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ABSOLUTE BOREL AND SOUSLIN SETS

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The theory of analytic, Borelian and absolutely Baire spaces is applied to the theory of absolutely Souslin and Borel Sets with respect to the class of all metrizable spaces. Several intrinsic characterizations of absolutely Souslin and Borel sets are given.

The basic idea is that if there is given a class $\mathscr{A}$ of respectable sets in a separable theory, then the corresponding class $\mathscr{C}$ in the associated nonseparable theory consists of all spaces P such that $P = A \cap G$ in βP where $A \in \mathscr{A}$ and G is a G_δ set in βP. If the elements of $\mathscr{A}$ are characterized by existence of a complete structure of certain type, then the elements of $\mathscr{C}$ are characterized by the existence of a complete bi-structure $\langle \alpha, \beta \rangle$ where α is closely related to the structure defining $\mathscr{A}$, and β is closely related to the structure characterizing absolute G_δ spaces. This approach to the nonseparable theory is discussed for analytic, Borelian and bi-analytic spaces. The theory is applied to absolutely Borel and Souslin sets in the class of all metrizable spaces.

Call a space P an absolute Borel space (an absolute Souslin space) if P is metrizable, and a Borel (Souslin) set in every metrizable $Q \supset P$. By a Souslin set in a space Q we mean a Souslin set derived from the closed sets of Q; a full definition is given below. Since every closed set in a metrizable space is a G_δ, a set P is Borel (Souslin) in $Q \supset P$ if and only if P is Borel (Souslin) in the closure of P in Q. A set X in a metrizable space Q is a Borel set (Souslin set) if (and only if) X is Borel (Souslin) in the closure of X in Q, or in any Borel set $Y \supset X$, $Y \subset Q$.

There is a rather extensive and deep theory of separable absolute Borel and Souslin sets. The basic facts from the separable theory were generalized to more general spaces by several authors, and a closed separable theory in the class of all uniformizable spaces is developed in [2], [3], [4]; for a survey see [5][1]. On the other hand the nonseparable theory is still waiting for new ideas. The best information can be found in A. H. Stone's papers. The results in nonseparable theory strongly resemble the separable theory although more careful proofs are needed. The purpose of this paper is to describe internal characterizations of absolute Borel and Souslin sets

[1] For an up-to-date survey we refer to the author's *A survey of descriptive theory of sets and spaces*, to appear in the first issue of Czech. Math. J., 1970. All the results needed are discussed in this paper.