A THEOREM ON THE REPRESENTATION THEORY OF JORDAN ALGEBRAS

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1. Introduction. Let J be a Jordan algebra over a field Φ of characteristic neither 2 nor 3. Let $a \longrightarrow S_a$ be a (general) representation of J. If α is an algebraic element of J, then S_α is an algebraic element. The object of this paper is to determine the polynomial identity* satisfied by S_α. The polynomial obtained depends only on the minimal polynomial of α and the characteristic of Φ. It is the minimal polynomial of S_α if the associative algebra U generated by the S_a is the universal associative algebra of J and if J is generated by α.

2. Preliminaries. A (nonassociative) commutative algebra J over a field Φ is called a *Jordan algebra* if

$$(a^2 b)a = a^2(ba) \tag{1}$$

holds for all $a, b \in J$. In this paper it will be assumed that the characteristic of Φ is neither 2 nor 3.

It is well known that the Jordan algebra J is *power associative*;** that is, the subalgebra generated by any single element a is associative. An immediate consequence is that if $f(x)$ is a polynomial with no constant term then $f(a)$ is uniquely defined.

Let R_a be the multiplicative mapping in J, $a \longrightarrow xa = ax$, determined by the element a. From (1) it can be shown that we have

$$[R_a R_{bc}] + [R_b R_{ac}] + [R_c R_{ab}] = 0$$

and

$$R_a R_b R_c + R_c R_b R_a + R_{(ac)b} = R_a R_{bc} + R_b R_{ac} + R_c R_{ab}$$

for all $a, b, c \in J$, where $[AB]$ denotes $AB - BA$. Since the characteristic of Φ is not 3, either of these relations and the commutative law imply (1). Let

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* This problem was proposed by N. Jacobson.

** See, for example, Albert [1].

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