STRUCTURED THEOREMS FOR RELATIVELY COMPLEMENTED LATTICES

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Introduction. In a previous paper [3] a study was made of the projectivities between the points of a simple relatively complemented lattice of finite dimension. It was shown that for a given dimension there is an upper bound for the number of transposes required to establish the projectivities between the points. The examples given in which this upper bound is attained have a particularly simple structure — they are closely related to a direct union. We shall prove here some general structure theorems for relatively complemented lattices and then apply these to the case of maximal projectivities.

The notation will be that of [3]. The lattice L to which we refer is always relatively complemented.

1. Structure Theorems. Our arguments depend heavily upon the simplicity or indecomposability [2] of L, and it is convenient to have the following characterization of a direct union:

THEOREM 1.1. *If L has dimension n, and a, b are two elements of L, then $L \cong a/z \vee b/z$ if and only if*

(1) $\rho(a) + \rho(b) \leq n$, *and*

(2) $p \subseteq a$ *if and only if* $p \not\subseteq b$ *for all points* $p \in L$.

Proof. Certainly if $L \cong a/z \vee b/z$, conditions (1) and (2) will hold. Suppose (1) and (2) hold in L. We shall proceed by induction on n. The theorem is true when $n = 1, 2$. Suppose it is true for all lattices of dimension less than n, but $L \not\cong a/z \vee b/z$.

It is clear that

$$x = (a \cap x) \cup (b \cap x)$$

for all $x \in L$. Consider the mapping

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