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ON THE CONVERGENCE OF A TRIGONOMETRIC INTEGRAL

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In the present paper, we shall first establish a theorem concerning the convergence of a trigonometric integral. Then in the final section, we shall evaluate some known definite integrals with the help of our theorem.

1. DEFINITION. We say that the integral $\int_0^\infty a(u)du$ is summable $(C, 1)$ to sum S, if

$$\lim_{\lambda\to\infty}\int_0^\lambda\left(1-\frac{u}{\lambda}\right)a(u)du = S\,.$$

In [1], a result regarding the $(C, 1)$ summability of a trigonometric integral was proved which is equivalent to

THEOREM A. *Let $f(t)$ be L over $(0, \infty)$. Then, for $0 < \alpha < 1$, the integral*

$$\int_0^\infty u^\alpha du\int_0^\infty f(t)\sin utdt$$

is summable $(C, 1)$ to

$$\Gamma(\alpha+1)\,\mathrm{Cos}\,\frac{1}{2}\alpha\pi\int_{\to 0}^\infty \frac{f(t)}{t^{1+\alpha}}dt$$

whenever this integral exists and whenever

$$f(t) = 0(t^\alpha) \quad as \quad t \to 0\,.$$

In § 2 of the present paper we establish the following theorem.

THEOREM. *Let $t^{-\alpha}f(t)(0 < \alpha < 1)$ be of bounded variation over $(0, \infty)$ and tend to zero both as $t \to 0$ and $t \to \infty$. If the integral $\int_0^{\to\infty} f(t)\sin utdt$ is uniformly convergent with respect to u over $0 < \mu \leqq u \leqq \lambda < \infty$, for every μ and λ, then*

$$\int_{\to 0}^{\to\infty} u^\alpha du\int_0^{\to\infty} f(t)\sin utdt = \Gamma(\alpha+1)\cos\frac{1}{2}\alpha\pi\int_{\to 0}^{\to\infty}\frac{f(t)}{t^{1+\alpha}}dt \tag{1.1}$$

whenever the last integral exists.

In the present problem $f(t)$ is not necessarily L over $(0, \infty)$. In