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# A COHOMOLOGY COMPLEX FOR MANIFOLDS WITH BOUNDARY

Kung ching Chang — Jiaquan Liu

*Dedicated to Professor Ky Fan*

Morse theory is an important part of critical point theory. A fashionable version of Morse theory, which implies Morse inequalities as consequences, describes a Morse function on an oriented compact differentiable manifold without boundary by a cohomology complex or a chain complex $\{C_k, \partial\}$. In J. Milnor [Mi], $C_k = \bigoplus \mathbb{Z}\langle x\rangle$, where $x$ is a critical point with Morse index $k$, and $\partial$ is the boundary operator, i.e., $\partial^2 = 0$, determined by the matrix of intersection numbers of oriented right hand spheres with left hand spheres having oriented normal bundles. And in E. Witten [Wi], $C_k$ is the linear space of the $k$-"harmonic" forms of a certain Laplacian related to the given function, and $\partial$ is a certain exterior differential operator. This version of Morse theory was generalized to infinite-dimensional manifolds by Floer in his study of symplectic geometry [Fl].

However, Morse inequalities for manifolds with boundary have been known to be useful in applications. The main purpose of this paper is to extend Witten's approach to that situation, i.e., we shall prove

Theorem. *Suppose that $f$ is a Morse function defined on an oriented compact manifold $M$ with boundary. Define*

$$d_t^p = tdf \wedge \cdot + d^p$$

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