COMMENTS ON THE PRECEDING PAPER BY HERZOG AND PIRANIAN

P. C. ROSENBLOOM

1. Our main purpose here is to extract and formulate explicitly the general principle underlying the construction of Herzog and Piranian. The results in this note are implicitly contained in the computations on pp. 535 and 537 of their paper, and the full credit belongs to them.

2. We use the notation $M(r,f) = \max |f(z)|$ on $|z| = r$.

THEOREM 1. *Let f_n be analytic in $|z| \leq 1$, let r_n be increasing, $0 < r_n \longrightarrow 1$ as $n \longrightarrow \infty$, let $a_n > 0$,*

$$A = \sum_{n=1}^{\infty} a_n < +\infty,$$

let $R(t) = \sum a_k$ over all k such that $r_k \geq t$, and let $g = \sum_{n=1}^{\infty} f_n$. If

(a) $M(r_n, f_{n+1}) \leq a_n$,

and

(b) $M(1, f_n') \leq a_n (1 - r_n)^{-1}$

for all n, then g is analytic in $|z| < 1$, and for $|z| \leq 1$, $r_{n-1} \leq r \leq r_n$, we have

$$(1) \qquad \left| g(rz) - \sum_{1}^{n-1} f_k(z) - f_n(rz) \right| \leq A(1-r)^{1/2} + R(1-(1-r)^{1/2}),$$

$$(2) \qquad |g(r_n z) - g(r_{n-1} z) - f_n(z)| \leq 2A(1 - r_{n-1})^{1/2} + 2R(1 - (1 - r_{n-1})^{1/2}) + R(r_n).$$

Proof. We have

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