SUBGROUPS OF FREE PRODUCTS

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1. Introduction. It was first shown by Kurosch [3] that a subgroup H of a free product

$$G = \prod_{\nu}^{*} A_{\nu}$$

is itself a free product

$$H = F * \prod_{j}^{*} x_j^{-1} U_j x_j$$

of a free group F and conjugates $x_j^{-1} U_j x_j$ of subgroups U_j of free factors A. The original proof of Kurosch involved constructing the free factors of H one at a time, and both the proof and the construction depended on transfinite induction. A later proof by Baer and Levi [1] was topological in nature. More recently a proof has been given by Kuhn [2].

The present paper gives a new proof of this theorem, which, apart from the use of well-ordering, is purely algebraic. It is shorter and simpler than the Kurosch proof. In terms of a semi-alphabetical ordering of G, a set K of elements generating H is found, and it is then shown from the properties of K that H is a free product of factors as stated above.

2. The theorem of Kurosch. The result is the following:

THEOREM OF KUROSCH. *A subgroup $H \neq 1$ of a free product*

$$G = \prod_{\nu}^{*} A_{\nu}$$

is itself a free product

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