A SPACE OF MULTIPLIERS OF TYPE $L^p(-\infty, \infty)$

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1. Introduction. Let $V(G)$ denote the set of all functions having finite variation on G. Set $G = (-\infty, \infty) = \hat{G}$, and let $V_\infty(G)$ be the Banach space of all functions in $V(G)$ which vanish at infinity. If $f \in V_\infty(G)$, then there exists a bounded linear operator $(t_p f)$ on $L^p(\hat{G})$ such that

$$(\mathrm{i}_0) \qquad (\text{Fourier transform of } (t_p f)x) = (\text{Fourier transform of } x) \cdot f$$

for all x in $L^p(\hat{G})$. This will be shown in 7.2. In the terminology of Hille [3, p. 566], functions f having property (i_0) are called "factor functions for Fourier transforms of type (L_p, L_p)".

Suppose $1 < p < \infty$. When $f \in L^1(G) \cap V(G) \subset V_\infty(G)$, then $(t_p f)$ is a singular integral operator: for all x in $L^p(\hat{G})$ it is found that $(t_p f)x$ has the form

$$[(t_p f)x]_\lambda = \frac{1}{2\pi i}\int_{-\infty}^{\infty} x(\theta)\frac{F(\theta - \lambda)}{\theta - \lambda}d\theta \qquad (\lambda \in \hat{G}) ,$$

where the integral is taken in the Cauchy principal value sense.

In 6.2 will be defined a set $\blacktriangle(L^p(\hat{G}))$ which contains all factor functions for Fourier transforms of type (L_p, L_p); the set $\blacktriangle(L^p(\hat{G}))$ is a slight extension of what Mihlin [6] calls "multipliers of Fourier integrals". We will find a number N_p such that

(i) *if* $f \in V_\infty(G)$ *then* $f \in \blacktriangle(L^p(\hat{G}))$ *and* $\|(t_p f)\| \leqq N_p \cdot \|f\|_v$,

where $\|f\|_v$ denotes the total variation on G of the function f. Let F_* be the mapping $\{x \to x * F\}$, where $x * F$ is the convolution of the functions x and F;

$$[x * F]_\lambda = \int_{-\infty}^{\infty} x(\theta) \cdot F(\theta - \lambda)d\theta \qquad (\lambda \in \hat{G}).$$

Let (Yf) denote the Fourier transform of the function f:

(ii) *if* $f \in L^1(G) \cap V(G)$, *then the transformation* $(Yf)_*$ *is a densely defined bounded operator, and* $(t_p f)$ *is its continuous linear extension to the whole space* $L^p(\hat{G})$.

Let us for a moment call $G = \{0, \pm 1, \pm 2, \cdots\}$ and $\hat{G} = [0, 1]$. In

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