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MULTIPLICITY OF FORCED OSCILLATIONS FOR THE SPHERICAL PENDULUM

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1. Introduction

In this paper we deal with the forced oscillations of a particle, of mass m, constrained to a two dimensional sphere S, with radius r, and acted on by the sum of three forces: a vector field depending only on the position, a possible friction and a T-periodic forcing term. More precisely, we are concerned with the following second order differential equation on S:

$$m\,\ddot{x} = -\frac{m\,|\dot{x}|^2}{r^2}\,x + h(x) - \eta\dot{x} + \lambda\varphi(t,x,\dot{x}), \quad \lambda \geq 0 \tag{1}$$

where $h : S \to \mathbb{R}^3$ is C^1 and tangent to S, $\eta \geq 0$ and $\varphi : \mathbb{R} \times TS \to \mathbb{R}^3$ is continuous, T-periodic in t, and such that $\varphi(t,q,v) \in T_qS$ for any $(t,q,v) \in \mathbb{R} \times TS$.

In the case when φ does not depend on $\dot{x}$, the problem of the existence of T-periodic solutions of (1), for any value of λ, has been positively solved in [2] and [3], and extended to the case of even dimensional spheres in [5]. In this paper, in order to get multiplicity results for forced oscillations of (1), we combine the methods used in [3] and [5] with a result of [7] about the set of harmonic solutions of periodically perturbed first order autonomous ODE's.

A physically relevant example is when h is the tangential component of the gravitational force. In this case we prove that the forced gravitational spherical

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