Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 3, 1994, 221–233

ON NIRENBERG'S PROBLEM AND RELATED TOPICS

YanYan Li[1]

Dedicated to Jean Leray

0. Introduction

Let $(\mathbf{S}^n, g_0)$ be the standard n-sphere. The following question was raised by Professor L. Nirenberg. Which function $K(x)$ on $\mathbf{S}^2$ is the Gauss curvature of a metric g on $\mathbf{S}^2$ conformally equivalent to g_0? Naturally, one may ask a similar question in higher dimensional cases, namely, which function $K(x)$ on $\mathbf{S}^n$ is the scalar curvature of a metric g on $\mathbf{S}^n$ conformally equivalent to g_0? In [20]–[22] we have given some existence results on the Nirenberg problem for $n \geq 4$ which are quite natural extensions of previous results of A. Chang & P. Yang ([7]) and A. Bahri & J. M. Coron ([2]) for $n = 2, 3$. A related critical exponent equation in $\mathbb{R}^n$ has also been studied and some existence results have been given in [17]–[22]. In this note we summarize the main results in [17]–[22] and outline the proofs. For $n = 2$, if we write $g = e^{2v} g_0$, the Nirenberg problem is equivalent to finding a function v on $\mathbf{S}^2$ which satisfies the equation

$$-\Delta_{g_0} v + 1 = K(x) e^{2v}, \tag{0.1}$$

where Δ_{g_0} denotes the Laplace-Beltrami operator associated with the metric g_0. For $n \geq 3$, if we write $g = v^{4/(n-2)} g_0$, the problem is equivalent to finding a

[1]Partially supported by the Alfred P. Sloan Foundation Research Fellowship and NSF grant DMS-9104293.