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A PARABOLIC LITTLEWOOD-PALEY INEQUALITY WITH APPLICATIONS TO PARABOLIC EQUATIONS

N. V. Krylov

Dedicated to Jean Leray

0. Introduction

In [2] we have used a "parabolic" version of the Littlewood-Paley inequality for the heat equation. It turns out that convolutions with the heat kernel can be replaced by convolutions with more general functions. Here we present the corresponding result. We also give its extension to parabolic equations with coefficients depending only on t, extension based on one rather general principle, which might be of independent interest.

The need in the parabolic version of the Littlewood-Paley inequality can be seen from the following. In $\mathbb{R}^d$ consider the simplest stochastic Cauchy problem

$$du(t,x) = \frac{1}{2}\Delta u(t,x)\,dt + g(t,x)\,dw_t, \qquad t>0, \quad u(0,x)=0,$$

where w_t is a one-dimensional Wiener process. The solution of this problem is known to be

$$u(t,x) = \int_0^t T_{t-s}g(s,\cdot)(x)\,dw_t,$$

where

$$T_t h(x) = t^{-d/2}\phi(x/\sqrt{t}) * g(x), \qquad \phi(x) = \frac{1}{(2\pi)^{d/2}}e^{-\frac{1}{2}|x|^2} \tag{0.1}$$

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