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ON HOPFIAN GROUPS

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A group G is said to be hopfian if every surjective endomorphism of G is an automorphism of G. Some authors have investigated the problem of forming new hopfian groups in some familiar way from given hopfian groups. This investigation is continued in this paper. Section 1 contains a statement of our main results and a discussion of these results. Sections 2 and 3 contain the proof of these results. We list our main results below.

Let the group G be a semi-direct product of its subgroups H and F; that is, $H \triangle G, G = HF$ and $H \cap F = 1$. If H is a hopfian abelian group, we show that G is hopfian if either of the following holds:

(a) F obeys the maximal condition for subgroups and H does not have an infinite cyclic direct factor.

(b) F is a free abelian group of finite rank.

Let H be a hopfian abelian normal subgroup of G (which is not necessarily a split extension of H). Suppose G/H satisfies the maximal condition for subgroups. Then G is hopfian if any of the following holds:

(c) H is a torsion group.

(d) H is of finite rank and has a hopfian torsion group.

(e) $G = H \cdot F$ where F is a finite group.

Let A be a hopfian group and let $A \times B$ be the direct product of A and B. We will show $A \times B$ is hopfian if either of the following holds:

(f) B is a finite solvable group with exactly n proper normal subgroups which form a chain.

(g) B is a finite group of cube free order.

(h) B is a finite group of order p^3, p a prime.

Finally we give some conditions on $Z(A)$, the center of A, which will guarantee the hopficity of $A \times B$ if B is an infinite cyclic group.

In some respects, the property of hopficity is strange. For example, Gilbert Baumslag and Donald Solitar have constructed a nonhopfian group defined by two generators and a single defining relation and a two generator hopfian group with a normal subgroup of finite index which is not hopfian [2]. A.L.S. Corner has shown that if A is an abelian hopfian group, $A \times A$ need not be hopfian [3].

In seeking to construct new hopfian groups from given hopfian groups one naturally looks at familiar group constructions. For example, I. Dey has shown that under certain conditions the free product of hopfian groups is hopfian [4]. In considering the direct product