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PERIODIC SOLUTIONS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH MULTIPLE STATE-DEPENDENT TIME LAGS

JOHN MALLET-PARET[1] — ROGER D. NUSSBAUM[2] — PANAGIOTIS PARASKEVOPOULOS[3]

Dedicated to Jean Leray

0. Introduction

Suppose that $h : \mathbb{R}^2 \to \mathbb{R}$ satisfies a "negative feedback condition": $h(\xi_0, \xi_1) < 0$ if $\xi_0 \geq 0$ and $\xi_1 > 0$ and $h(\xi_0, \xi_1) > 0$ if $\xi_0 \leq 0$ and $\xi_1 < 0$. Let $r : \mathbb{R} \to [0, M]$ be a locally Lipschitz map with $r(0) = \tau_0 > 0$. Consider the differential-delay equation

$$x'(t) = h(x(t), x(t - r(x(t)))). \tag{0.1}$$

Under further natural hypotheses it has been proved in [12] and [13] that equation (0.1) has a "slowly oscillating periodic solution" or "SOP solution" (see Definition 3.1 in Section 3 below).

More generally, suppose that $h : \mathbb{R}^{m+1} \to \mathbb{R}$ satisfies a negative feedback condition analogous to that above. For $1 \leq j \leq m$, let $r_j : \mathbb{R} \to [0, M]$ be a locally Lipschitz map. Assume (and this is crucial) that $r_j(0) = \tau_0$ for $1 \leq j \leq m$.

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