \gamma_\mu \left| \delta_{1n}^\mu - x_n \right| \Bigg], \\
&\qquad \sup_{n \ge T_L}\Bigg[(1 - \beta_\mu - \gamma_\mu)\left| y_n^\mu - y_n \right| + \beta_\mu \\
&\qquad \times \left| \frac{L}{p_{2n+\tau_2}} - \frac{y_{n+\tau_2}^\mu}{p_{2n+\tau_2}} + \frac{1}{p_{2n+\tau_2}} \sum_{m=n}^{\infty}\left[f_2\left(m, x_{c_{1m}}^\mu, \dots, x_{c_{hm}}^\mu, y_{d_{1m}}^\mu, \dots, y_{d_{km}}^\mu\right) - q_{2m} \right] - y_n \right| \\
&\qquad + \gamma_\mu \left| \delta_{2n}^\mu - y_n \right| \Bigg] \Bigg\}
\end{aligned}$$

$$\begin{aligned}
&\le \max\Bigg\{\sup_{n\ge T_L}\Big[(1-\beta_\mu-\gamma_\mu)\|x^\mu-x\|+\beta_\mu\Big|S_{1L}\Big(x_n^\mu,y_n^\mu\Big)-S_{1L}(x_n,y_n)\Big|+2M\gamma_\mu\Big],\\
&\qquad\qquad \sup_{n\ge T_L}\Big[(1-\beta_\mu-\gamma_\mu)\|y^\mu-y\|+\beta_\mu\Big|S_{2L}\Big(x_n^\mu,y_n^\mu\Big)-S_{2L}(x_n,y_n)\Big|+2M\gamma_\mu\Big]\Bigg\}\\
&\le (1-\beta_\mu(1-\theta_L))\left\|(x^\mu,y^\mu)-(x,y)\right\|_1+2M\gamma_\mu,\quad \forall\mu\ge 0,
\end{aligned} \tag{2.75}$$

which implies (2.7). Thus, Lemma 1.1 and (2.7)–(2.9) imply that $\lim_{\mu\to\infty}\|(x^\mu,y^\mu)-(x,y)\|_1=0$.

(b) Let $L_1,L_2\in(N,M)$ and $L_1\ne L_2$. Similar to the proof of (a), we conclude that for each $i\in\Lambda_2$, there exist a constant $\theta_{L_i}\in(0,1)$, a positive integer $T_{L_i}\ge\max\{\tau_1,\tau_2\}+n_1+|\alpha|$, and mappings S_{L_i}, S_{1L_i}, S_{2L_i} satisfying (2.14) and (2.68)–(2.71), where L, θ_L, and T_L are replaced by L_i, θ_{L_i}, and T_{L_i}, respectively, and the contraction mappings S_{L_1} and S_{L_2} have the unique fixed points $(u,v)=(\{u_n\}_{n\in\mathbb{Z}_\alpha},\{v_n\}_{n\in\mathbb{Z}_\alpha})$, $(w,z)=(\{x_n\}_{n\in\mathbb{Z}_\alpha},\{y_n\}_{n\in\mathbb{Z}_\alpha})\in A(N,M)$, which are bounded positive solutions of System (1.14) in $A(N,M)$, respectively. In order to show that System (1.14) possesses uncountably many bounded positive solutions in $A(N,M)$, we prove only that $(u,v)\ne(w,z)$. In light of (2.14) and (2.68)–(2.71), we have

$$\begin{aligned}
&\|(u,v)-(w,z)\|_1\\
&=\max\{\|u-w\|,\|v-z\|\}\\
&\ge\max\left\{\sup_{n\ge\max\{T_{L_1},T_{L_2}\}}|u_n-w_n|,\ \sup_{n\ge\max\{T_{L_1},T_{L_2}\}}|v_n-z_n|\right\}\\
&\ge\max\Bigg\{\sup_{n\ge\max\{T_{L_1},T_{L_2}\}}\Bigg\{\frac{|L_1-L_2|}{|p_{1n+\tau_1}|}-\frac{|u_{n+\tau_1}-w_{n+\tau_1}|}{|p_{1n+\tau_1}|}\\
&\qquad\qquad -\frac{1}{|p_{1n+\tau_1}|}\sum_{m=n+\tau_1}^{\infty}\Big|f_1(m,u_{a_{1m}},\ldots,u_{a_{hm}},v_{b_{1m}},\ldots,v_{b_{km}})\\
&\qquad\qquad\qquad -f_1\ (m,w_{a_{1m}},\ldots,w_{a_{hm}},z_{b_{1m}},\ldots,z_{b_{km}})\Big|\Bigg\},\\
&\qquad \sup_{n\ge\max\{T_{L_1},T_{L_2}\}}\Bigg\{\frac{|L_1-L_2|}{|p_{2n+\tau_2}|}-\frac{|u_{n+\tau_2}-w_{n+\tau_2}|}{|p_{2n+\tau_2}|}-\frac{1}{|p_{2n+\tau_2}|}\\
&\qquad\qquad\times\sum_{m=n+\tau_2}^{\infty}\Big|f_2(m,u_{c_{1m}},\ldots,u_{c_{hm}},v_{d_{1m}},\ldots,v_{d_{km}})\\
&\qquad\qquad\qquad -f_2\ (m,w_{c_{1m}},\ldots,w_{c_{hm}},z_{d_{1m}},\ldots,z_{d_{km}})\Big|\Bigg\}\Bigg\}
\end{aligned}$$

$$
\begin{aligned}
&\geq \max\Bigg\{ \frac{|L_1 - L_2|}{\overline{p}} - \frac{\|u-w\|}{\underline{p}} \\
&\qquad - \frac{1}{\underline{p}} \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{1m} \max\Big\{ |u_{a_{lm}} - w_{a_{lm}}|, \Big| v_{b_{jm}} - z_{b_{jm}} \Big| : l \in \Lambda_h, j \in \Lambda_K \Big\}, \\
&\qquad \frac{|L_1 - L_2|}{\overline{p}} - \frac{\|v-z\|}{\underline{p}} \\
&\qquad - \frac{1}{\underline{p}} \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{2m} \max\Big\{ |u_{c_{lm}} - w_{c_{lm}}|, \Big| v_{d_{jm}} - z_{d_{jm}} \Big| : l \in \Lambda_h, j \in \Lambda_K \Big\} \Bigg\} \\
&\geq \max\Bigg\{ \frac{|L_1 - L_2|}{\overline{p}} - \frac{\|u-w\|}{\underline{p}} - \frac{1}{\underline{p}} \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{1m} \max\{\|u-w\|, \|v-z\|\}, \\
&\qquad \frac{|L_1 - L_2|}{\overline{p}} - \frac{\|v-z\|}{\underline{p}} - \frac{1}{\underline{p}} \sum_{m=\max\{T_{L_1},T_{L_2}\}}^{\infty} t_{2m} \max\{\|u-w\|, \|v-z\|\} \Bigg\} \\
&\geq \frac{|L_1 - L_2|}{\overline{p}} - \frac{1}{\underline{p}} (1 + \max\{\theta_{L_1}, \theta_{L_2}\}) \|(u,v) - (w,z)\|_1,
\end{aligned}
\tag{2.76}
$$

which yields that

$$
\|(u,v) - (w,z)\|_1 \geq \frac{\underline{p}|L_1 - L_2|}{\overline{p}\left(\underline{p} + \max\{\theta_{L_1}, \theta_{L_2}\}\right)} > 0,
\tag{2.77}
$$

that is, $(u,v) \neq (w,z)$. This completes the proof. □

Remark 2.6. Let (a_1, b_1) and (a_2, b_2) be two arbitrary intervals in $\mathbb{R}$. It is easy to see that

$$
(a_1, b_1) \cap (a_2, b_2) \neq \emptyset \Longleftrightarrow \max\{a_1, a_2\} < \min\{b_1, b_2\}.
\tag{2.78}
$$

From Remark 2.6 and Theorems 2.1–2.5, we can obtain the following.

Theorem 2.7. *Assume that there exist constants $M, N \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying* (2.1), (2.2),

$$\sum_{n=n_0}^{\infty} n \max\{r_{1n}, t_{1n}, |q_{1n}|\} < +\infty, \tag{2.79}$$

$$\sum_{n=n_0}^{\infty} \max\{r_{2n}, t_{2n}, |q_{2n}|\} < +\infty, \tag{2.80}$$

$$N < M, \qquad p_{1n} = -1, \quad \forall n \geq n_1, \tag{2.81}$$

$$p_{2n} = 1, \quad \forall n \geq n_1. \tag{2.82}$$

Then,

(a) *for each $L \in (N, M)$, there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ such that for any $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, the Mann iterative sequence with errors $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ generated by Schemes* (2.5) *and* (2.27) *converges to a bounded positive solution $(x, y) \in A(N, M)$ of System* (1.14) *and has the error estimate* (2.7)*, where $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$ and $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in $A(N, M)$.*

Theorem 2.8. *Assume that there exist constants $M, N \in \mathbb{R}^+ \setminus \{0\}$, $\overline{p}, \underline{p} \in \mathbb{R}^+$, $n_1 \in \mathbb{N}_{n_0}$, and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ satisfying* (2.1), (2.2), (2.79)–(2.81),

$$N < \left(1 - \underline{p} - \overline{p}\right) M, \quad \underline{p} + \overline{p} < 1, \quad -\underline{p} \leq p_{2n} \leq \overline{p}, \quad \forall n \geq n_1. \tag{2.83}$$

Then,

(a) *for each $L \in (N, M) \cap (N + \overline{p}M, M(1 - \underline{p}))$, there exist $\theta_L \in (0,1)$ and $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ such that for any $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, the Mann iterative sequence with errors $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ generated by Schemes* (2.5) *and* (2.41) *converges to a bounded positive solution $(x, y) \in A(N, M)$ of System* (1.14) *and has the error estimate* (2.7)*, where $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$ and $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in $A(N, M)$.*

Theorem 2.9. *Assume that there exist constants $M, N, \overline{p}, \underline{p} \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ and four nonnegative sequences $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$ and $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$, satisfying* (2.1), (2.2), (2.79)–(2.81),

$$0 < N\left(\underline{p} - 1\right) < M(\overline{p} - 1), \quad -\underline{p} \leq p_{2n} \leq -\overline{p}, \quad \forall n \geq n_1, \tag{2.84}$$

$$\max\left\{N, N\left(\underline{p} - 1\right)\right\} < \min\{M, M(\overline{p} - 1)\}. \tag{2.85}$$

Then,

(a) *for each* $L \in (N, M) \cap (N(\underline{p}-1), M(\overline{p}-1))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.5) *and* (2.54) *converges to a bounded positive solution* $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$ *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Theorem 2.10. *Assume that there exist constants* $M, N, M_0, N_0, \overline{p}_0, \underline{p}_0 \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2) *with* $i = 1$, (2.79)–(2.81),

$$\left|f_2(n, u_1, \dots, u_h, v_1, \dots, v_k)\right| \leq r_{2n}, \quad \forall (n, u_l, v_j) \in \mathbb{N}_{n_1} \times [N_0, M_0]^2,\ l \in \Lambda_h,\ j \in \Lambda_k, \tag{2.86}$$

$$\begin{aligned}&\left|f_2(n, u_1, \dots, u_h, v_1, \dots, v_k) - f_2(n, w_1, \dots, w_h, z_1, \dots, z_k)\right| \\ &\quad \leq t_{2n} \max\{|u_l - w_l|, |v_j - z_j| : l \in \Lambda_h, j \in \Lambda_k\}, \\ &\quad \forall (n, u_l, w_l, v_j, z_j) \in \mathbb{N}_{n_1} \times [N_0, M_0]^4,\ l \in \Lambda_h,\ j \in \Lambda_k,\end{aligned} \tag{2.87}$$

$$\frac{N_0\left(\overline{p}_0^2 - \underline{p}_0\right)}{\overline{p}_0} < \frac{M_0\left(\underline{p}_0^2 - \overline{p}_0\right)}{\underline{p}_0}, \quad 1 < \underline{p}_0 \leq p_{2n} \leq \overline{p}_0 < \underline{p}_0^2, \quad \forall n \geq n_1, \tag{2.88}$$

$$\max\left\{N, \overline{p}_0\left(\frac{M_0}{\underline{p}_0} + N_0\right)\right\} < \min\left\{M, \underline{p}_0\left(\frac{N_0}{\overline{p}_0} + M_0\right)\right\}. \tag{2.89}$$

Then,

(a) *for each* $L \in (N, M) \cap (\overline{p}_0(M_0/\underline{p}_0 + N_0), \underline{p}_0(N_0/\overline{p}_0 + M_0))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M, N_0, M_0)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.5) *and* (2.67) *converges to a bounded positive solution* $(x, y) \in A(N, M, N_0, M_0)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M, N_0, M_0)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *the system* (1.14) *has uncountably many bounded positive solutions in* $A(N, M, N_0, M_0)$.

Theorem 2.11. *Assume that there exist constants* $M, N \in \mathbb{R}^+ \setminus \{0\}$, $\overline{p}, \underline{p} \in \mathbb{R}^+$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2), (2.24), (2.83) *and*

$$N < M, \qquad p_{1n} = 1, \quad \forall n \geq n_1. \tag{2.90}$$

Then,

(a) *for each* $L \in (N, M) \cap (N + \overline{p}M, M(1 - \underline{p}))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.26) *and* (2.41) *converges to a bounded positive solution* $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Theorem 2.12. *Assume that there exist constants* $M, N, \overline{p}, \underline{p} \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2), (2.24), (2.84), (2.85), *and* (2.90). *Then,*

(a) *for each* $L \in (N, M) \cap (N(\underline{p} - 1), M(\overline{p} - 1))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.26) *and* (2.54) *converges to a bounded positive solution* $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Theorem 2.13. *Assume that there exist* $M, N, M_0, N_0, \overline{p}_0, \underline{p}_0 \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2) *with* $i = 1$, (2.24), *and* (2.86)–(2.90). *Then,*

(a) *for each* $L \in (N, M) \cap (\overline{p}_0(M_0/\underline{p}_0 + N_0), \underline{p}_0(N_0/\overline{p}_0 + M_0))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M, N_0, M_0)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.26) *and* (2.67) *converges to a bounded positive solution* $(x, y) \in A(N, M, N_0, M_0)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M, N_0, M_0)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M, N_0, M_0)$.

Theorem 2.14. *Assume that there exist constants* $M, N, \overline{p}, \underline{p} \in \mathbb{R}^+ \setminus \{0\}$, $\overline{p}_1, \underline{p}_1 \in \mathbb{R}^+$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}$, $\{r_{2n}\}_{n\in\mathbb{N}_{n_0}}$, $\{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2), (2.24), (2.84),

$$N < \left(1 - \underline{p}_1 - \overline{p}_1\right)M, \quad \underline{p}_1 + \overline{p}_1 < 1, \quad -\underline{p}_1 \leq p_{1n} \leq \overline{p}_1, \quad \forall n \geq n_1, \tag{2.91}$$

$$\max\left\{N + M\overline{p}_1, N\left(\underline{p} - 1\right)\right\} < \min\left\{M\left(1 - \underline{p}_1\right), M(\overline{p} - 1)\right\}. \tag{2.92}$$

Then,

(a) *for each* $L \in (N + M\overline{p}_1, M(1 - \underline{p}_1)) \cap (N(\underline{p} - 1), M(\overline{p} - 1))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.41) *and* (2.54) *converges to a bounded positive solution* $(x, y) \in A(N, M)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M)$ *are arbitrary sequences with* (2.8) *and* (2.9),

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M)$.

Theorem 2.15. *Assume that there exist constants* $M, N, M_0, N_0, \overline{p}_0, \underline{p}_0, \overline{p}_1, \underline{p}_1 \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}, \{r_{2n}\}_{n\in\mathbb{N}_{n_0}}, \{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2) *with* $i = 1$, (2.24), (2.86)–(2.88), (2.91), *and*

$$\max\left\{N + M\overline{p}_1, \overline{p}_0\left(\frac{M_0}{\underline{p}_0} + N_0\right)\right\} < \min\left\{M\left(1 - \underline{p}_1\right), \underline{p}_0\left(\frac{N_0}{\overline{p}_0} + M_0\right)\right\}. \tag{2.93}$$

Then,

(a) *for each* $L \in (N + M\overline{p}_1, M(1 - \underline{p}_1)) \cap (\overline{p}_0(M_0/\underline{p}_0 + N_0), \underline{p}_0(N_0/\overline{p}_0 + M_0))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M, N_0, M_0)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.41) *and* (2.67) *converges to a bounded positive solution* $(x, y) \in A(N, M, N_0, M_0)$ *of System* (1.14) *and has the error estimate* (2.7), *where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$, *and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M, N_0, M_0)$ *are arbitrary sequences with* (2.8) *and* (2.9)

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M, N_0, M_0)$.

Theorem 2.16. *Assume that there exist constants* $M, N, M_0, N_0, \overline{p}_0, \underline{p}_0, \overline{p}_1, \underline{p}_1 \in \mathbb{R}^+ \setminus \{0\}$, $n_1 \in \mathbb{N}_{n_0}$ *and four nonnegative sequences* $\{r_{1n}\}_{n\in\mathbb{N}_{n_0}}, \{r_{2n}\}_{n\in\mathbb{N}_{n_0}}, \{t_{1n}\}_{n\in\mathbb{N}_{n_0}}$, *and* $\{t_{2n}\}_{n\in\mathbb{N}_{n_0}}$ *satisfying* (2.1), (2.2) *with* $i = 1$, (2.24), (2.86)–(2.88),

$$0 < N\left(\underline{p}_1 - 1\right) < M(\overline{p}_1 - 1), \quad -\underline{p}_1 \leq p_{1n} \leq -\overline{p}_1, \quad \forall n \geq n_1, \tag{2.94}$$

$$\max\left\{N\left(\underline{p}_1 - 1\right), \overline{p}_0\left(\frac{M_0}{\underline{p}_0} + N_0\right)\right\} < \min\left\{M(\overline{p}_1 - 1), \underline{p}_0\left(\frac{N_0}{\overline{p}_0} + M_0\right)\right\}. \tag{2.95}$$

Then,

(a) *for each* $L \in (N(\underline{p}_1 - 1), M(\overline{p}_1 - 1)) \cap (\overline{p}_0(M_0/\underline{p}_0 + N_0), \underline{p}_0(N_0/\overline{p}_0 + M_0))$, *there exist* $\theta_L \in (0,1)$ *and* $T_L \geq \max\{\tau_1, \tau_2\} + n_1 + |\alpha|$ *such that for any* $(x^0, y^0) = (\{x_n^0\}_{n\in\mathbb{Z}_\alpha}, \{y_n^0\}_{n\in\mathbb{Z}_\alpha}) \in A(N, M, N_0, M_0)$, *the Mann iterative sequence with errors* $\{(x^\mu, y^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{x_n^\mu\}_{n\in\mathbb{Z}_\alpha}, \{y_n^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0}$ *generated by Schemes* (2.54) *and* (2.67) *converges to a bounded positive solution* $(x, y) \in A(N, M, N_0, M_0)$ *of System* (1.14) *and has* *the error estimate* (2.7)*, where* $\{\beta_\mu\}_{\mu\in\mathbb{N}_0}, \{\gamma_\mu\}_{\mu\in\mathbb{N}_0} \subset [0,1]$*, and* $\{(\delta_1^\mu, \delta_2^\mu)\}_{\mu\in\mathbb{N}_0} = \{(\{\delta_{1n}^\mu\}_{n\in\mathbb{Z}_\alpha}, \{\delta_{2n}^\mu\}_{n\in\mathbb{Z}_\alpha})\}_{\mu\in\mathbb{N}_0} \subset A(N, M, N_0, M_0)$ *are arbitrary sequences with* (2.8) *and* (2.9)*,*

(b) *System* (1.14) *has uncountably many bounded positive solutions in* $A(N, M, N_0, M_0)$*.*

Remark 2.17. Theorems 2.1–2.5 extend and improve [14, Theorem 1] and [27, Theorems 2.1–2.7].

3. Examples

In this section we construct fifteen examples to explain the results presented in Section 2.

Example 3.1. Consider the nonlinear difference system with multiple delays:

$$\begin{aligned} &\Delta(x_n - x_{n-\tau_1}) + \frac{x_{n-3}^3 y_{n-6} - \sqrt{n}\, x_{n-4} y_{n-5}^2}{n^3 + n x_{n-3}^2 + (n^2+1) x_{n-4}^4 y_{n-6}^2} = \frac{(-1)^{n-1}}{n^4 + \sqrt{n+1}}, \\ &\Delta(y_n - y_{n-\tau_2}) + \frac{(n+1) x_{n-2}^2 y_{n-5} + (-1)^n y_{n-2}}{n^4 + (n+2) x_{n-1}^2} = \frac{\sqrt{n+1}}{n^3 + (n+2)\ln n}, \quad n \ge 1, \end{aligned} \tag{3.1}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$\begin{gathered} n_0 = n_1 = 1, \quad h = k = 2, \quad M > N > 0, \quad \alpha = \min\{1-\tau_1, 1-\tau_2, -5\}, \quad a_{1n} = n-3, \\ a_{2n} = n-4, \quad b_{1n} = n-6, \quad b_{2n} = n-5, \quad c_{1n} = n-2, \quad c_{2n} = n-1, \quad d_{1n} = n-5, \\ d_{2n} = n-2, \quad p_{1n} = p_{2n} = -1, \quad q_{1n} = \frac{(-1)^{n-1}}{n^4 + \sqrt{n+1}}, \quad q_{2n} = \frac{\sqrt{n+1}}{n^3 + (n+2)\ln n}, \\ f_1(n, u_1, u_2, v_1, v_2) = \frac{u_1^3 v_1 - \sqrt{n}\, u_2 v_2^2}{n^3 + n u_1^2 + (n^2+1) u_2^4 v_1^2}, \quad f_2(n, u_1, u_2, v_1, v_2) = \frac{(n+1) u_1^2 v_2 + (-1)^n v_1}{n^4 + (n+2) u_2^2}, \\ r_{1n} = \frac{M^3(M + \sqrt{n})}{n^3 + nN^2 + (n^2+1)N^6}, \quad r_{2n} = \frac{M(1 + (n+1)M^2)}{n^4 + (n+2)N^2}, \\ t_{1n} = \frac{nM^2(4n^2 M + 3n^{5/2} + 2M^3 + 5\sqrt{n}M^2 + 16nM^7 + 14\sqrt{n}M^6)}{(n^3 + nN^2 + (n^2+1)N^6)^2}, \\ t_{2n} = \frac{1}{n^7}\left(6n^4 M^2 + n^3 + 30nM^4 + 9M^2\right), \quad \forall (n, u_1, u_2, v_1, v_2) \in \mathbb{N}_{n_0} \times \mathbb{R}^4. \end{gathered} \tag{3.2}$$

It is easy to see that (2.1)–(2.4) are satisfied. Thus Theorem 2.1 implies that System (3.1) possesses uncountably many bounded positive solutions in $A(N, M)$. But [14, Theorem 1] and [27, Theorem 2.1] are not valid for System (3.1).

Example 3.2. Consider the nonlinear difference system with multiple delays:

$$\Delta\left(x_n + x_{n-\tau_1}\right) + \frac{nx_{n-1}^3 - (n-1)y_{n-4}}{n^3 + x_{n-1}^2 y_{n-4}^2} = \frac{n^2-1}{n^4+2n+1},$$
$$\Delta\left(y_n + y_{n-\tau_2}\right) + \frac{n^9 x_{n-3} y_{n-2}^2}{n^{12} + (n^6-1)x_{n-3}^2} = \frac{(-1)^n n^2 \ln^3 n}{(n^2+1)^2}, \quad n \ge 1, \tag{3.3}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$n_0 = n_1 = 1, \qquad h = k = 1, \qquad M > N > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -3\}, \qquad a_{1n} = n-1,$$
$$b_{1n} = n-4, \qquad c_{1n} = n-3, \qquad d_{1n} = n-2, \qquad p_{1n} = p_{2n} = 1, \qquad q_{1n} = \frac{n^2-1}{n^4+2n+1},$$
$$q_{2n} = \frac{(-1)^n n^2 \ln^3 n}{(n^2+1)^2}, \qquad f_1(n,u,v) = \frac{nu^3-(n-1)v}{n^3+u^2v^2}, \qquad f_2(n,u,v) = \frac{n^9uv^2}{n^{12}+(n^6-1)u^2},$$
$$r_{1n} = \frac{nM(M+1)}{n^3+N^4}, \qquad r_{2n} = \frac{n^9M^3}{n^{12}+(n^6-1)N^2}, \qquad t_{1n} = \frac{n\left(n^3(2M+1)+3M^4+2M^5\right)}{(n^3+N^4)^2},$$
$$t_{2n} = \frac{3n^{15}(n^6+1)M^2}{\left(n^{12}+(n^6-1)N^2\right)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2. \tag{3.4}$$

It is easy to verify that (2.1), (2.2), (2.24), and (2.25) are fulfilled. Thus Theorem 2.2 implies that System (3.3) possesses uncountably many bounded positive solutions in $A(N,M)$. But in [14, Theorem 1] and [27, Theorem 2.2] are unapplicable for System (3.3).

Example 3.3. Consider the nonlinear difference system with multiple delays:

$$\Delta\left(x_n + \frac{n^2}{2n^2+3}x_{n-\tau_1}\right) + \frac{x_{n-2}-2y_{n-6}}{n^2+\sin\left(x_{n-1}y_{n-6}\right)} = \frac{\sqrt{n-2}\cos\left(n^3-3n^2+6\right)}{n^3+3n+1},$$
$$\Delta\left(y_n + \frac{(-1)^n}{3n+1}y_{n-\tau_2}\right) + \frac{nx_{n-1}+(2n-1)y_{n-3}^2}{n^3+x_{n-1}^2} = \frac{(n^2-3)\sin\left(n^5-2n+1\right)}{n^4+\ln^3 n}, \quad n \ge 2, \tag{3.5}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$
\begin{gathered}
n_0 = n_1 = 2, \qquad h = k = 1, \qquad M > 6N > 0, \\
\alpha = \min\{2-\tau_1, 2-\tau_2, -4\}, \qquad a_{1n} = n-2, \\
b_{1n} = n-6, \qquad c_{1n} = n-1, \qquad d_{1n} = n-3, \qquad p_{1n} = \frac{n^2}{2n^2+3}, \\
p_{2n} = \frac{(-1)^n}{3n+1}, \qquad \overline{p} = \frac{1}{2}, \qquad \underline{p} = \frac{1}{3}, \\
q_{1n} = \frac{\sqrt{n-2}\cos(n^3-3n^2+6)}{n^3+3n+1}, \qquad q_{2n} = \frac{(n^2-3)\sin(n^5-2n+1)}{n^4+\ln^3 n}, \\
f_1(n,u,v) = \frac{u-2v}{n^2+\sin(uv)}, \\
f_2(n,u,v) = \frac{nu+(2n-1)v^2}{n^3+u^2}, \qquad r_{1n} = \frac{3M}{n^2-1}, \qquad r_{2n} = \frac{nM(1+2M)}{n^3+N^2}, \\
t_{1n} = \frac{6(n^2+M^2)}{(n^2-1)^2}, \quad t_{2n} = \frac{n((1+4M)n^3+M^2+8M^3)}{(n^3+N^2)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.
\end{gathered}
\tag{3.6}
$$

It is easy to see that (2.1), (2.2), (2.24), and (2.39) hold. Thus, Theorem 2.3 ensures that System (3.5) possesses uncountably many bounded positive solutions in $A(N, M)$. But [14, Theorem 1] and [27, Theorems 2.3, 2.5, and 2.6] are not valid for System (3.5).

Example 3.4. Consider the nonlinear difference system with multiple delays:

$$
\begin{gathered}
\Delta\left(x_n + \left(-4+\sin\left(2n^2\right)\right)x_{n-\tau_1}\right) + \frac{n^5 x_{n-1}^2 y_{n-2}^5}{n^{10}+5n^8+1} = \frac{(-1)^n n^3 \ln(1+n)}{n^9+n^5+1}, \\
\Delta\left(y_n + \left(-4+\cos\left(5n^3\right)\right)y_{n-\tau_2}\right) + \frac{n^2 x_{n-2} - (n+2)y_{n-3}^2}{n^8+1+nx_{n-2}^2+y_{n-3}^2} = \frac{n^5-6}{(n+1)^7}, \quad n \geq 1,
\end{gathered}
\tag{3.7}
$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$
\begin{gathered}
n_0 = n_1 = h = k = 1, \qquad M > 2N > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -2\}, \\
a_{1n} = n-1, \qquad b_{1n} = n-2, \\
c_{1n} = n-2, \qquad d_{1n} = n-3, \qquad p_{1n} = -4 + \sin\left(2n^2\right), \\
p_{2n} = -4 + \cos\left(5n^3\right), \qquad \overline{p} = 3, \qquad \underline{p} = 5, \\
q_{1n} = \frac{(-1)^n n^3 \ln(1+n)}{n^9 + n^5 + 1}, \qquad q_{2n} = \frac{n^5 - 6}{(n+1)^7}, \qquad f_1(n,u,v) = \frac{n^5 u^2 v^5}{n^{10} + 5n^8 + 1}, \\
f_2(n,u,v) = \frac{n^2 u - (n+2)v^2}{n^8 + 1 + nu^2 + v^2}, \qquad r_{1n} = \frac{M^7}{n^5}, \\
r_{2n} = \frac{n^2 M + (n+2)M^2}{n^8 + 1 + nN^2 + N^2}, \qquad t_{1n} = \frac{7M^6}{n^5}, \\
t_{2n} = \frac{n^2\left(2n^8 + 8n^7 M + nM^2 + 8M^3 + 3M^2\right)}{\left(n^8 + 1 + nN^2 + N^2\right)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.
\end{gathered}
\tag{3.8}
$$

Clearly (2.1), (2.2), (2.24), and (2.52) hold. Thus, Theorem 2.4 means that System (3.7) possesses uncountably many bounded positive solutions in $A(N, M)$. But [14, Theorem 1] and [27, Theorem 2.4] are inapplicable for System (3.7).

Example 3.5. Consider the nonlinear difference system with multiple delays:

$$
\begin{gathered}
\Delta\left(x_n + \frac{8n^2+1}{n^2+1} x_{n-\tau_1}\right) + \frac{n - (n-1) y_{b_{1n}}^3}{n^3 + x_{a_{1n}}^2} = \frac{\sqrt{n} - (-1)^{n(n+1)/2}\sqrt{3n-2}}{n^2 + \sqrt{2n+1}}, \\
\Delta\left(y_n + \frac{1 + 24\ln^3 n}{2 + 3\ln^3 n} y_{n-\tau_2}\right) + \frac{2n x_{c_{1n}} - 3\sqrt{n+1}\, y_{d_{1n}}^3}{n^4 + x_{c_{1n}}^2 y_{d_{1n}}^2} = \frac{1 + n\ln\left(1+n^2\right)}{n^3 \ln^3\left(1+n^2\right)}, \quad n \geq 1,
\end{gathered}
\tag{3.9}
$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$
\begin{gathered}
a_{1n} = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even}, \\ n-2 & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} n-1 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \\
c_{1n} = \begin{cases} n-2 & \text{if } n \text{ is even}, \\ n-1 & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} \dfrac{n}{2} - 1 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}. \end{cases}
\end{gathered}
\tag{3.10}
$$

Let

$$
\begin{gathered}
n_0 = 1, \qquad n_1 = 10, \qquad h = k = 1, \qquad M > \frac{399}{328} N > 0, \\
\alpha = \min\{1-\tau_1, 1-\tau_2, -1\}, \qquad p_{1n} = \frac{8n^2+1}{n^2+1}, \\
p_{2n} = \frac{1+24\ln^3 n}{2+3\ln^3 n}, \qquad \overline{p} = 8, \qquad \underline{p} = 7, \\
q_{1n} = \frac{\sqrt{n} - (-1)^{n(n+1)/2}\sqrt{3n-2}}{n^2+\sqrt{2n+1}}, \qquad q_{2n} = \frac{1+n\ln(1+n^2)}{n^3\ln^3(1+n^2)}, \\
f_1(n,u,v) = \frac{n-(n-1)v^3}{n^3+u^2}, \qquad f_2(n,u,v) = \frac{2nu - 3\sqrt{n+1}v^3}{n^4+u^2v^2}, \\
r_{1n} = \frac{n(1+M^3)}{n^3+N^2}, \\
r_{2n} = \frac{nM(2+3M^2)}{n^4+N^4}, \qquad t_{1n} = \frac{nM(2+3n^3M+5M^3)}{(n^3+N^2)^2}, \\
t_{2n} = \frac{n(2n^4+6M^2+9n^4M^2+9M^6)}{(n^4+N^4)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.
\end{gathered} \tag{3.11}
$$

Obviously (2.1), (2.2), (2.24), and (2.65) hold. Thus, Theorem 2.5 implies that System (3.9) possesses uncountably many bounded positive solutions in $A(N,M)$. But [14, Theorem 1] and [27, Theorem 2.7] are useless for System (3.9).

Example 3.6. Consider the nonlinear difference system with multiple delays:

$$
\begin{gathered}
\Delta(x_n - x_{n-\tau_1}) + \frac{2n^2 x_{a_{1n}}^3 + n - 1}{n^5 + n(nx_{a_{1n}} - (3n+2)y_{b_{1n}})^2} = \frac{(-1)^{(n-1)n(n+1)/3}\sqrt{2n^3+4}}{n^4+\sin^2(2n^4+1)}, \\
\Delta(y_n + y_{n-\tau_2}) + \frac{(-1)^n x_{c_{1n}}^2 - ny_{d_{1n}}^3}{(n+1)^2(\sqrt{n}+1)} = \frac{(-1)^{n-1}\ln(n^3+2n+1)}{n^{3/2}+\ln^3(1+n^2)}, \quad n \ge 1,
\end{gathered} \tag{3.12}
$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$
\begin{gathered}
a_{1n} = \begin{cases} n-1 & \text{if } n \ge 100, \\ n-2 & \text{if } 1 \le n < 100, \end{cases} \qquad b_{1n} = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \\
c_{1n} = \begin{cases} \dfrac{n}{2} - 2 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} n-3 & \text{if } n \ge 100, \\ n & \text{if } 1 \le n < 100. \end{cases}
\end{gathered} \tag{3.13}
$$

Let

$$n_0 = n_1 = h = k = 1, \qquad M > N > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -1\}, \qquad p_{1n} = -1,$$

$$p_{2n} = 1, \qquad q_{1n} = \frac{(-1)^{(n-1)n(n+1)/3}\sqrt{2n^3+4}}{n^4 + \sin^2(2n^4+1)}, \qquad q_{2n} = \frac{(-1)^{n-1}\ln(n^3+2n+1)}{n^{3/2} + \ln^3(1+n^2)},$$

$$f_1(n,u,v) = \frac{2n^2u^3 + n - 1}{n^5 + n(nu - (3n+2)v)^2}, \qquad f_2(n,u,v) = \frac{(-1)^n u^2 - nv^3}{(n+1)^2(\sqrt{n}+1)},$$

$$r_{1n} = \frac{1+2nM^3}{n^4}, \qquad r_{2n} = \frac{M^2 + nM^3}{n^{5/2}}, \tag{3.14}$$

$$t_{1n} = \frac{6M(16 + n^3M + 8(2n+M)M^2 + 3nM^3)}{n^6},$$

$$t_{2n} = \frac{2M + 3nM^2}{n^{5/2}}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.$$

It is easy to see that (2.1), (2.2), and (2.79)–(2.82) are satisfied. Thus, Theorem 2.7 implies that System (3.12) possesses uncountably many bounded positive solutions in $A(\mathrm{N}, M)$.

Example 3.7. Consider the nonlinear difference system with multiple delays:

$$\Delta(x_n - x_{n-\tau_1}) + \frac{nx^3_{a_{1n}} - y_{b_{1n}}}{(n+1)^4 + (\sqrt{n}+1)y^2_{b_{1n}}} = \frac{(-1)^{n(n+1)/2}(1-\sqrt{3n+1})}{n^3 + \sqrt{3n+2}},$$

$$\Delta\left(y_n + \frac{n^2-1}{3n^2+2}y_{n-\tau_2}\right) + \frac{\sqrt{n}(n+1)y^4_{d_{1n}}}{(n+2)^3 + nx^2_{c_{1n}}} = \frac{\sqrt{2n+1}(n+3)^2\ln(1+n^4)}{(n+1)^4}, \quad n \geq 1, \tag{3.15}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$a_{1n} = \begin{cases} n-4 & \text{if } n \text{ is even}, \\ n-1 & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} n-2 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases}$$

$$c_{1n} = \begin{cases} n-1 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even}, \\ n-2 & \text{if } n \text{ is odd}. \end{cases} \tag{3.16}$$

Let

$$
\begin{gathered}
n_0 = n_1 = h = k = 1, \qquad M > 6N > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -2\}, \\
p_{1n} = -1, \qquad p_{2n} = \frac{n^2-1}{3n^2+2}, \\
\overline{p} = \frac{1}{3}, \qquad \underline{p} = \frac{1}{2}, \qquad q_{1n} = \frac{(-1)^{n(n+1)/2}\left(1-\sqrt{3n+1}\right)}{n^3+\sqrt{3n+2}}, \\
q_{2n} = \frac{\sqrt{2n+1}(n+3)^2\ln\left(1+n^4\right)}{(n+1)^4}, \\
f_1(n,u,v) = \frac{nu^3-v}{(n+1)^4+\left(\sqrt{n}+1\right)v^2}, \qquad f_2(n,u,v) = \frac{\sqrt{n}(n+1)v^4}{(n+2)^3+nu^2}, \\
r_{1n} = \frac{\left(nM^2+1\right)M}{(n+1)^4}, \\
r_{2n} = \frac{\sqrt{n}(n+1)M^4}{(n+2)^3+nN^2}, \qquad t_{1n} = \frac{(n+1)^4\left(3nM^2+1\right)+\left(5nM^2+1\right)\left(\sqrt{n}+1\right)M^2}{\left((n+1)^4+\left(\sqrt{n}+1\right)N^2\right)^2}, \\
t_{2n} = \frac{2\sqrt{n}(n+1)M^3\left(2(n+2)^3+3nM^2\right)}{\left((n+2)^3+nN^2\right)^2}, \qquad \forall (n,u,v) \in \mathbb{N}_{n_0}\times\mathbb{R}^2.
\end{gathered} \tag{3.17}
$$

Clearly (2.1), (2.2), (2.79)–(2.81), and (2.83) are satisfied. Thus, Theorem 2.8 guarantees that System (3.15) possesses uncountably many bounded positive solutions in $A(N, M)$.

Example 3.8. Consider the nonlinear difference system with multiple delays:

$$
\begin{gathered}
\Delta\left(x_n - x_{n-\tau_1}\right) + \frac{n^7 - x_{a_{1n}}^2}{n^{10}+2y_{b_{1n}}^2} = \frac{n^{20}-(-1)^{(n+1)(n+2)/2}\left(3n^5+1\right)^2}{n^{23}+(n+5)^5\ln^8\left(n^2+1\right)}, \\
\Delta\left(y_n - \frac{6n^5}{2n^5+1}y_{n-\tau_2}\right) + \frac{n^3 x_{c_{1n}} y_{d_{1n}}^2}{n^6+(n+1)x_{c_{1n}}^4} = \frac{n^8-(-1)^{n-1}\left(n^3+1\right)^2}{(n+1)^{11}}, \qquad n \geq 2,
\end{gathered} \tag{3.18}
$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$
\begin{gathered}
a_{1n} = \begin{cases} \dfrac{n}{2}-1 & \text{if } n \text{ is even}, \\ \dfrac{n-3}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} n-1 & \text{if } n \text{ is even}, \\ n-2 & \text{if } n \text{ is odd}, \end{cases} \\
c_{1n} = \begin{cases} \dfrac{n}{2}+1 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} n-5 & \text{if } n \text{ is even}, \\ n-3 & \text{if } n \text{ is odd}. \end{cases}
\end{gathered} \tag{3.19}
$$

Let

$$n_0 = n_1 = 2, \qquad h = k = 1, \qquad M > 6N > 0, \qquad \alpha = \min\{2-\tau_1, 2-\tau_2, -3\}, \qquad p_{1n} = -1,$$
$$p_{2n} = -\frac{6n^5}{2n^5+1}, \qquad \overline{p} = 2, \qquad \underline{p} = 3, \qquad q_{1n} = \frac{n^{20} - (-1)^{(n+1)(n+2)/2}\left(3n^5+1\right)^2}{n^{23} + (n+5)^5 \ln^8\left(n^2+1\right)},$$
$$q_{2n} = \frac{n^8 - (-1)^{n-1}\left(n^3+1\right)^2}{(n+1)^{11}}, \qquad f_1(n,u,v) = \frac{n^7 - u^2}{n^{10} + 2v^2}, \qquad f_2(n,u,v) = \frac{n^3 u v^2}{n^6 + (n+1)u^4},$$
$$r_{1n} = \frac{n^7 + M^2}{n^{10} + 2N^2}, \qquad r_{2n} = \frac{n^3 M^3}{n^6 + (n+1)N^4}, \qquad t_{1n} = \frac{2M\left(2n^7 + n^{10} + 4M^2\right)}{\left(n^{10} + 2N^2\right)^2},$$
$$t_{2n} = \frac{3n^3 M^2\left(n^6 + (n+1)M^4\right)}{\left(n^6 + (n+1)N^4\right)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2. \tag{3.20}$$

Obviously (2.1), (2.2), (2.79)–(2.81), (2.84), and (2.85) hold. Thus, Theorem 2.9 implies that System (3.18) possesses uncountably many bounded positive solutions in $A(N, M)$.

Example 3.9. Consider the nonlinear difference system with multiple delays:

$$\begin{gathered}
\Delta\left(x_n - x_{n-\tau_1}\right) + \frac{n x_{a_{1n}}^2 - y_{b_{1n}}^3}{n^3 \ln^2 n + 1} = \frac{n^2 - (-1)^{n-1}(3n+1)}{n^5 + 8(n+1)^4 + 1}, \\
\Delta\left(y_n + \frac{8n^2}{2n^2+3} y_{n-\tau_2}\right) + \frac{n^2}{n^4 + \left|x_{c_{1n}} y_{d_{1n}}\right|} = \frac{(-1)^{n-1}\sin\left(n^5+1\right)}{n^2+1}, \quad n \geq 1,
\end{gathered} \tag{3.21}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$\begin{gathered}
a_{1n} = \begin{cases} n-2 & \text{if } n \text{ is even}, \\ \dfrac{n-3}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \\
c_{1n} = \begin{cases} n-1 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} n-4 & \text{if } n \text{ is even}, \\ n-2 & \text{if } n \text{ is odd}. \end{cases}
\end{gathered} \tag{3.22}$$

Let

$$\begin{gathered}
n_0 = 1, \quad n_1 = 3, \quad h = k = 1, \quad M = 3000, \\
N = 2000, \quad M_0 = 1200, \quad N_0 = 40, \\
\alpha = \min\{1-\tau_1, 1-\tau_2, -2\}, \quad p_{1n} = -1, \quad p_{2n} = \frac{8n^2}{2n^2+3}, \quad \overline{p}_0 = 4, \quad \underline{p}_0 = 3, \\
q_{1n} = \frac{n^2 - (-1)^{n-1}(3n+1)}{n^5 + 8(n+1)^4 + 1}, \quad q_{2n} = \frac{(-1)^{n-1}\sin(n^5+1)}{n^2+1}, \quad f_1(n,u,v) = \frac{nu^2 - v^3}{n^3\ln^2 n + 1}, \\
f_2(n,u,v) = \frac{n^2}{n^4 + |uv|}, \quad r_{1n} = \frac{M^2(n+M)}{n^3\ln^2 n + 1}, \quad r_{2n} = \frac{n^2}{n^4 + N_0^2}, \\
t_{1n} = \frac{M(2n+3M)}{n^3\ln^2 n + 1}, \quad t_{2n} = \frac{2n^2 M_0}{\left(n^4 + N_0^2\right)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.
\end{gathered} \tag{3.23}$$

It is easy to verify that (2.1), (2.2) with $i = 1$, (2.79)–(2.81), and (2.86)–(2.89) are satisfied. Thus, Theorem 2.10 reveals that System (3.21) possesses uncountably many bounded positive solutions in $A(N, M, N_0, M_0)$.

Example 3.10. Consider the nonlinear difference system with multiple delays:

$$\begin{gathered}
\Delta(x_n + x_{n-\tau_1}) + \frac{1 - n^5 x_{a_{1n}}^2}{n^7 + (n+3)\ln\left(1 + y_{b_{1n}}^2\right)} = \frac{n^4 - 4n^3 + n - 1}{n^6 + (n-1)^4}, \\
\Delta\left(y_n - \frac{2n^2}{5n^2+6} y_{n-\tau_2}\right) + \frac{n^2 y_{d_{1n}}}{n^4 + \sin^2\left(x_{c_{1n}}^2 + n y_{d_{1n}}^2\right)} = \frac{(-1)^n\sqrt{n-1}}{n^3 + 2n^2 + 1}, \quad n \geq 2,
\end{gathered} \tag{3.24}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$\begin{gathered}
a_{1n} = \begin{cases} n-3 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} \dfrac{n}{2}+1 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}, \end{cases} \\
c_{1n} = \begin{cases} n-1 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} n-2 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}. \end{cases}
\end{gathered} \tag{3.25}$$

Let

$$n_0 = n_1 = 2, \qquad h = k = 1, \qquad M > \frac{5N}{3} > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -1\}, \qquad p_{1n} = 1,$$
$$p_{2n} = -\frac{2n^2}{5n^2+6}, \qquad \overline{p} = 0, \qquad \underline{p} = \frac{2}{5}, \qquad q_{1n} = \frac{n^4 - 4n^3 + n - 1}{n^6 + (n-1)^4}, \qquad q_{2n} = \frac{(-1)^n\sqrt{n-1}}{n^3+2n^2+1},$$
$$f_1(n,u,v) = \frac{1-n^5u^2}{n^7+(n+3)\ln(1+v^2)}, \qquad f_2(n,u,v) = \frac{n^2 v}{n^4 + \sin^2(u^2+nv^2)},$$
$$r_{1n} = \frac{1+n^5M^2}{n^7+(n+3)\ln(1+N^2)}, \qquad r_{2n} = \frac{M}{n^2},$$
$$t_{1n} = \frac{1}{(n^7+(n+3)\ln(1+N^2))^2}\left[\frac{2\mathrm{M}(n+3)}{1+N^2} + n^{12} + n^5(n+3)\left(\frac{2M^2}{1+N^2} + \ln\left(1+M^2\right)\right)\right],$$
$$t_{2n} = \frac{n^4 + 4nM^2 + 4M^2 + 1}{n^6}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2. \tag{3.26}$$

Obviously (2.1), (2.2), (2.24), (2.83), and (2.90) hold. Thus, Theorem 2.11 gives that System (3.24) possesses uncountably many bounded positive solutions in $A(N,M)$.

Example 3.11. Consider the nonlinear difference system with multiple delays:

$$\begin{aligned} &\Delta(x_n + x_{n-\tau_1}) + \frac{n^2 - x^2_{a_{1n}} y_{b_{1n}}}{n^4 + (n+1)y^4_{b_{1n}}} = \frac{n^6 - 9n^5 + (-1)^n n^4 - n^3 + n - 1}{n^8 + 3n^7 + n^5 + 4n^4 + n^2 + 1}, \\ &\Delta\left(y_n - \frac{3n^3}{n^3+1} y_{n-\tau_2}\right) + \frac{n y^2_{d_{1n}}}{n^3 + x^2_{c_{1n}}} = \frac{(-1)^{n(n+1)/2}(2n-5)\ln^2(1+n^2)}{n^3+3n^2+5n+1}, \quad n \geq 1, \end{aligned} \tag{3.27}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed and

$$\begin{aligned} a_{1n} &= \begin{cases} n-2 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad b_{1n} = \begin{cases} \dfrac{n}{2} - 1 & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2} & \text{if } n \text{ is odd}, \end{cases} \\ c_{1n} &= \begin{cases} n-3 & \text{if } n \text{ is even}, \\ \dfrac{n-3}{2} & \text{if } n \text{ is odd}, \end{cases} \qquad d_{1n} = \begin{cases} n-4 & \text{if } n \text{ is even}, \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd}. \end{cases} \end{aligned} \tag{3.28}$$

Let

$$n_0 = n_1 = h = k = 1, \qquad M > 4N > 0, \qquad \alpha = \min\{1-\tau_1, 1-\tau_2, -2\}, \qquad p_{1n} = 1,$$

$$p_{2n} = -\frac{3n^3}{n^3+1}, \qquad \overline{p} = \frac{3}{2}, \qquad \underline{p} = 3, \qquad q_{1n} = \frac{n^6 - 9n^5 + (-1)^n n^4 - n^3 + n - 1}{n^8 + 3n^7 + n^5 + 4n^4 + n^2 + 1},$$

$$q_{2n} = \frac{(-1)^{n(n+1)/2}(2n-5)\ln^2(1+n^2)}{n^3+3n^2+5n+1}, \qquad f_1(n,u,v) = \frac{n^2 - u^2 v}{n^4 + (n+1)v^4}, \tag{3.29}$$

$$f_2(n,u,v) = \frac{nv^2}{n^3+u^2}, \qquad r_{1n} = \frac{n^2 + M^3}{n^4 + (n+1)N^4}, \qquad r_{2n} = \frac{nM^2}{n^3 + N^2},$$

$$t_{1n} = \frac{nM^2(8n^2M + 3n^3 + 10M^4)}{(n^4 + (n+1)N^4)^2}, \quad t_{2n} = \frac{2M(n^3 + 2M^2)}{(n^3+N^2)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.$$

Clearly (2.1), (2.2), (2.24), (2.84), (2.85), and (2.90) hold. Thus, Theorem 2.12 ensures that System (3.27) possesses uncountably many bounded positive solutions in $A(N,M)$.

Example 3.12. Consider the nonlinear difference system with multiple delays:

$$\begin{gathered}
\Delta(x_n + x_{n-\tau_1}) + \frac{n^{25} - (n^{15}+1)y_{n-1}^2}{n^{30} + (n^{18}+1)x_{n-2}^2} = \frac{n^{38} - 19n^{25} + 30n^{18} - 3n^9 + 2n - 1}{n^{40} + 8n^{30} + n^{15} + 6n^2 + 1}, \\
\Delta\left(y_n + \frac{16n^3+1}{4n^3+131} y_{n-\tau_2}\right) + \frac{x_{n-4}^3 y_{n-3}^2}{n\ln^{10} n + 1} = \frac{n^{34} - 6n^{32} - 7n^{15} + (-1)^n n^7 - 1}{n^{36} + 6n^{30} + 5n^{12} + 6n^6 + n^3 + 1}, \quad n \geq 2,
\end{gathered} \tag{3.30}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$n_0 = 2, \qquad n_1 = 4, \qquad h = k = 1, \qquad M = 800N_0,$$

$$N = 500N_0, \qquad M_0 = 300N_0 > 0,$$

$$\alpha = \min\{2-\tau_1, 2-\tau_2, -2\}, \qquad a_{1n} = n-2, \qquad b_{1n} = n-1,$$

$$c_{1n} = n-4, \qquad d_{1n} = n-3,$$

$$p_{1n}=1,\qquad p_{2n}=\frac{16n^3+1}{4n^3+131},\qquad \overline{p}_0=4,\qquad \underline{p}_0=3,$$
$$q_{1n}=\frac{n^{38}-19n^{25}+30n^{18}-3n^9+2n-1}{n^{40}+8n^{30}+n^{15}+6n^2+1},$$
$$q_{2n}=\frac{n^{34}-6n^{32}-7n^{15}+(-1)^n n^7-1}{n^{36}+6n^{30}+5n^{12}+6n^6+n^3+1},\qquad f_1(n,u,v)=\frac{n^{25}-\left(n^{15}+1\right)v^2}{n^{30}+\left(n^{18}+1\right)u^2},$$
$$f_2(n,u,v)=\frac{u^3v^2}{n\ln^{10}n+1},\qquad r_{1n}=\frac{n^{25}+\left(n^{15}+1\right)M^2}{n^{30}+\left(n^{18}+1\right)N^2},\qquad r_{2n}=\frac{M_0^5}{n\ln^{10}n+1},$$
$$t_{1n}=\frac{4n^{33}M\left(n^{10}+n^{12}+4M^2\right)}{\left(n^{30}+\left(n^{18}+1\right)N^2\right)^2},\qquad t_{2n}=\frac{5M_0^4}{\left(n\ln^{10}n+1\right)^2},\qquad \forall(n,u,v)\in\mathbb{N}_{n_0}\times\mathbb{R}^2. \tag{3.31}$$

It is easy to verify that (2.1), (2.2) with $i=1$, (2.24), and (2.86)–(2.90) hold. Thus, Theorem 2.13 guarantees that System (3.30) possesses uncountably many bounded positive solutions in $A(N,M,N_0,M_0)$.

Example 3.13. Consider the nonlinear difference system with multiple delays:

$$\begin{aligned}\Delta\left(x_n+\frac{3(-1)^n n^{10}+2n+1}{9n^{10}+4n^8+1}x_{n-\tau_1}\right)+\frac{x_{n-3}-ny_{n-1}}{n^4+x_{n-3}^2}&=\frac{n^4-\sqrt{2n+5}\ln^5 n}{n^6+6n^2+1},\\ \Delta\left(y_n-\frac{(4+(-1)^n)n^4+5}{n^4+1}y_{n-\tau_2}\right)+\frac{n^3x_{n-2}y_{n-4}}{n^5+(n+1)y_{n-4}^2}&=\frac{n^7-(-1)^n n^4-1}{n^9+3n^6+5},\quad n\geq 2,\end{aligned} \tag{3.32}$$

where $\tau_1,\tau_2\in\mathbb{N}$ are fixed. Let

$$n_0=2,\qquad n_1=4,\qquad h=k=1,\qquad M>10N>0,\qquad \alpha=\min\{2-\tau_1,2-\tau_2,-2\},$$
$$a_{1n}=n-3,\qquad b_{1n}=n-1,\qquad c_{1n}=n-2,\qquad d_{1n}=n-4,\qquad p_{1n}=\frac{3(-1)^n n^{10}+2n+1}{9n^{10}+4n^8+1},$$
$$p_{2n}=-\frac{(4+(-1)^n)n^4+5}{n^4+1},\qquad \overline{p}_1=\underline{p}_1=\frac{1}{3},\qquad \overline{p}=3,\qquad \underline{p}=5,\qquad q_{1n}=\frac{n^4-\sqrt{2n+5}\ln^5 n}{n^6+6n^2+1},$$
$$q_{2n}=\frac{n^7-(-1)^n n^4-1}{n^9+3n^6+5},\qquad f_1(n,u,v)=\frac{u-nv}{n^4+u^2},\qquad f_2(n,u,v)=\frac{n^3uv}{n^5+(n+1)v^2},$$
$$r_{1n}=\frac{M(n+1)}{n^4+N^2},\qquad r_{2n}=\frac{n^3M^2}{n^5+(n+1)N^2},\qquad t_{1n}=\frac{n^4+M^2+n^5+3nM^2}{\left(n^4+N^2\right)^2},$$
$$t_{2n}=\frac{2n^3M\left(n^5+(n+1)M^2\right)}{\left(n^5+(n+1)N^2\right)^2},\qquad \forall(n,u,v)\in\mathbb{N}_{n_0}\times\mathbb{R}^2. \tag{3.33}$$

Obviously (2.1), (2.2), (2.24), (2.84), (2.91), and (2.92) hold. Thus, Theorem 2.14 means that System (3.32) possesses uncountably many bounded positive solutions in $A(N, M)$.

Example 3.14. Consider the nonlinear difference system with multiple delays:

$$\Delta\left(x_n + \frac{(-1)^n n + 1}{4n + 100} x_{n-\tau_1}\right) + \frac{x_{n-2} - n y_{n-1}}{n^3 + x_{n-2}^2 y_{n-1}^2} = \frac{n^5 \ln^9 n - (-1)^n}{n^8 + 5n^7 + 1},$$
$$\Delta\left(y_n + \frac{64n^8 + 1}{8n^8 + 3n^4 + 7} y_{n-\tau_2}\right) + \frac{n x_{n-5}^2 - y_{n-3} \sin n}{n^2 \ln^2 n + 1} = \frac{n^6 - (-1)^n n^2 + 1}{n^8 + 7n^7 + 2}, \quad n \geq 2, \tag{3.34}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$n_0 = n_1 = 2, \quad h = k = 1, \quad M = 40000, \quad N = 5000, \quad M_0 = 4480, \quad N_0 = 80,$$
$$\alpha = \min\{2 - \tau_1, 2 - \tau_2, -3\}, \quad a_{1n} = n - 2, \quad b_{1n} = n - 1, \quad c_{1n} = n - 5, \quad d_{1n} = n - 3,$$
$$p_{1n} = \frac{(-1)^n n + 1}{4n + 100}, \quad p_{2n} = \frac{64n^8 + 1}{8n^8 + 3n^4 + 7}, \quad \overline{p}_1 = \underline{p}_1 = \frac{1}{4}, \quad \overline{p}_0 = 8, \quad \underline{p}_0 = 7,$$
$$q_{1n} = \frac{n^5 \ln^9 n - (-1)^n}{n^8 + 5n^7 + 1}, \quad q_{2n} = \frac{n^6 - (-1)^n n^2 + 1}{n^8 + 7n^7 + 2}, \quad f_1(n, u, v) = \frac{u - nv}{n^3 + u^2 v^2},$$
$$f_2(n, u, v) = \frac{nu^2 - v \sin n}{n^2 \ln^2 n + 1}, \quad r_{1n} = \frac{M(n+1)}{n^3 + N^4}, \quad r_{2n} = \frac{M_0(M_0 + 1)}{n^2 \ln^2 n + 1},$$
$$t_{1n} = \frac{n^3 + 3M^4 + n^4 + 3nM^2}{\left(n^3 + N^4\right)^2}, \quad t_{2n} = \frac{2M_0 + 1}{\left(n^2 \ln^2 n + 1\right)^2}, \quad \forall (n, u, v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2. \tag{3.35}$$

It is easy to see that (2.1), (2.2) with $i = 1$, (2.24), (2.86)–(2.88), (2.91), and (2.93) hold. Thus, Theorem 2.15 implies that System (3.34) possesses uncountably many bounded positive solutions in $A(N, M, N_0, M_0)$.

Example 3.15. Consider the nonlinear difference system with multiple delays:

$$\Delta\left(x_n + \frac{-20n + 1}{4n + 9} x_{n-\tau_1}\right) + \frac{n x_{n-1}^3}{n^3 + y_{n-2}^2} = \frac{\sqrt{n^3 + 3n^2\sqrt{n-1} + 1}}{n^5 + 3n^4 + 1},$$
$$\Delta\left(y_n + \frac{6n^2 + 1}{2n^2 + 5n + 3} y_{n-\tau_2}\right) + \frac{x_{n-2} y_{n-4}}{n^2 + \cos(n^2 + 1)} = \frac{n^4 + 5n^2 - 7}{n^6 + 4n^3 + 1}, \quad n \geq 2, \tag{3.36}$$

where $\tau_1, \tau_2 \in \mathbb{N}$ are fixed. Let

$$
\begin{gathered}
n_0 = 2, \quad n_1 = 10, \quad h = k = 1, \quad M = 2N_0, \quad N = N_0 > 0, \quad M_0 = 30N_0, \\
\alpha = \min\{2-\tau_1, 2-\tau_2, -2\}, \quad a_{1n} = n-1, \quad b_{1n} = n-2, \quad c_{1n} = n-2, \quad d_{1n} = n-4, \\
p_{1n} = \frac{-20n+1}{4n+9}, \quad p_{2n} = \frac{6n^2+1}{2n^2+5n+3}, \quad \overline{p}_1 = 4, \quad \underline{p}_1 = 5, \quad \overline{p}_0 = 3, \quad \underline{p}_0 = 2, \\
q_{1n} = \frac{\sqrt{n^3+3n^2\sqrt{n-1}+1}}{n^5+3n^4+1}, \quad q_{2n} = \frac{n^4+5n^2-7}{n^6+4n^3+1}, \quad f_1(n,u,v) = \frac{nu^3}{n^3+v^2}, \\
f_2(n,u,v) = \frac{uv}{n^2+\cos(n^2+1)}, \quad r_{1n} = \frac{nM^3}{n^3+N^2}, \quad r_{2n} = \frac{M_0^2}{n^2-1}, \\
t_{1n} = \frac{nM^2(3n^3+5M^2)}{(n^3+N^2)^2}, \quad t_{2n} = \frac{2M_0}{(n^2-1)^2}, \quad \forall (n,u,v) \in \mathbb{N}_{n_0} \times \mathbb{R}^2.
\end{gathered}
\tag{3.37}
$$

Clearly (2.1), (2.2) with $i = 1$, (2.24), (2.86)–(2.88), (2.94), and (2.95) hold. Thus, Theorem 2.16 implies that System (3.36) possesses uncountably many bounded positive solutions in $A(N, M, N_0, M_0)$.

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