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Research Article

On a Cubic Equation and a Jensen-Quadratic Equation

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We obtain the general solutions of the cubic functional equation $3[g(x+y)+g(x-y)+6g(x)] = 2g(2x+y)+2g(2x-y)+g(-x-y)+g(-x+y)+6g(-x)$ and the Jensen-quadratic functional equation $f((x+y)/2, z+w)+f((x+y)/2, z-w) = f(x,z)+f(x,w)+f(y,z)+f(y,w)$.

1. Introduction

Throughout this paper, let X and Y be vector spaces.

A mapping $g : X \to Y$ is called a *Jensen* (resp., *quadratic*) mapping if g satisfies the functional equation $2g((x+y)/2) = g(x)+g(y)$ (resp., $g(x+y)+g(x-y) = 2g(x)+2g(y)$).

Definition 1.1. A mapping $f : X \times X \to Y$ is called *Jensen-quadratic* if f satisfies the system of equations

$$\begin{gathered} 2f\left(\frac{x+y}{2}, z\right) = f(x,z)+f(y,z), \\ f(x,y+z)+f(x,y-z) = 2f(x,y)+2f(x,z). \end{gathered} \tag{1.1}$$

When $X = Y = \mathbb{R}$, the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ given by $f(x,y) := axy^2 + by^2$ is a solution of (1.1). In particular, letting $x = y$, we get a cubic function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) := f(x,x) = ax^3 + bx^2$.

For a mapping $g: X \to Y$, consider the cubic functional equation

$$
\begin{aligned}
3&[g(x+y)+g(x-y)+6g(x)] \\
&= 2g(2x+y)+2g(2x-y)+g(-x-y)+g(-x+y)+6g(-x).
\end{aligned}
\tag{1.2}
$$

For a mapping $f: X \times X \to Y$, consider the functional equation

$$
f\left(\frac{x+y}{2}, z+w\right)+f\left(\frac{x+y}{2}, z-w\right) = f(x,z)+f(x,w)+f(y,z)+f(y,w). \tag{1.3}
$$

In this paper, we find out the general solutions of (1.2) and (1.3), and investigate the relation between them.

For more detailed definitions of the functional equation and the Hyers-Ulam stability, we refer the reader to [1–4].

2. Solution of the cubic functional equation (1.2)

Lemma 2.1. *A mapping $g: X \to Y$ satisfies*

$$
g(x+y)+g(x-y) = 2g(x)+2g(y) \tag{2.1}
$$

for all $x, y \in X$ if and only if

$$
g(2x+y)+g(2x-y) = g(x+y)+g(x-y)+6g(x) \tag{2.2}
$$

for all $x, y \in X$.

Proof. If g satisfies (2.1), then $g(0) = 0$. Letting $y = x$ in (2.1), we get $g(2x) = 4g(x)$ for all $x \in X$. Thus one can easily see that

$$
\begin{aligned}
g(2x+y)+g(2x-y) &= 2g(2x)+2g(y) \\
&= 8g(x)+2g(y) \\
&= 2g(x)+2g(y)+6g(x) \\
&= g(x+y)+g(x-y)+6g(x)
\end{aligned}
\tag{2.3}
$$

for all $x, y \in X$.

Conversely, if g satisfies (2.2), then $g(0) = 0$. Setting $y = 0$ and $y = x$ in (2.2), respectively, we gain

$$
g(2x) = 4g(x), \qquad g(3x) = 9g(x) \tag{2.4}
$$

for all $x \in X$. Letting $y = 2x$ in (2.2) and using (2.4), we know that g is even. Replacing y by $2y$ in (2.2), we get

$$
g(2x+2y)+g(2x-2y) = g(x+2y)+g(x-2y)+6g(x) \tag{2.5}
$$

for all $x, y \in X$. By (2.2) and the above equality, we have

$$\begin{aligned} g(2x+2y)+g(2x-2y) &= g(2y+x)+g(2y-x)+6g(x) \\ &= g(y+x)+g(y-x)+6g(y)+6g(x) \\ &= g(x+y)+g(x-y)+6g(x)+6g(y) \end{aligned} \tag{2.6}$$

for all $x,y \in X$. By (2.4), we obtain that g satisfies (2.1). □

Theorem 2.2. *A mapping $g : X \to Y$ satisfies (1.2) if and only if there exist a symmetric multiadditive mapping $M : X \times X \times X \to Y$ and a symmetric biadditive mapping $B : X \times X \to Y$ such that $g(x) = M(x,x,x) + B(x,x)$ for all $x \in X$.*

Proof. Assume that g satisfies (1.2). Let $g_e(x) := (g(x) + g(-x))/2$ and $g_o(x) := (g(x) - g(-x))/2$ for all $x \in X$. Then g_e and g_o also satisfy (1.2). Since g_e satisfies (1.2) and is even, g_e satisfies (2.2). By Lemma 2.1, g_e satisfies (2.1). By [5], there exists a symmetric biadditive mapping $B : X \times X \to Y$ such that $g_e(x) = B(x,x)$ for all $x \in X$, where the mapping B is given by

$$B(x,y) = \frac{1}{4}\left[g_e(x+y) - g_e(x-y)\right] \tag{2.7}$$

for all $x, y \in X$. Since g_o satisfies (1.2) and is odd,

$$g_o(2x+y) + g_o(2x-y) = 2g_o(x+y) + 2g_o(x-y) + 12g_o(x) \tag{2.8}$$

for all $x \in X$. By [6], there exists a symmetric multiadditive mapping $M : X \times X \times X \to Y$ such that $g_o(x) = M(x,x,x)$ for all $x \in X$, where the mapping B is given by

$$M(x,y,z) = \frac{1}{24}\left[g_o(x+y+z) - g_o(x+y-z) - g_o(x-y+z) + g_o(x-y-z)\right] \tag{2.9}$$

for all $x, y, z \in X$. Obviously, we obtain that

$$g(x) = g_o(x) + g_e(x) = M(x,x,x) + B(x,x) \tag{2.10}$$

for all $x \in X$.

Conversely, we may assume that there exist a symmetric multiadditive mapping $M : X \times X \times X \to Y$ and a symmetric biadditive mapping $B : X \times X \to Y$ such that $g(x) = M(x, x,x) + B(x,x)$ for all $x \in X$.

Then we obtain that

$$\begin{aligned}
&3[g(x+y)+g(x-y)+6g(x)]\\
&\quad= 3[M(x+y,x+y,x+y)+B(x+y,x+y)\\
&\qquad\quad +M(x-y,x-y,x-y)+B(x-y,x-y)+6M(x,x,x)+6B(x,x)]\\
&\quad= 24M(x,x,x)+18M(x,y,y)+24B(x,x)+6B(y,y)\\
&\quad= 2M(2x+y,2x+y,2x+y)+2B(2x+y,2x+y)\\
&\qquad +2M(2x-y,2x-y,2x-y)+2B(2x-y,2x-y)\\
&\qquad +M(-x-y,-x-y,-x-y)+B(-x-y,-x-y)\\
&\qquad +M(-x+y,-x+y,-x+y)+B(-x+y,-x+y)\\
&\qquad +6M(-x,-x,-x)+6B(-x,-x)\\
&\quad= 2g(2x+y)+2g(2x-y)+g(-x-y)+g(-x+y)+6g(-x)
\end{aligned} \tag{2.11}$$

for all $x, y \in X$. □

3. Solution of the Jensen-quadratic functional equation (1.3)

Lemma 3.1. *If a mapping $g: X \to Y$ satisfies (2.1) for all $x, y \in X$, then it also satisfies*

$$g(x+y+z)-g(x+y-z)=g(x+z)-g(x-z)+g(y+z)-g(y-z) \tag{3.1}$$

for all $x, y \in X$.

Proof. Replacing x and y by $x+y$ and z in (2.1), respectively, we gain

$$g(x+y+z)+g(x+y-z)=2g(x+y)+2g(z) \tag{3.2}$$

for all $x, y, z \in X$. Replacing y by $y-z$ in (2.1), we get

$$g(x+y-z)+g(x-y+z)=2g(x)+2g(y-z) \tag{3.3}$$

for all $x, y, z \in X$. Subtracting the latter from the former, we have

$$g(x+y+z)-g(x-y+z)=2g(x+y)+2g(z)-2g(x)-2g(y-z) \tag{3.4}$$

for all $x, y, z \in X$. Exchanging y and z in the above equation and using the fact that g is even, we obtain

$$\begin{aligned}
&g(x+y+z)-g(x+y-z)\\
&\quad= 2g(x+z)+2g(y)-2g(x)-2g(y-z)\\
&\quad= g(x+z)-g(x-z)+g(y+z)-g(y-z)\\
&\qquad +g(x+z)+g(x-z)+2g(y)-g(y+z)-g(y-z)-2g(x)\\
&\quad= g(x+z)-g(x-z)+g(y+z)-g(y-z)\\
&\qquad +2g(x)+2g(z)+2g(y)-2g(y)-2g(z)-2g(x)\\
&\quad= g(x+z)-g(x-z)+g(y+z)-g(y-z)
\end{aligned} \tag{3.5}$$

for all $x, y, z \in X$. □

THEOREM 3.2. *A mapping $f : X \times X \to Y$ satisfies (1.1) if and only if there exist a multiadditive mapping $M : X \times X \times X \to Y$ and a symmetric biadditive mapping $B : X \times X \to Y$ such that $f(x,y) = M(x,y,y) + B(y,y)$ and $M(x,y,z) = M(x,z,y)$ for all $x,y,z \in X$.*

Proof. We first assume that f is a solution of (1.1). Let $x \in X$ be arbitrarily fixed. Define $g_x : X \to Y$ by $g_x(y) = f(x,y)$ for all $y \in X$. Then g_x is a quadratic mapping. By [5], there exists a symmetric biadditive mapping $B_x : X \times X \to Y$ such that $g_x(y) = B_x(y,y)$, where the mapping B_x is given by

$$B_x(y,z) = \frac{1}{4}[g_x(y+z) - g_x(y-z)] \tag{3.6}$$

for all $y,z \in X$.

Define

$$M(x,y,z) := B_x(y,z) - B_0(y,z) \tag{3.7}$$

for all $x,y,z \in X$. Replacing x by $2x$ and letting $y = 0$ in the first equation in (1.1), we get

$$f(2x,z) = 2f(x,z) - f(0,z) \tag{3.8}$$

for all $x,z \in X$. By the first equation in (1.1) and the above equation, we know that

$$\begin{aligned} M(x+y,z,w) &\\ &= B_{x+y}(z,w) - B_0(z,w) \\ &= \frac{1}{4}[g_{x+y}(z+w) - g_{x+y}(z-w)] - \frac{1}{4}[g_0(z+w) - g_0(z-w)] \\ &= \frac{1}{4}[f(x+y,z+w) - f(x+y,z-w)] - \frac{1}{4}[f(0,z+w) - f(0,z-w)] \\ &= \frac{1}{8}[f(2x,z+w) + f(2y,z+w) - f(2x,z-w) - f(2y,z-w)] \\ &\quad - \frac{1}{4}[f(0,z+w) - f(0,z-w)] \\ &= \frac{1}{4}[f(x,z+w) - f(x,z-w)] + \frac{1}{4}[f(y,z+w) - f(y,z-w)] \\ &\quad - \frac{1}{2}[f(0,z+w) - f(0,z-w)] \\ &= \frac{1}{4}[g_x(z+w) - g_x(z-w)] + \frac{1}{4}[g_y(z+w) - g_y(z-w)] \\ &\quad - \frac{1}{2}[g_0(z+w) - g_0(z-w)] \\ &= B_x(z,w) + B_y(z,w) - 2B_0(z,w) \\ &= M(x,z,w) + M(y,z,w) \end{aligned} \tag{3.9}$$

for all $x, y, z, w \in X$. By the second equation in (1.1) and Lemma 3.1, we see that

$$\begin{aligned}
M(x, y+z, w) &= B_x(y+z, w) - B_0(y+z, w)\\
&= \frac{1}{4}[g_x(y+z+w) - g_x(y+z-w)] - \frac{1}{4}[g_0(y+z+w) - g_0(y+z-w)]\\
&= \frac{1}{4}[f(x, y+z+w) - f(x, y+z-w)] - \frac{1}{4}[f(0, y+z+w) - f(0, y+z-w)]\\
&= \frac{1}{4}[f(x, y+w) - f(x, y-w) + f(x, z+w) - f(x, z-w)]\\
&\quad - \frac{1}{4}[f(0, y+w) - f(0, y-w) + f(0, z+w) - f(0, z-w)]\\
&= \frac{1}{4}[g_x(y+w) - g_x(y-w)] + \frac{1}{4}[g_x(z+w) - g_x(z-w)]\\
&\quad - \frac{1}{4}[g_0(y+w) - g_0(y-w)] - \frac{1}{4}[g_0(z+w) - g_0(z-w)]\\
&= B_x(y, w) + B_x(z, w) - B_0(y, w) - B_0(z, w)\\
&= M(x, y, w) + M(x, z, w)
\end{aligned} \tag{3.10}$$

for all $x, y, z, w \in X$. Since g_x is quadratic for all $x \in X$, one can easily obtain that $M(x, y, z) = M(x, z, y)$ for all $x, y, z \in X$. Thus M is multiadditive.

Conversely, we assume that there exist a multiadditive mapping $M : X \times X \times X \to Y$ and a symmetric biadditive mapping $B : X \times X \to Y$ such that $f(x, y) = M(x, y, y) + B(y, y)$ and $M(x, y, z) = M(x, z, y)$ for all $x, y, z \in X$. Since M is additive in the first variable,

$$\begin{aligned}
2f\left(\frac{x+y}{2}, z\right) &= 2M\left(\frac{x+y}{2}, z, z\right) + 2B(z, z)\\
&= M(x, z, z) + M(y, z, z) + 2B(z, z)\\
&= f(x, z) + f(y, z)
\end{aligned} \tag{3.11}$$

for all $x, y, z \in X$. Since M is multiadditive and odd in each variable,

$$\begin{aligned}
f(x, y+z) + f(x, y-z) &= M(x, y+z, y+z) + M(x, y-z, y-z)\\
&\quad + B(y+z, y+z) + B(y-z, y-z)\\
&= 2M(x, y, y) + 2M(x, z, z) + 2B(y, y) + 2B(z, z)\\
&= 2f(x, y) + 2f(x, z)
\end{aligned} \tag{3.12}$$

for all $x, y, z \in X$. □

Theorem 3.3. *A mapping $f : X \times X \to Y$ satisfies (1.1) if and only if it satisfies (1.3).*

Proof. If f satisfies (1.1), then

$$\begin{aligned} f\left(\frac{x+y}{2}, z+w\right) &+ f\left(\frac{x+y}{2}, z-w\right) \\ &= \frac{1}{2}[f(x,z+w)+f(y,z+w)] + \frac{1}{2}[f(x,z-w)+f(y,z-w)] \\ &= \frac{1}{2}[f(x,z+w)+f(x,z-w)] + \frac{1}{2}[f(y,z+w)+f(y,z-w)] \\ &= f(x,z)+f(x,w)+f(y,z)+f(y,w) \end{aligned} \tag{3.13}$$

for all $x, y, z, w \in X$.

Conversely, assume that f satisfies (1.3). Choosing $x = y = z = w = 0$ in (1.3), we have $f(0,0) = 0$. Letting $y = x$ and $z = w = 0$ in (1.3), we get $f(x,0) = 0$ for all $x \in X$. Putting $w = 0$ in (1.3), we obtain

$$2f\left(\frac{x+y}{2}, z\right) = f(x,z)+f(y,z) \tag{3.14}$$

for all $x, y, z \in X$. Taking $y = x$ in (1.3) and replacing z by y and w by z, we have

$$f(x,y+z)+f(x,y-z) = 2f(x,y)+2f(x,z) \tag{3.15}$$

for all $x, y, z \in X$. □

4. The relation between (1.2) and (1.3)

Theorem 4.1. *Let $g : X \to Y$ be a mapping satisfying (1.2) and let $f : X \times X \to Y$ be the mapping given by*

$$f(x,y) := \frac{1}{6}[g(x+y)+g(x-y)-2g(x)+2g(y)+2g(-y)] \tag{4.1}$$

for all $x, y \in X$. Then f satisfies (1.3) and

$$g(x) = f(x,x) \tag{4.2}$$

for all $x \in X$.

Proof. Letting $x = y = 0$ in (1.2), $g(0) = 0$. Putting $y = 0$ in (1.2), we have

$$g(2x) = 6g(x)-2g(-x) \tag{4.3}$$

for all $x \in X$. Setting $y = x$ in (4.1), (4.2) holds by (4.3). By Theorem 2.2, there exist a symmetric multiadditive mapping $M : X \times X \times X \to Y$ and a symmetric biadditive mapping $B : X \times X \to Y$ such that

$$g(x) = M(x,x,x)+B(x,x) \tag{4.4}$$

for all $x \in X$. By (4.4), we obtain that

$$
\begin{aligned}
& g\left(\frac{x+y}{2}+z+w\right)+g\left(\frac{x+y}{2}-z-w\right)-2g\left(\frac{x+y}{2}\right)+2g(z+w)+2g(-z-w) \\
& \quad +g\left(\frac{x+y}{2}+z-w\right)+g\left(\frac{x+y}{2}-z+w\right)-2g\left(\frac{x+y}{2}\right)+2g(z-w)+2g(-z+w) \\
& = M\left(\frac{x+y}{2}+z+w,\frac{x+y}{2}+z+w,\frac{x+y}{2}+z+w\right)+B\left(\frac{x+y}{2}+z+w,\frac{x+y}{2}+z+w\right) \\
& \quad +M\left(\frac{x+y}{2}-z-w,\frac{x+y}{2}-z-w,\frac{x+y}{2}-z-w\right)+B\left(\frac{x+y}{2}-z-w,\frac{x+y}{2}-z-w\right) \\
& \quad -2M\left(\frac{x+y}{2},\frac{x+y}{2},\frac{x+y}{2}\right)-2B\left(\frac{x+y}{2},\frac{x+y}{2}\right) \\
& \quad +2M(z+w,z+w,z+w)+2B(z+w,z+w) \\
& \quad +2M(-z-w,-z-w,-z-w)+2B(-z-w,-z-w) \\
& \quad +M\left(\frac{x+y}{2}+z-w,\frac{x+y}{2}+z-w,\frac{x+y}{2}+z-w\right)+B\left(\frac{x+y}{2}+z-w,\frac{x+y}{2}+z-w\right) \\
& \quad +M\left(\frac{x+y}{2}-z+w,\frac{x+y}{2}-z+w,\frac{x+y}{2}-z+w\right)+B\left(\frac{x+y}{2}-z+w,\frac{x+y}{2}-z+w\right) \\
& \quad -2M\left(\frac{x+y}{2},\frac{x+y}{2},\frac{x+y}{2}\right)-2B\left(\frac{x+y}{2},\frac{x+y}{2}\right) \\
& \quad +2M(z-w,z-w,z-w)+2B(z-w,z-w) \\
& \quad +2M(-z+w,-z+w,-z+w)+2B(-z+w,-z+w) \\
& = 6\left[M(x,z,z)+M(x,w,w)+M(y,z,z)+M(y,w,w)+2B(z,z)+2B(w,w)\right] \\
& = M(x+z,x+z,x+z)+B(x+z,x+z)+M(x-z,x-z,x-z)+B(x-z,x-z) \\
& \quad -2M(x,x,x)-2B(x,x)+2M(z,z,z)+2B(z,z)+2M(-z,-z,-z)+2B(-z,-z) \\
& \quad +M(x+w,x+w,x+w)+B(x+w,x+w)+M(x-w,x-w,x-w)+B(x-w,x-w) \\
& \quad -2M(x,x,x)-2B(x,x)+2M(w,w,w)+2B(w,w)+2M(-w,-w,-w)+2B(-w,-w) \\
& \quad +M(y+z,y+z,y+z)+B(y+z,y+z)+M(y-z,y-z,y-z)+B(y-z,y-z) \\
& \quad -2M(y,y,y)-2B(y,y)+2M(z,z,z)+2B(z,z)+2M(-z,-z,-z)+2B(-z,-z) \\
& \quad +M(y+w,y+w,y+w)+B(y+w,y+w)+M(y-w,y-w,y-w)+B(y-w,y-w) \\
& \quad -2M(y,y,y)-2B(y,y)+2M(w,w,w)+2B(w,w)+2M(-w,-w,-w)+2B(-w,-w)
\end{aligned}
$$

$$
\begin{aligned}
&= g(x+z)+g(x-z)-2g(x)+2g(z)+2g(-z)\\
&\quad +g(x+w)+g(x-w)-2g(x)+2g(w)+2g(-w)\\
&\quad +g(y+z)+g(y-z)-2g(y)+2g(z)+2g(-z)\\
&\quad +g(y+w)+g(y-w)-2g(y)+2g(w)+2g(-w)
\end{aligned}
\tag{4.5}
$$

for all $x,y,z,w \in X$.

By (4.1) and the above equality, f satisfies (1.3). □

Theorem 4.2. *Let $f : X \times X \to Y$ be a mapping satisfying (1.3) and let $g : X \to Y$ be the mapping given by (4.2) for all $x, y \in X$. If f and g satisfy (4.1) for all $x, y \in X$, then g satisfies (1.2).*

Proof. Letting $x = y = z = w = 0$ in (1.3) and then using (4.2), we have $g(0) = 0$. Putting $y = x$ in (4.1) and then using (4.2), we obtain that g satisfies (4.3). By (1.3) and (4.1), we know that

$$
\begin{aligned}
&g\left(\frac{x+y}{2}+z+w\right)+g\left(\frac{x+y}{2}-z-w\right)-2g\left(\frac{x+y}{2}\right)+2g(z+w)+2g(-z-w)\\
&+g\left(\frac{x+y}{2}+z-w\right)+g\left(\frac{x+y}{2}-z+w\right)-2g\left(\frac{x+y}{2}\right)+2g(z-w)+2g(-z+w)\\
&\qquad = g(x+z)+g(x-z)-2g(x)+2g(z)+2g(-z)\\
&\qquad\quad +g(x+w)+g(x-w)-2g(x)+2g(w)+2g(-w)\\
&\qquad\quad +g(y+z)+g(y-z)-2g(y)+2g(z)+2g(-z)\\
&\qquad\quad +g(y+w)+g(y-w)-2g(y)+2g(w)+2g(-w)
\end{aligned}
\tag{4.6}
$$

for all $x,y,z,w \in X$. Replacing x, y, z, and w by 0, 0, x, and y, respectively, in (4.6), we obtain that

$$
g(x+y)+g(x-y)+g(-x+y)+g(-x-y) = 2\left[g(x)+g(-x)+g(y)+g(-y)\right] \tag{4.7}
$$

for all $x, y \in X$. Replacing x, y, z, and w by $2x$, 0, y, and 0, respectively, in (4.6), we see that, by (4.3),

$$
2\left[g(x+y)+g(x-y)+4g(x)\right] = g(2x+y)+g(2x-y)+g(y)+g(-y)+4g(-x) \tag{4.8}
$$

for all $x, y \in X$. From the above two equalities, we conclude that

$$
3\left[g(x+y)+g(x-y)+6g(x)\right] = 2g(2x+y)+2g(2x-y)+g(-x-y)+g(-x+y)+6g(-x) \tag{4.9}
$$

for all $x, y \in X$. □

References

[1] S. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific, River Edge, NJ, USA, 2002.
[2] D. H. Hyers, G. Isac, and Th. M. Rassias, *Stability of Functional Equations in Several Variables*, vol. 34 of *Progress in Nonlinear Differential Equations and Their Applications*, Birkhäuser, Boston, Mass, USA, 1998.
[3] S.-M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Palm Harbor, Fla, USA, 2001.
[4] Th. M. Rassias, *Functional Equations and Inequalities*, vol. 518 of *Mathematics and Its Applications*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2000.
[5] J. Aczél and J. Dhombres, *Functional Equations in Several Variables*, vol. 31 of *Encyclopedia of Mathematics and Its Applications*, Cambridge University Press, Cambridge, UK, 1989.
[6] K.-W. Jun and H.-M. Kim, "The generalized Hyers-Ulam-Rassias stability of a cubic functional equation," *Journal of Mathematical Analysis and Applications*, vol. 274, no. 2, pp. 867–878, 2002.

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