ON THE STABILITY OF BOUNDARY COMPONENTS

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I. PRESENTATION OF THE PROBLEM

1. Definitions.

1. A *boundary component* of a plane region $D \subset (|z| \leqq \infty)$ is a component of the boundary ∂D of D, i.e., a connected subset of ∂D which is not a proper subset of any connected subset of ∂D.

There is an alternate definition. Let $\{\Omega_n\}_{n=1}^{\infty}$ be a sequence of subregions of D such that

(i) $\Omega_1 \supset \Omega_2 \supset \cdots$,

(ii) the relative boundary $\partial\Omega_n \cap D$ consists of one closed analytic curve in D,

(iii) $\bigcap_{n=1}^{\infty} \Omega_n = \phi$. Two sequences $\{\Omega_n\}$ and $\{\Omega'_n\}$ are said to be equivalent if, for any n, there exists m such that $\Omega_m \subset \Omega'_n$ and $\Omega'_m \subset \Omega_n$. A *boundary component* of D is an equivalence class of $\{\Omega_n\}$.

These two definitions are equivalent in the following sense:

(i) Given a sequence $\{\Omega_n\}$, the set $\bigcap_{n=1}^{\infty} \bar{\Omega}_n$ is a component of ∂D and, for two sequences, these sets coincide if and only if the sequences are equivalent.

(ii) Given a component Γ of ∂D, there exists a sequence such that $\Gamma = \bigcap_{n=1}^{\infty} \bar{\Omega}_n$.

For a boundary component Γ, the sequence $\{\Omega_n\}$ such that $\Gamma = \bigcap_{n=1}^{\infty} \bar{\Omega}_n$ is called a *defining sequence of* Γ.

Let $w = f(z)$ be a topological mapping of D onto a plane region D'. Then we can immediately see from the second definition that f gives a one-to-one correspondence between the boundary components of D and D'. We shall speak of the *image of a boundary component* Γ *under* f in this sense and denote it by $f(\Gamma)$.

2. Let D^c denote the complement of D with respect to the extended plane $|z| \leqq \infty$. For a boundary component Γ, there exists a uniquely determined component of D^c whose boundary coincides with Γ. We call it the *component of* D^c *corresponding to* Γ and denote it by Γ^*.

If D does not contain the point $z = \infty$, the boundary component Γ

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