FORCES ON THE BOUNDARY OF A DIELECTRIC

G. POWER

1. Introduction. It has been shown [1, ch. VII] that the component parallel to the axis of x of the resultant force on the matter inside any closed surface S_1 drawn in a medium of specific inductive capacity K is given by

$$X = -\iint_{S_1} (lP_{xx} + mP_{xy} + nP_{xz})\, dS,$$

where (l, m, n) are the direction-cosines of the normal to the surface,

$$P_{xx} = \frac{K}{8\pi} (\overline{X}^2 - \overline{Y}^2 - \overline{Z}^2),$$

$$P_{xy} = \frac{K}{4\pi} \overline{X}\,\overline{Y},$$

$$P_{xz} = \frac{K}{4\pi} \overline{X}\,\overline{Z},$$

and $\overline{X}$, $\overline{Y}$, $\overline{Z}$ are given in terms of the potential by $-\partial\phi/\partial x$, $-\partial\phi/\partial y$, $-\partial\phi/\partial z$, respectively, provided the effect of electrostriction is neglected.

If any other surface S_2 is taken, surrounding S_1, and if

$$\frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} + \frac{\partial P_{xz}}{\partial z} = 0$$

at all points between the surfaces, that is to say provided $\nabla^2\phi = 0$ at all such points, then, by Green's theorem,

$$X = -\iint_{S_2} (lP_{xx} + mP_{xy} + nP_{xz})\, dS,$$

and similarly for the other components of the resultant force on the matter inside S_1.

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