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ON THE LERAY-SCHAUDER ALTERNATIVE

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Dedicated to the memory of Juliusz Schauder

1. Introduction

In 1933, Jean Leray and Juliusz Schauder discovered [9] that the problem of solvability of an equation $x = Tx$, for a completely continuous operator T in a Banach space, reduces to finding a priori bounds on all possible solutions for the family of equations $x = \lambda Tx$, where $\lambda \in (0,1)$. Since then, this fact, known as the Leray-Schauder Alternative, and its various extensions and modifications, have played a basic role in various applications to nonlinear problems.

In this note, we elucidate and complement the above result. We introduce a class of nonlinear operators of the *Leray-Schauder type* and discuss its properties both in the fixed point and the coincidence setting. By elementary means and using only some known fixed point results, we show that many of the currently used nonlinear operators are of the Leray-Schauder type.

We begin with some notation and terminology. By *space* we shall understand a metric space and by a *map* a set-valued transformation.

Given a map $T : X \to Y$ between spaces, the sets Tx are the *values* of T and the set

$$\Gamma_T = \{(x,y) \in X \times Y : \; y \in Tx\}$$

is the *graph* of T. Two maps $S, T : X \to Y$ are said to have a *coincidence* provided $\Gamma_S \cap \Gamma_T \neq \emptyset$; if $T : A \to X$, where $A \subset X$, then x is a *fixed point* for T, provided $x \in Tx$.

By an *operator* we shall understand an upper semicontinuous map with non-empty compact values. An operator is said to be *compact* provided its range