SOME THEOREMS CONCERNING ABSOLUTE NEIGHBORHOOD RETRACTS

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1. **Introduction.** If X is a subset of a topological space X^*, and there exists a mapping (continuous function) $\rho: X^* \longrightarrow X$ such that $\rho(x) = x$ for $x \in X$, then X is called a *retract* of X^*, and ρ is called a *retraction.* If U is a neighborhood of X in X^* (that is, a set open in X^* and containing X), and there exists a retraction $\rho': U \longrightarrow X$, then X is called a *neighborhood retract* of X^*. If X is a separable metric space such that every homeomorphic image of X as a closed subset of a separable metric space M is a neighborhood retract of M, then X is called an *absolute neighborhood retract* or an *ANR*. It is with such spaces that we shall be principally concerned. They are of particular interest because of their usefulness in homotopy theory, and also because every locally-finite polyhedron is an *ANR* [5].

Section 2 is concerned with two theorems of point-set topological nature. For many other theorems along this line, see [3, §10].

In §3, we are concerned with spaces of mappings into *ANR*'s, and with homotopy problems involving *ANR*'s; in particular, we obtain certain restrictions on the cardinality of collections of homotopy classes of mappings into *ANR*'s. For closely related results, see [4], especially the corollary to Theorem 5.

2. **Theorems in point-set-topology.** The following theorem is a slight generalization of a standard result.

THEOREM 1. *If a subset X_0 of an ANR X is a neighborhood retract of X, then X_0 is an ANR.*

This generalization consists in not requiring that X_0 be closed. For the more restricted theorem and its proof (which in actuality establishes Theorem 1), see [3, §10.1].

An interesting corollary of Theorem 1 is that *an open subset of an ANR is an ANR.*

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