CAPACITY, VIRTUAL MASS, AND GENERALIZED SYMMETRIZATION

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1. **Introduction.** A body of revolution B can be symmetrized with respect to its axis of symmetry in a number of ways. One of these is the Schwarz symmetrization, which preserves the volume of B. Another is the Steiner symmetrization of the meridian section of B, which preserves the area of this section but in general decreases the volume. The influence of the Schwarz symmetrization on the capacity has been investigated by G. Pólya and G. Szegő, [1]. More recently P. R. Garabedian and D. C. Spencer [2] discussed the same question for the virtual mass of bodies of revolution. In the present paper we shall study by a different and simpler method the behavior of the capacity and virtual mass under a more general type of symmetrization, which includes the Schwarz and Steiner symmetrizations as particular cases.

2. **Definitions.** Let the (x, y)-plane be the meridian plane of B, the x-axis being the axis of symmetry. The part of the meridian section of B which lies in the upper half plane $y \geq 0$ is denoted by D. The complement of D in the half plane is designated as E. We assume that D is simply connected and that E is a connected domain. The boundary of D consists in general of a segment of the x-axis and a line L. We exclude the case where L is a closed curve and lies entirely above the x-axis, as is the case in which B is a torus. We assume L to have at most a finite number ot angular points.

We shall use in this paper some recent results of axially symmetric potential theory in n-dimensional space. This theory which is of mathematical interest in itself will be used here mainly as a tool to obtain results for bodies of revolution in three dimensions.

Let us henceforth consider our (x, y)-plane as the meridian plane of a body of revolution $B[n]$ in n-dimensions, $n = 3, 4, 5, \cdots$. We assume that $B[n]$ has the same meridian section D as our three-dimensional body $B = B[3]$. All quantities considered hereafter are defined in the meridian plane and therefore are functions of x and y only. Actually we shall never use $B[n]$ but only its meridian section.

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