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REGULAR SEQUENCES AND LIFTING PROPERTY

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Let A be a commutative noetherian ring, E a finite A-module and let M be an arbitrary A-module. Let $\varphi: E \to M$ be a homomorphisn of A-modules.

In this note we prove in an elementary way that an M-sequence $\underset{\sim}{x} = (x_1, \cdots, x_n)$ being taken to lie in the (Jacobson-) radical $\operatorname{rad}(A)$ of A, is also an E-sequence if $\underset{\sim}{x}E$ is the contraction $\varphi^{-1}(\underset{\sim}{x}M)$ of $\underset{\sim}{x}M$ in E.

As a corollary of this lifting property we obtain very easily the so-called delocalization-lemma for regular sequences (also **[2]**, Cor. 1 for local rings A and **[4]** Chap. I, §4). Then we exemplify that the condition $\varphi^{-1}(\underset{\sim}{x}M) = \underset{\sim}{x}E$ is not necessary for the statement of our theorem (see Example 3); otherwise it is easily seen that generally the theorem (especially Corollary 2) becomes false without any additional condition (see Examples 1 and 2).

Recall that a sequence $x_1, \cdots, x_n$ of elements of A is said to be (M-regular or) an M-sequence if, for each $0 \leqq i \leqq n-1$, a_{i+1} is a non-zerodivisor on $M/(x_1, \cdots, x_i)M$ and $M \neq (x_1, \cdots, x_n)M$.

2. First we consider the case $n = 1$.

LEMMA. *The notations being as above. Let x be a M-regular element in the radical* $\operatorname{rad}(A)$ *of A and suppose that*

$$\ker \varphi \subseteq xE^1. \tag{1}$$

Then x is an E-regular element too and φ is injective.

Proof. We put $F = \ker \varphi$. Clearly x is E/F-regular, hence $xE \cap F = xF$, hence $F = xF$ by (1). Therefore we get $F = 0$ by Nakayama's lemma, hence φ is injective and x is E-regular.

THEOREM. *Let E be a finite A-module, M an arbitrary A-module and $\varphi: E \to M$ a module-homomorphism. Let $\underset{\sim}{x} = (x_1, \cdots, x_n)$ be an M-sequence in* $\operatorname{rad}(A)$ *and suppose that*

[1] We denote by xE or $\underset{\sim}{x}E$ the product $(x)E$ or $(\underset{\sim}{x})E$ respectively, where (x) or $(\underset{\sim}{x})$ is the ideal generated by x or $x_1, \cdots, x_n$ respectively.