Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 10, 1997, 47–78

A MASLOV-TYPE INDEX THEORY FOR SYMPLECTIC PATHS

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1. Introduction

In this paper, we extend the Maslov-type index theory defined in [7], [15], [10], and [18] to all continuous degenerate symplectic paths, give a topological characterization of this index theory for all continuous symplectic paths, and study its basic properties.

Suppose $\tau > 0$. We consider an τ-periodic symmetric continuous $2n \times 2n$ matrix function $B(t)$, i.e. $B \in C(S_\tau, \mathcal{L}_s(\mathbb{R}^{2n}))$ with $S_\tau = \mathbb{R}/(\tau\mathbb{Z})$, $\mathcal{L}(\mathbb{R}^{2n})$ being the set of all real $2n\times 2n$ matrices, and $\mathcal{L}_s(\mathbb{R}^{2n})$ being the subset of all symmetric matrices. It is well-known that the fundamental solution γ of the linear first order Hamiltonian system

$$\dot{y} = JB(t)y, \quad y \in \mathbb{R}^{2n}, \tag{1.1}$$

yields a path in the symplectic group $\mathrm{Sp}(2n) = \{M \in \mathcal{L}(\mathbb{R}^{2n}) \mid M^T JM = J\}$ starting from the identity matrix, where $J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}$, I_n is the identity matrix on $\mathbb{R}^n$. When there is no confusion we shall omit the subindex n of I_n. Define $\mathrm{Sp}(2n)^0 = \{M \in \mathrm{Sp}(2n) \mid \det(M - I) = 0\}$ and $\mathrm{Sp}(2n)^* = \mathrm{Sp}(2n)\backslash\mathrm{Sp}(2n)^0$. In order to study such problems, we introduce the following families of paths in

1991 *Mathematics Subject Classification.* 58F05, 58E05, 34C25.

Key words and phrases. Maslov-type index, paths, topology, symplectic group, Hamiltonian systems.

Partially supported by the NNSF and MCSEC of China and Qiu Shi Sci. Tech. Foundation.