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REMARKS ON TOPOLOGICAL SOLITONS

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Dedicated to Louis Nirenberg on the occasion of his 70th birthday

1. A generalization of the sine-Gordon equation

In this paper we deal with soliton solutions of Lorentz invariant equations. Roughly speaking, a soliton is a solution of a field equation whose energy travels as a localized packet and which preserves its form under perturbations. In this respect soliton solutions have a particle-like behaviour and they occur in many questions of mathematical physics, such as classical and quantum field theory, nonlinear optics, fluid mechanics and plasma physics (see [7], [8], [11], [13]).

In general the solitonic behaviour arises when one of the following circumstances occurs:

- existence of infinitely many first integrals of motion (e.g. KdV equation);
- existence of topological constraints which characterize the solutions.

In this paper we deal with the second case, namely we shall study topological solitons which are solutions of Lorentz invariant equations in more than one space dimensions. A classical interesting one-dimensional model is the sine-Gordon equation

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