CONTRIBUTIONS TO THE THEORY OF DIVERGENT SERIES

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INTRODUCTION

We shall be concerned throughout with methods of summability consistent with the method of analytic continuation (P). We recall that two methods are called *consistent* if whenever they are simultaneously effective they yield identical generalized sums. This property is not a consequence of regularity.

The purpose of this paper is to examine in a general way the class of methods consistent with (P) and subject to the following agreements.

1. For greater generality, regularity of the methods will not be assumed.

2. The class shall be closed with respect to multiplication; that is, the result of successive application of two methods of the class shall represent a method of the class. To this end consistency is postulated, not only for the case in which the series is summed, but in a general way (see postulates below).

We are thus led to consider the class of matrix transformations defined by the following postulates.

(I) Any series $\sum u_n$, summable (P), is transformed into a series $\sum v_k$, summable (P)

(II) $\sum u_n\ (P) = \sum v_k\ (P)$.

The class defined by these conditions is the exact analogue of the class of regular matrices, which may be defined in a similar fashion if "summable (P)" and "(P)" in (I) and (II) are replaced by "convergent" and "convergence", respectively.

1. THE CLASS $\mathfrak{C}_{aa}$

1.1. Notations and Definitions. The partial sums of a series $\sum u_n$ will be denoted by U_n:

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