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GLOBAL BIFURCATION OF PERIODIC SOLUTIONS

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Dedicated to the memory of Karol Borsuk

0. Introduction

In this paper we study periodic solutions of a family of autonomous differential equations

$$u'(t) = \phi(u(t), a)$$

where $a \in \mathbb{R}$, $U \subset \mathbb{R}^m$ is an open subset and, the function $\phi : U \times \mathbb{R} \to \mathbb{R}^m$, is assumed to be of class C^1 and to satisfy some natural conditions.

This work was inspired by a paper by Mallet-Paret and Yorke [15]. Our main results are contained in Theorems 1.3, 1.5 and 1.8. The major difference between Mallet-Paret and Yorke's result and our Theorem 1.5 is that we discuss the general case, and not only some generic one. This is possible owing to purely topological methods of proof (see also Fiedler [7]for a complete review of previous works).

The principal tools used in the present paper are the S^1-equivariant degree (defined in [5]) and the complementary function method (introduced by Ize [10], [11]).

Let us briefly illustrate the geometric essence of the method, using the classical Brouwer degree. Assume that $\Omega \subset \mathbb{R}^{n+1}$ is an open bounded set and $f : \overline{\Omega} \to \mathbb{R}^n$ a continuous map. Further, assume U_+, U_- are disjoint open subsets of $\partial\Omega$ such that $f^{-1}(0) \cap \partial\Omega \subset U_+ \cup U_-$. We call $\theta : \overline{\Omega} \to \mathbb{R}$ a complementing function if $\theta(x) < 0$ for $x \in U_-$ and $\theta(x) > 0$ for $x \in U_+$. Setting

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