# ON THE ZEROS OF SOLUTIONS OF SOME LINEAR COMPLEX DIFFERENTIAL EQUATIONS

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**Introduction.** In this paper Green's function methods are used to investigate the distribution on the real axis of zeros of solutions of the complex differential equations

$$(p(x)y')' + f(x)y = 0 \tag{1}$$

and

$$y''' + f(x)y = 0 \,. \tag{2}$$

In both cases the coefficient $f(x)$ is assumed to be complex-valued and continuous on a half-line $I: x_0 \leq x < \infty$, while $p(x)$ in equation (1) is assumed to belong to a special class of complex-valued functions to be defined in Section I.

Equation (1) or equation (2) is said to be *nonoscillatory* on a set $E$ if no nontrivial solution has an infinite number of zeros in $E$. In what follows a solution shall mean a nontrivial solution. Suppose in equation (1) $x$ is a complex variable and $p(x)$ and $f(x)$ are analytic in a simply-connected region $R$. Consider the well known Green's function $g(x, s)$ for the differential system

$$(p(x)y')' = 0, \quad y(a) = y(b) = 0 \,, \tag{3}$$

where $a$ and $b$ are distinct points of $R$[1]. If $a$ and $b$ are zeros of a solution of equation (1), then

$$1 \leq \int_a^b |g(x, s)| \, |f(x)| \, |dx| \,,$$

where the integral is taken along a path $C$ in $R$ and $s$ is an interior point of $C$. Starting with this inequality and imposing various bounds on $|f(x)|$, Z. Nehari [7] and P. R. Beesack [3] have obtained nonoscillation theorems for $y'' + f(x)y = 0$ in various regions of the complex plane where $f(x)$ is analytic. By the same methods the author [2] has extended some of these theorems and obtained similar results for equation (1). The methods used in this paper are essentially those employed in the sources mentioned above. However, by restricting the independent variable to the real axis the condition of analyticity is relaxed and

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[1] Sufficient conditions for the existence of $g(x, s)$ are given in [**2**, p 15].