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# MULTIPLE POSITIVE SOLUTIONS FOR SOME NONLINEAR ELLIPTIC SYSTEMS

Kazunaga Tanaka

## 0. Introduction

In this paper we study, via variational methods, the existence and multiplicity of positive solutions of the following systems of nonlinear elliptic equations:

$$k_1\Delta u + V_u(u,v) = 0 \qquad \text{in } \Omega, \tag{0.1}$$

$$k_2\Delta v + V_v(u,v) = 0 \qquad \text{in } \Omega, \tag{0.2}$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \qquad \text{on } \partial\Omega, \tag{0.3}$$

$$u(x) > 0, \quad v(x) > 0 \qquad \text{in } \Omega, \tag{0.4}$$

where $k_1$, $k_2 > 0$ are positive constants, $\Omega \subset \mathbb{R}^N$ is a bounded domain with a smooth boundary $\partial\Omega$ and $V(u,v) \in C^2(\mathbb{R}^2, \mathbb{R})$. We refer to [CdFM], [CM], [dFF], [dFM] and [HvV] for variational study of such elliptic systems. However, it seems that the multiplicity of positive solutions for such elliptic systems is not well studied.

Here, we study a case related to some models (with diffusion) in mathematical biology, ecology, etc., and we consider the case where (0.1)–(0.3) have 4 constant non-negative solutions $(0,0)$, $(a,0)$, $(0,b)$, $(u_0,v_0) \in \mathbb{R}^2$ ($a$, $b$, $u_0$, $v_0 > 0$), that is, solutions of $V_u(u,v) = V_v(u,v) = 0$, and 2 constant solutions $(a,0)$, $(0,b)$ are

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