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EQUIVARIANT DEGREE FOR ABELIAN ACTIONS
PART I: EQUIVARIANT HOMOTOPY GROUPS

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Dedicated to the memory of Juliusz Schauder

0. Introduction

The classical degree theories and their Leray-Schauder extensions to infinite dimensions have been very useful in the study of nonlinear problems. The presence of symmetries in such problems, which restricts the class of maps and sets, gives a richer structure to the possible degrees. In our previous papers [6] and [7] we have defined a degree theory for such maps. In particular, we have studied and applied the degree for the case of a S^1-action.

Let E and F be two Banach spaces and Γ be a compact Lie group acting linearly, via isometries, on both of them. Let Ω be a bounded open invariant subset of E and f be an equivariant map defined on $\overline{\Omega}$ with values in F, that is, $f(\gamma x) = \widetilde{\gamma} f(x)$, for all x in $\overline{\Omega}$ and γ in $\Gamma, \widetilde{\gamma}$ representing the action on F.

If $f(x) \neq 0$ on $\partial\Omega$, then the Γ-degree is constructed as follows: take a large ball B centered at the origin and containing Ω and let $\widetilde{f} : B \to F$ be a Γ-equivariant continuous extension of f, with the usual compactness properties. Let N be a

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