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HEAT CONTENT ASYMPTOTICS OF NON-MINIMAL OPERATORS

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Dedicated to Jean Leray

0. Introduction

Let M be a compact smooth Riemannian manifold of dimension m with smooth boundary ∂M. Let V be a smooth unitary vector bundle over M and let P be a second order partial differential operator on $C^\infty(V)$ with positive definite leading symbol. We impose suitable boundary conditions $\mathcal{B}$ for P and assume $P_\mathcal{B}$ is strongly elliptic and self-adjoint. Let $f \in C^\infty(V)$. To study the short time behavior of the fundamental solution to the heat equation $e^{-tP_\mathcal{B}}f$, we introduce an auxiliary smooth test function $\tilde{f} \in C^\infty(V)$ and define

$$\beta(f, \tilde{f}, P, \mathcal{B})(t) := \int_M (e^{-tP_\mathcal{B}}f, \tilde{f})\,dx. \tag{0.1}$$

Standard elliptic methods, see for example the discussion in [9, Lemma 1.3] show that as $t \downarrow 0^+$ there is an asymptotic series of the form

$$\beta(f, \tilde{f}, P, \mathcal{B})(t) \sim \sum_{n=0}^{\infty} \beta_n(f, \tilde{f}, P, \mathcal{B})t^{n/2}. \tag{0.2}$$

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