A NOTE ON THE COMPUTATION OF ALDER'S POLYNOMIALS

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In two recent papers [2, 3] I deduced and used the general transformation

$$(1) \qquad 1+\sum_{s=1}^{\infty}(-1)^s k^{Ms} x^{\frac{1}{2}s\{(2M+1)s-1\}}(1-kx^{2s})\frac{(kx;s-1)}{(x;s)}$$
$$=\prod_{n=1}^{\infty}(1-kx^n)\sum_{t=0}^{\infty}\frac{k^t G_{M,t}(x)}{(x;t)}, \qquad (M=2,3,\cdots)$$

to prove certain generalized identities of the type

$$(2) \qquad \prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-s})(1-x^{(2M+1)n-(2M+1-s)})(1-x^{(2M+1)n})}{(1-x^n)}$$
$$=\sum_{t=0}^{\infty}\frac{A_s(x,t)G_{M,t}(x)}{(x;t)},$$

where $A_s(x,t)$ and $G_{M,t}(x)$ are polynomials. For $s=M$ and $s=1$ respectively in (2), we get Alder's generalizations of the well-known Rogers-Ramanujan identities

$$\prod_{n=1}^{\infty}\frac{(1-x^{5n-2})(1-x^{5n-3})(1-x^{5n})}{(1-x^n)}=\sum_{t=0}^{\infty}\frac{x^{t^2}}{(x;t)}$$

and

$$\prod_{n=1}^{\infty}\frac{(1-x^{5n-1})(1-x^{5n-4})(1-x^{5n})}{(1-x^n)}=\sum_{t=0}^{\infty}\frac{x^{t(t+1)}}{(x;t)}$$

in the form [1]

$$\prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-M})(1-x^{(2M+1)n-M-1})(1-x^{(2M+1)n})}{(1-x^n)}=\sum_{t=0}^{\infty}\frac{G_{M,t}(x)}{(x;t)}$$

and

$$\prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-1})(1-x^{(2M+1)n-2M})(1-x^{(2M+1)n})}{(1-x^n)}=\sum_{t=0}^{\infty}\frac{x^t G_{M,t}(x)}{(x;t)}.$$

For the Alder polynomials $G_{M,t}(x)$ in (1), I gave the general form

$$(3) \qquad G_{M,t}(x)=x^{t^2}\sum_{t_1=0}^{\left[\frac{M-2}{M-1}t\right]}\frac{(x^{t-2t_1+1};2t_1)x^{-2t_1(t-t_1)}}{(x;t_1)}\prod_{n=2}^{M-2}T_{n,M}$$

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