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ELLIPTIC EQUATIONS WITH DISCONTINUOUS NONLINEARITIES[1]

WALTER ALLEGRETTO — PAOLO NISTRI

(Submitted by L. Górniewicz)

Dedicated to the memory of Juliusz Schauder

1. Introduction

This paper deals with the existence or nonexistence of positive solutions for nonlinear elliptic equations with the nonlinear term discontinuous in the unknown function u. The prototype problem is illustrated by the equation:

$$\ell u \coloneqq -\Delta u + \sum_{j=1}^{n} b_j(x) D_j u = \lambda f(x,u) \tag{1}$$

in a domain Ω of $\mathbb{R}^n$ with $n \geq 3$ and

$$f(x,u) = \begin{cases} g(x,u), & u > c, \\ 0, & u < c, \end{cases} \tag{2}$$

for some nonnegative smooth function g, monotone in u, and positive constant c. We always assume that $u = 0$ on $\partial\Omega$ (resp. u vanishes at ∞ for Ω unbounded). Observe that $f(x,c)$ is not specified and indeed it will be our purpose to obtain solutions

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