Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 10, 1997, 249–282

ASYMPTOTIC ANALYSIS OF THE NAVIER–STOKES EQUATIONS IN THIN DOMAINS

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Dedicated to O. A. Ladyzhenskaya

0. Introduction

We are interested in this article with the Navier–Stokes equations of viscous incompressible fluids in three dimensional thin domains. Let Ω_ε be the thin domain $\Omega_\varepsilon = \omega \times (0, \varepsilon)$, where ω is a suitable domain in $\mathbb{R}^2$ and $0 < \varepsilon < 1$.

Our aim is to derive an asymptotic expansion of the strong solution u^ε of the Navier–Stokes equations in the thin domain Ω_ε when ε is small, which is valid uniformly in time. This study should give a better understanding of the global existence results in thin domains obtained previously; see [15]–[17] and [23], [22]. We consider in this work two types of boundary conditions: the Dirichlet-periodic boundary condition and the purely periodic condition. For the first type of boundary condition we derive an asymptotic expansion of the solution u^ε in terms of the solution of the associated Stokes problem. More precisely, we prove that the solution can be written, for ε small, as

$$u^\varepsilon(t) = w^\varepsilon + \overline{u}^\varepsilon \exp\left(-\frac{\nu t}{2\varepsilon^2}\right), \quad \forall t > 0,$$

where w^ε is the solution of the associated Stokes problem and $\overline{u}^\varepsilon$ is a bounded (in time) function depending on the initial data. We also give a new proof and an improvement of the global existence result obtained in [23].

1991 *Mathematics Subject Classification.* 34C35, 35Q30, 76D05.
Key words and phrases. Navier–Stokes equations, global existence of strong solutions, asymptotic analysis.