CONGRUENT IMBEDDING IN F-METRIC SPACES

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1. Introduction. An F-metric space arises by associating with each pair x, y of elements ("points") of an abstract set S an element xy^2 (the "squared-distance") of a field F. It is required of the association merely that $xy^2 = yx^2$, $xx^2 = 0$, and if $x \neq y$ then $xz^2 \neq yz^2$ for at least one point z of S. In this note we establish some fundamental distance-geometric properties of the two F-metric spaces F_n, $F_n(a_1, \cdots, a_n)$ obtained by attaching to each two elements

$$x = (x_1, x_2, \cdots, x_n), \quad y = (y_1, y_2, \cdots, y_n)$$

of the set of ordered n-tuples of F the elements

$$xy^2 = \sum_{i=1}^{n} (x_i - y_i)^2 \quad \text{and} \quad xy^2 = \sum_{i=1}^{n} a_i (x_i - y_i)^2$$

Received April 24, 1953. This paper was left by the late Professor Abraham Wald as a manuscript (in German). It was translated and edited by Professor Leonard M. Blumenthal of the University of Missouri, whose friendship with Professor Wald began at Vienna during the period when the latter was making distinguished contributions to Distance Geometry.

Translator's note. This article was written while Professor Wald was at the University of Vienna, probably in 1934. He had previously proved similar metric characterization theorems for the space of all n-tuples of *complex* numbers with

$$xy^2 = \sum_{i=1}^{n} (x_i - y_i)^2$$

(*Ergebnisse eines mathematischen Kolloquiums* (Wien), Heft 5 (1933), pp. 32-42). It seems that the present paper was intended to follow one in that journal by Olga Taussky (Mrs. John Todd) in which the same problems were solved in the more abstract setting obtained upon replacing the complex number field by any field of characteristic zero *in which every element is a square.* (See footnote 1.) It was announced in Heft 6 of the *Ergebnisse* that Wald's paper (which complements Mrs. Todd's by treating the problems in *formally real fields*) would appear in Heft 7, but for some reason this intention was not carried out. Nor is it contained in Heft 8, the last number of the Vienna *Ergebnisse* that was published.

The remaining footnotes in this paper are comments by the translator.

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