REMARKS ON THE SPACE H^p

STANLEY S. WALTERS

1. Introduction. The space H^p is the collection of all single-valued complex functions f which are regular on the interior of the unit circle in the complex plane, and for which

$$\sup_{0 \leq r < 1} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta < \infty .$$

In [6] it was shown that H^p, $0 < p < 1$, is a linear topological space in which the metric is $\|f - g\|^p$, where we define

$$\|f\| = \sup_{0 \leq r < 1} \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta \right)^{1/p} .$$

It was moreover shown that $(H^p)^*$, the conjugate of H^p, has sufficiently many elements (linear functionals on H^p) so as to distinguish elements in H^p, in the sense that if $f \neq 0$ is in H^p, then there is a $\gamma \in (H^p)^*$ such that $\gamma(f) \neq 0$.

In the present paper it will be shown that *if γ is in $(H^p)^*$, $0 < p < 1$, then there exists a unique function G which is regular in the open unit circle, continuous on the closed circle,*[1] *and such that*

$$\gamma(f) = \lim_{r=1} \frac{1}{2\pi} \int_0^{2\pi} f(\rho e^{i\theta}) G\left(\frac{r}{\rho} e^{-i\theta}\right) d\theta , \qquad r < \rho < 1 ,$$

for every f in H^p. It is further shown that the following is true of G:

Received November 29, 1950.

The author is indebted to Professor Angus E. Taylor for his valuable assistance in the preparation of this paper.

[1] The author wishes to acknowledge the fact that the referee suggested the plausibility that G is continuous on the closed unit circle.

Pacific J. Math. 1 (1951), 455-471.