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Sharp estimates for maximal operators associated to the wave equation

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Abstract. The wave equation, $\partial_{tt}u=\Delta u$, in $\mathbb{R}^{n+1}$, considered with initial data $u(x,0)=f\in H^s(\mathbb{R}^n)$ and $u'(x,0)=0$, has a solution which we denote by $\frac{1}{2}(e^{it\sqrt{-\Delta}}f+e^{-it\sqrt{-\Delta}}f)$. We give almost sharp conditions under which $\sup_{0<t<1}|e^{\pm it\sqrt{-\Delta}}f|$ and $\sup_{t\in\mathbb{R}}|e^{\pm it\sqrt{-\Delta}}f|$ are bounded from $H^s(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)$.

1. Introduction

The Schrödinger equation, $i\partial_t u+\Delta u=0$, in $\mathbb{R}^{n+1}$, with initial datum f contained in a Sobolev space $H^s(\mathbb{R}^n)$, has solution $e^{it\Delta}f$ which can be formally written as

$$e^{it\Delta}f(x)=\int \hat{f}(\xi)e^{2\pi i(x\cdot\xi-2\pi t|\xi|^2)}\,d\xi. \tag{1}$$

The minimal regularity of f under which $e^{it\Delta}f$ converges almost everywhere to f, as t tends to zero, has been studied extensively. By standard arguments, the problem reduces to the minimal value of s for which

$$\Big\|\sup_{0<t<1}|e^{it\Delta}f|\Big\|_{L^q(\mathbb{B}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)} \tag{2}$$

holds, where $\mathbb{B}^n$ is the unit ball in $\mathbb{R}^n$.

In one spatial dimension, L. Carleson [4] (see also [9]) showed that (2) holds when $s\geq\frac{1}{4}$, and B. E. J. Dahlberg and C. E. Kenig [6] showed that this is sharp in the sense that it is not true when $s<\frac{1}{4}$. In two spatial dimensions, significant contributions have been made by J. Bourgain [1] and [2], A. Moyua, A. Vargas and L. Vega [11] and [12], and T. Tao and Vargas [22] and [23]. The best known result

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is due to S. Lee [10] who showed that (2) holds when $s>\frac{3}{8}$. In higher dimensions, P. Sjölin [16] and L. Vega [24] independently showed that (2) holds when $s>\frac{1}{2}$.

Replacing the unit ball $\mathbb{B}^n$ in (2) by the whole space $\mathbb{R}^n$, there has also been significant interest (see [3], [5], [13], [14], [17], [22] and [23]) in the global bounds

$$\Big\|\sup_{0<t<1}|e^{it\Delta}f|\Big\|_{L^q(\mathbb{R}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)}$$

and

$$\Big\|\sup_{t\in\mathbb{R}}|e^{it\Delta}f|\Big\|_{L^q(\mathbb{R}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)},$$

sometimes in connection with the well-posedness with certain initial value problems (see [8]). In one spatial dimension there are almost sharp bounds (see [7], [8], [15], [18] and [25]), but in higher dimensions the problem remains open.

The wave equation, $\partial_{tt}u=\Delta u$, in $\mathbb{R}^{n+1}$, considered with initial data $u(\,\cdot\,,0)=f$ and $u'(\,\cdot\,,0)=0$, has solution which can be formally written as

$$\frac{1}{2}\big(e^{it\sqrt{-\Delta}}f+e^{-it\sqrt{-\Delta}}f\big)=\int\hat{f}(\xi)e^{2\pi ix\cdot\xi}\cos 2\pi t|\xi|\,d\xi,$$

where

$$e^{\pm it\sqrt{-\Delta}}f(x)=\int\hat{f}(\xi)e^{2\pi i(x\cdot\xi\pm t|\xi|)}\,d\xi. \tag{3}$$

Mainly we will be concerned with the global bounds

$$\Big\|\sup_{0<t<1}\big|e^{\pm it\sqrt{-\Delta}}f\big|\Big\|_{L^q(\mathbb{R}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)} \tag{4}$$

and

$$\Big\|\sup_{t\in\mathbb{R}}\big|e^{\pm it\sqrt{-\Delta}}f\big|\Big\|_{L^q(\mathbb{R}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)}. \tag{5}$$

We note that (5) is simply a mixed norm Strichartz estimate.

Everything that will follow is true for the solution to the wave equation with initial derivative equal to zero, however, for notational convenience, we will write things in terms of the one-sided solutions $e^{\pm it\sqrt{-\Delta}}f$.

Let

$$s_{n,q}=\max\left\{n\left(\frac{1}{2}-\frac{1}{q}\right),\frac{n+1}{4}-\frac{n-1}{2q}\right\}\quad\text{and}\quad q_n=\frac{2(n+1)}{n-1}.$$

We will prove the following almost sharp theorems. The positive part of Theorem 1, when $q=2$, is due to M. Cowling [5].

Theorem 1. *If* $q\in[2,\infty]$ *and* $s>s_{n,q}$, *then* (4) *holds. If* $q<2$ *or* $s<s_{n,q}$, *then* (4) *does not hold.*

Theorem 2. *If* $q\in[q_n,\infty]$ *and* $s>n(1/2-1/q)$, *then* (5) *holds. If* $q<q_n$ *or* $s<n(1/2-1/q)$, *then* (5) *does not hold.*

We will also briefly consider the local bounds

$$\Big\|\sup_{0<t<1}\big|e^{\pm it\sqrt{-\Delta}}f\big|\Big\|_{L^q(\mathbb{B}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)}. \tag{6}$$

and

$$\Big\|\sup_{t\in\mathbb{R}}\big|e^{\pm it\sqrt{-\Delta}}f\big|\Big\|_{L^q(\mathbb{B}^n)}\le C_{n,q,s}\|f\|_{H^s(\mathbb{R}^n)}. \tag{7}$$

That (6) and (7) hold when $q\in[1,2]$ and $s>\frac{1}{2}$ is due to Vega [24] and [25], and that this is not true when $s\le\frac{1}{2}$ is due to B. G. Walther [26].

In the following theorem we prove that (6) does not hold when

$$s<\frac{n+1}{4}-\frac{n-1}{2q}$$

which is an improvement of the fact that (6) does not hold when $s<n/4-(n-1)/2q$, due to Sjölin [19].

Theorem 3. *If* $q\in[1,\infty]$ *and* $s>\max\{\frac{1}{2},s_{n,q}\}$, *then* (6) *and* (7) *hold. If* $s<\max\{\frac{1}{2},s_{n,q}\}$, *then* (6) *and* (7) *do not hold.*

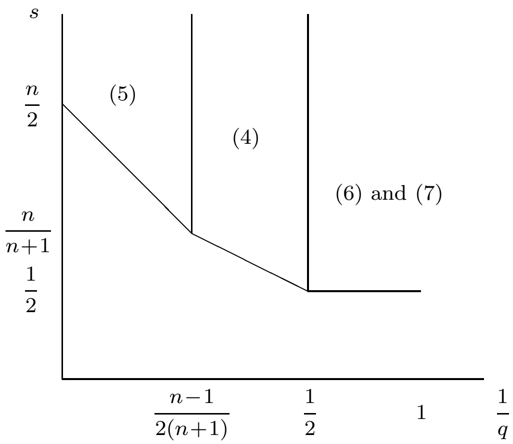

Figure 1. Region of boundedness for (4), (5), (6) and (7).

When $q=\infty$, there is a well-known example (see for example [20]), which shows that $s>n/2$ is necessary for (4), (5), (6) and (7) to hold. We also note that, by the counterexample of Walther [26], $s>\frac{1}{2}$ is necessary for (4) to hold when $q=2$. We will not discuss these endpoint cases further.

Throughout, C will denote an absolute constant whose value may change from line to line.

2. The positive results

As usual, we define ∂_t^α by $\widehat{\partial_t^\alpha g}(\tau)=(2\pi|\tau|)^\alpha \hat{g}(\tau)$, where $\alpha\geq 0$. By the following theorem and Sobolev imbedding, we see that (4) and (5) hold when $q\geq q_n$ and $s>n(1/2-1/q)$.

Theorem 4. *Let $q\in[q_n,\infty)$ and $s>n/2-(n+1)/q+\alpha$. Then there exists a constant $C_{n,q,\alpha,s}$ such that*

$$\big\|\partial_t^\alpha e^{\pm it\sqrt{-\Delta}} f\big\|_{L^q(\mathbb{R}^{n+1})}\leq C_{n,q,\alpha,s}\|f\|_{H^s(\mathbb{R}^n)}.$$

Proof. First we observe that $\partial_t^\alpha e^{\pm it\sqrt{-\Delta}} f=e^{\pm it\sqrt{-\Delta}} f_\alpha$, where we have $\hat{f}_\alpha(\xi)=(2\pi|\xi|)^\alpha \hat{f}(\xi)$. Thus, it will suffice to prove that

$$\big\|e^{\pm it\sqrt{-\Delta}} f_\alpha\big\|_{L^q(\mathbb{R}^{n+1})}\leq C_{n,q,\alpha,s}\|f_\alpha\|_{H^s(\mathbb{R}^n)},$$

where $q\geq q_n$ and $s>n/2-(n+1)/q$. By the standard Littlewood–Paley arguments, it will suffice to show that

$$\big\|e^{\pm it\sqrt{-\Delta}} g\big\|_{L^q(\mathbb{R}^{n+1})}\leq C_{n,q}N^{n/2-(n+1)/q}\|g\|_{L^2(\mathbb{R}^n)},$$

where $\operatorname{supp}\hat{g}\subset\{\xi:N/2\leq|\xi|\leq N\}$.

By scaling, this is equivalent to

$$\big\|e^{\pm it\sqrt{-\Delta}} g\big\|_{L^q(\mathbb{R}^{n+1})}\leq C_{n,q}\|g\|_{L^2(\mathbb{R}^n)},$$

where $\operatorname{supp}\hat{g}\subset\{\xi:\frac{1}{2}\leq|\xi|\leq 1\}$, which follows for all $q\geq q_n$ by the Strichartz inequality [21]. $\square$

It is tempting to try to increase the range of q above using bilinear restriction estimates on the cone as in [23]. Later we will see that this is not possible.

Corollary 1. *If $q\in[q_n,\infty)$ and $s>n(1/2-1/q)$, then* (4) *and* (5) *hold.*

The following theorem is a corollary of a more general result due to Cowling [5].

Theorem 5. *If* $q=2$ *and* $s>\frac{1}{2}$ *then* (4) *holds.*

Considering H^s to be a weighted L^2 space, we interpolate between Corollary 1 with $q=q_n$, and the previous theorem to get the following corollary.

Corollary 2. *If* $q\in[2,q_n]$ *and* $s>(n+1)/4-(n-1)/2q$, *then* (4) *holds.*

3. The negative results

Theorem 6. *If* (4) *holds, then* $q\in[2,\infty]$ *and* $s\geq s_{n,q}$. *If* (5) *holds, then* $q\in[q_n,\infty]$ *and* $s\geq n(1/2-1/q)$. *If* (6) *or* (7) *hold then* $s\geq\max\{\frac{1}{2},s_{n,q}\}$.

Proof. By a change of variables, it will suffice to consider $e^{-it\sqrt{-\Delta}}f$. First we obtain necessary conditions for $\sup_{t\in\mathbb{R}}\bigl|e^{-it\sqrt{-\Delta}}f\bigr|$, and then add the condition $t\in(0,1)$ to obtain necessary conditions for $\sup_{0<t<1}\bigl|e^{-it\sqrt{-\Delta}}f\bigr|$.

Let A be a set contained in the ball $B(0,N)$, where $N\gg1$, and define f_A by $f_A=\widehat{\chi}_A$. Recall that

$$\sup_{t\in\mathbb{R}}\bigl|e^{-it\sqrt{-\Delta}}f_A\bigr|=\sup_{t\in\mathbb{R}}\left|\int_A e^{2\pi i(x\cdot\xi-t|\xi|)}\,d\xi\right|.$$

The basic idea that we exploit, is to choose sets A and E for which a time $t(x)$ can be chosen, so that the phase $2\pi(x\cdot\xi-t(x)|\xi|)$ is almost zero for all $\xi\in A$ and $x\in E$. Then, as $\cos 2\pi(x\cdot\xi-t(x)|\xi|)\geq C$, we see that

$$\Bigl\|\sup_{t\in\mathbb{R}}\bigl|e^{-it\sqrt{-\Delta}}f_A\bigr|\Bigr\|_{L^q(\mathbb{R}^n)}\geq\left(\int_E(C|A|)^q\,dx\right)^{1/q}\geq C|A|\,|E|^{1/q}.$$

On the other hand,

$$\|f_A\|_{H^s(\mathbb{R}^n)}\leq\left(\int_A(1+|\xi|)^{2s}\,d\xi\right)^{1/2}\leq|A|^{1/2}(1+N)^s,$$

so that, as $\bigl\|\sup_{t\in\mathbb{R}}|e^{-it\sqrt{-\Delta}}f_A|\bigr\|_{L^q(\mathbb{R}^n)}\leq C\|f_A\|_{H^s(\mathbb{R}^n)}$, we have

$$|A|^{1/2}|E|^{1/q}\leq CN^s \tag{8}$$

for all $N\gg1$.

When $n=1$, we let $t(x)=x$, so that the phase is equal to zero for all $\xi\in[0,N]$ and $x\in\mathbb{R}$. Thus, substituting $|E|=|\mathbb{R}|$ in (8), we see that there can be no bound for $q<\infty$. When $q=\infty$, substituting $|A|=N$ into (8), we see that $s\geq\frac{1}{2}$, and we have the necessary conditions for (5). Substituting $|A|=N$ and $E=[0,1]$ into (8), we see that $s\geq\frac{1}{2}$, and we have the necessary conditions for (7).

Considering $\sup_{0<t<1}\big|e^{-it\sqrt{-\Delta}}f_A\big|$, we have the added constraint that we must choose $t(x)$ in the interval $(0,1)$. Choosing $t(x)=x$ again, and $E=(0,1)$, we see that $s\geq\frac{1}{2}$. We note that this is a necessary condition for (6) as well as (4). That (4) does not hold when $q<2$, follows from an example in [18].

When $n\geq 2$, define A by

$$A=\left\{\xi\in\mathbb{R}^n : |\theta_{\xi,e_n}|<\frac{N^{-\lambda}}{10} \text{ and } |\xi|<N\right\},$$

where $N\gg 1$, $\lambda\in[0,\infty)$ and θ_{ξ,e_n} denotes the angle between ξ and the standard basis vector e_n. Similarly we define E by

$$E=\{x\in\mathbb{R}^n : |\theta_{x,e_n}|<N^{-\lambda} \text{ and } |x|<N^{2\lambda-1}\},$$

and let $t(x)=|x|$. Given that

$$|\cos\theta_{\xi,x}-1|\leq\left(\frac{N^{-\lambda}}{5}\right)^2,$$

we have

$$\big|2\pi(x\cdot\xi-t(x)|\xi|)\big|=2\pi\big|\xi\|x\|\cos\theta_{\xi,x}-1\big|\leq 2\pi NN^{2\lambda-1}\left(\frac{N^{-\lambda}}{5}\right)^2\leq\frac{2\pi}{25},$$

so that the phase is always close to zero. Now as

$$|A|\geq C_nN(N^{1-\lambda})^{n-1} \quad \text{and} \quad |E|\geq C_nN^{2\lambda-1}(N^{\lambda-1})^{n-1},$$

we see from (8), that

$$N^s\geq C_nN^{(n-\lambda(n-1))/2}N^{((n+1)\lambda-n)/q}$$

for all $N\gg 1$, so that

$$s\geq n\left(\frac{1}{2}-\frac{1}{q}\right)-\lambda\left(\frac{n-1}{2}-\frac{n+1}{q}\right).$$

Letting $\lambda=0$, we see that $s\geq n(1/2-1/q)$. When $q<q_n$, we also have $(n-1)/2-(n+1)/q<0$, so that we can let $\lambda\to\infty$ to get a contradiction for all s. This completes the sufficient conditions for (5).

Considering $\sup_{0<t<1}\big|e^{-it\sqrt{-\Delta}}f_A\big|$, we have the added condition that $t(x)<1$. This is fulfilled if $\lambda\leq\frac{1}{2}$, so that $|x|<1$. Letting $\lambda=0$, we have $s\geq n(1/2-1/q)$ as before, and letting $\lambda=\frac{1}{2}$, we get $s\geq(n+1)/4-(n-1)/2q$. We note that these are also necessary conditions for the local bounds.

It remains to prove that $q\geq 2$ is necessary for the global boundedness of $\sup_{0<t<1}\left|e^{-it\sqrt{-\Delta}}f\right|$, and that $s\geq\frac{1}{2}$ is necessary for the local bounds. These will require separate constructions.

For the global bound, we consider A as defined before with E defined by

$$E=\left\{x\in\mathbb{R}^n : |\theta_{x,e}|\leq N^{-\lambda} \text{ for some } e\in\operatorname{span}\{e_1,...,e_{n-1}\}, \text{ and } |x|<\frac{N^{\lambda-1}}{10}\right\},$$

where $\lambda\in[0,\infty)$, and we let $t(x)=0$. Then

$$\begin{aligned}\left|2\pi(x\cdot\xi-t(x)|\xi|)\right| &= 2\pi\left|\xi\|x\|\cos\theta_{\xi,x}\right| \\ &= 2\pi\left|\xi\|x\|\sin(\pi/2-\theta_{\xi,x})\right|\leq 2\pi N\frac{N^{\lambda-1}}{10}2N^{-\lambda}\leq\frac{4\pi}{10},\end{aligned}$$

so that the phase is always close to zero. Now as

$$|A|\geq C_nN(N^{1-\lambda})^{n-1} \quad \text{and} \quad |E|\geq C_nN^{-1}(N^{\lambda-1})^{n-1},$$

we see from (8), that

$$N^s\geq C_nN^{(n-\lambda(n-1))/2}N^{(\lambda(n-1)-n)/q},$$

so that

$$s\geq n\left(\frac{1}{2}-\frac{1}{q}\right)-\lambda\left(\frac{n-1}{2}-\frac{n-1}{q}\right).$$

We see that when $q<2$, we can let $\lambda\to\infty$ to get a contradiction for all s.

Finally, for the local bounds, we define A and E by

$$\begin{aligned}A&=\left\{\xi\in\mathbb{R}^n : |\theta_{\xi,e_n}|<\frac{1}{N} \text{ and } |\xi|<N\right\},\\ E&=\left\{x\in\mathbb{R}^n : |\theta_{x,e_n}|<\frac{1}{100} \text{ and } |x|<1\right\},\end{aligned}$$

and let $t(x)=|x|\cos\theta_{x,e_n}$. Now using the inequality $|\cos x-\cos y|\leq|x^2-y^2|$ we have

$$\begin{aligned}\left|2\pi(x\cdot\xi-t(x)|\xi|)\right| &= 2\pi\left|\xi\|x\|\cos\theta_{x,\xi}-\cos\theta_{x,e_n}\right|\leq 2\pi N\left|\theta_{x,\xi}^2-\theta_{x,e_n}^2\right| \\ &= 2\pi N|\theta_{x,e_n}-\theta_{x,\xi}|\,|\theta_{x,e_n}+\theta_{x,\xi}|\leq 2\pi N\frac{1}{N}\left(\frac{1}{100}+\frac{1}{N}\right)\leq\frac{1}{3},\end{aligned}$$

so that the phase is always close to zero. Now as

$$|A|\geq C_nN \quad \text{and} \quad |E|\geq C_n,$$

we see from (8), that

$$N^s \geq C_n N^{1/2}$$

for all $N \gg 1$, so that $s \geq \frac{1}{2}$ and this completes the necessary conditions for local boundedness. $\square$

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