CHARACTERISTIC SUBGROUPS OF MONOMIAL GROUPS

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1. Introduction. Let U be a set, $o(U) = B = \aleph'_u$, $u \geqq 0$, where $o(U)$ means the number of elements of U. Let H be a fixed group. A monomial substitution y is a transformation that maps every x of U in a one-to-one fashion into an x of U multiplied on the left by an element h_x of H. Multiplication of substitutions means successive applications. The set of all monomial substitutions forms the monomial group Σ. Ore [5] has studied this group for finite U, and some of his results have been generalized to general U in [2], [3], and [4].

This paper determines the structure of the characteristic subgroups for the case when U is infinite in the cases where normal subgroups and automorphisms are known. The method used makes clear how corresponding theorems for the case where U is finite might be proved but does not list these results.

2. Definitions and preliminaries. Let d be the cardinal of the integers. Let B be an infinite cardinal; B^+, the successor of B; U, a set such that $o(U) = B$; and C such that $d \leqq C \leqq B^+$. Let H be a fixed group and e the identity of H. Denote by $\Sigma = \Sigma(H; B, d, C)$ the monomial group of U over H whose elements are of the form

$$y = \begin{pmatrix} \cdots, & x_\xi, & \cdots \\ \cdots, & h_\xi x_{i_\xi}, & \cdots \end{pmatrix} \tag{1}$$

where only a finite number of the h_ξ are not e and the number of x not mapped into themselves is less than C. Any element of Σ may be written in the form

$$y = \begin{pmatrix} \cdots, & x_\xi, & \cdots \\ \cdots, & h_\xi x_\xi, & \cdots \end{pmatrix} \begin{pmatrix} \cdots, & x_\xi, & \cdots \\ \cdots, & e x_{i_\xi}, & \cdots \end{pmatrix}$$

or $y = vs$ where v sends every x into itself and every h of s is e. Elements of the form of

$$v = \begin{pmatrix} \cdots, & x_\xi, & \cdots \\ \cdots, & h_\xi x_\xi, & \cdots \end{pmatrix} = [\cdots, h_\xi, \cdots]$$

are *multiplications* and all such elements form a normal subgroup, *the basis groups* $V(B, d) = V$ of Σ. The h_ξ of y are called the *factors* of y. Elements of the form of s are *permutations* and all such elements form a subgroup, the *permutation group*, $S(B, C) = S$ of $\Sigma(H; B, d, C)$. Cycles

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